

Soek jy 'n fantastiese tutor?

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basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE 12/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2023

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 23 pages./
*Hierdie nasienriglyne bestaan uit 23 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason)
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	Award a mark if statement AND reason are both correct
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/VRAAG 1

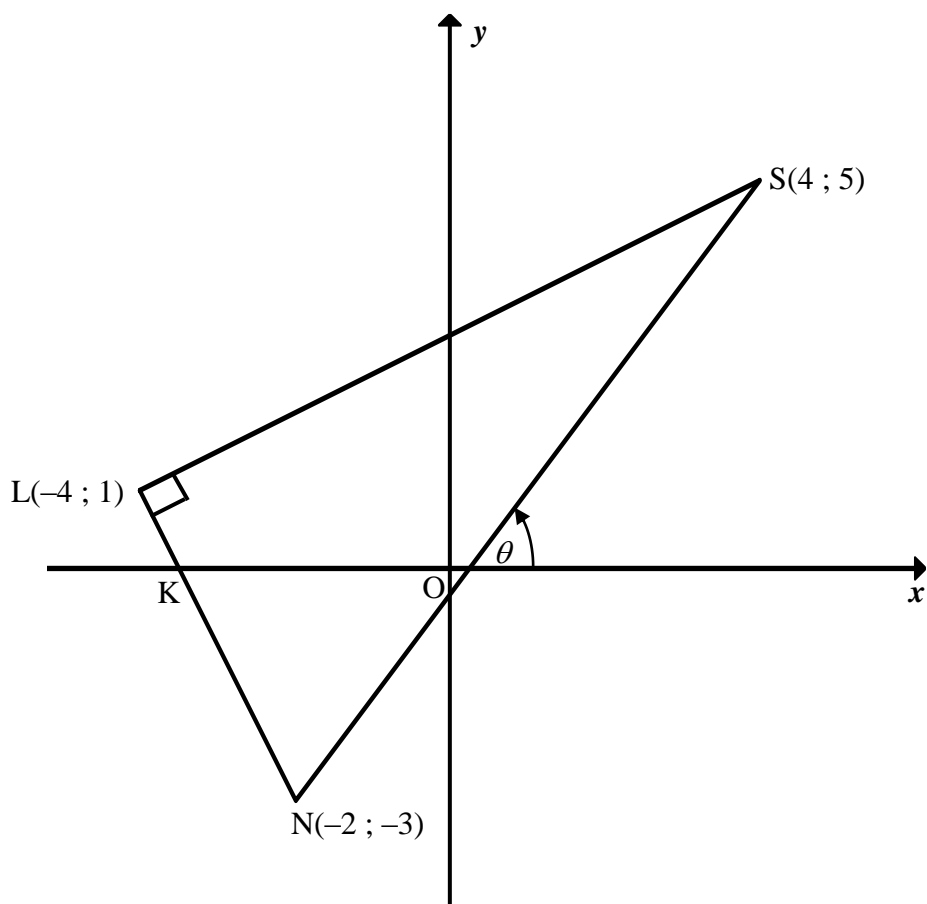
1.1	$a = -23,846\dots$ $b = 0,227\dots$ $\hat{y} = -23,85 + 0,23x$	✓ $a = -23,846\dots$ ✓ $b = 0,227\dots$ ✓ equation (3)
1.2	$\hat{y} = -23,85 + 0,23(550)$ $y = 102,65$ OR $y = 101,02$	✓ substitution of 550 ✓ answer (2) ✓✓ $y = 101,02$ (calculator) (2)
1.3	$r = 0,98$	✓ $r = 0,98$ (1)
1.4	Very strong positive correlation	✓ strong positive (1)

50	100	130	150	180	190	200	200
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1.5.1	$\bar{x} = \frac{1200}{8}$ $\bar{x} = 150$ OR $\bar{x} = 150$	✓ 1200 ✓ answer (2) ✓✓ $\bar{x} = 150$ (2)
1.5.2	$\sigma = 50,50$	✓ $\sigma = 50,50$ (1)
1.5.3	$\bar{x} - \sigma$ $= 150 - 50,50$ $= 99,50$ \therefore 1 stop	✓ calculation of $\bar{x} - \sigma$ ✓ answer (2)
		[12]

QUESTION/VRAAG 2

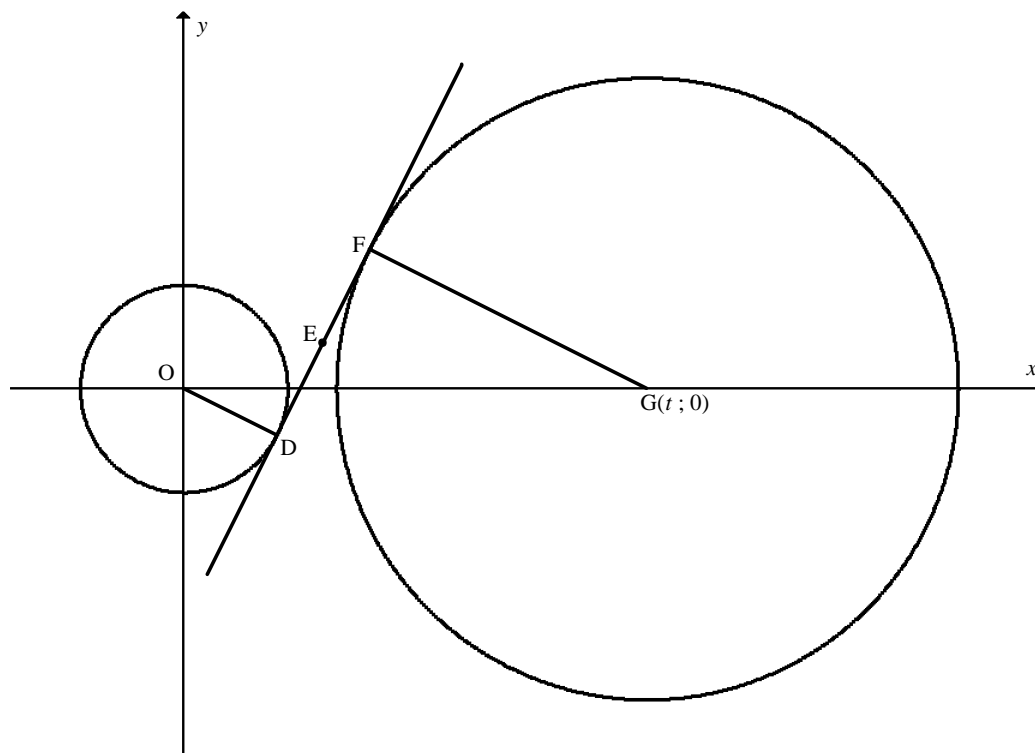
2.1	<table border="1"> <thead> <tr> <th>Number of glasses of water per day</th><th>Number of staff members</th><th>Cumulative frequency</th></tr> </thead> <tbody> <tr> <td>$0 \leq x < 2$</td><td>5</td><td>5</td></tr> <tr> <td>$2 \leq x < 4$</td><td>15</td><td>20</td></tr> <tr> <td>$4 \leq x < 6$</td><td>13</td><td>33</td></tr> <tr> <td>$6 \leq x < 8$</td><td>5</td><td>38</td></tr> <tr> <td>$8 \leq x < 10$</td><td>2</td><td>40</td></tr> </tbody> </table>	Number of glasses of water per day	Number of staff members	Cumulative frequency	$0 \leq x < 2$	5	5	$2 \leq x < 4$	15	20	$4 \leq x < 6$	13	33	$6 \leq x < 8$	5	38	$8 \leq x < 10$	2	40	<p>✓ 5; 20</p> <p>✓ 40</p> <p>(2)</p>
Number of glasses of water per day	Number of staff members	Cumulative frequency																		
$0 \leq x < 2$	5	5																		
$2 \leq x < 4$	15	20																		
$4 \leq x < 6$	13	33																		
$6 \leq x < 8$	5	38																		
$8 \leq x < 10$	2	40																		
2.2	40 staff members	<p>✓ answer</p> <p>(1)</p>																		
2.3	33 staff members	<p>✓ answer</p> <p>(1)</p>																		
2.4	$\bar{x} = \frac{\left(1 \times \left(5 + \frac{k}{2}\right)\right) + (3 \times 15) + \left(5 \times \left(13 + \frac{k}{2}\right)\right) + (7 \times 5) + (9 \times 2)}{40 + k} = 4$ $5 + \frac{k}{2} + 45 + 65 + \frac{5k}{2} + 35 + 18 = 160 + 4k$ $3k + 168 = 160 + 4k$ $k = 8$ <p>OR</p> $\bar{x} = \frac{(1 \times 5) + (15 \times 3) + (13 \times 5) + (5 \times 7) + (2 \times 9)}{40}$ $= 4,2$ $\bar{x}_{\text{old}} - \bar{x}_{\text{current}} = 4,2 - 4$ $= 0,2$ $\therefore 0,2 \times 40$ $= 8 \text{ teachers}$	<p>✓ answer from Q2.2 + k</p> <p>✓ $\left(1 \times \left(5 + \frac{k}{2}\right)\right)$</p> <p>✓ $\left(5 \times \left(13 + \frac{k}{2}\right)\right)$</p> <p>✓ answer</p> <p>(4)</p> <p>✓ 4,2</p> <p>✓ $\bar{x}_{\text{old}} - 4$</p> <p>✓ difference</p> <p>✓ answer</p> <p>(4)</p>																		
		[8]																		

QUESTION/VRAAG 3

3.1	$SL = \sqrt{(x_S - x_L)^2 + (y_S - y_L)^2}$ $SL = \sqrt{(4 - (-4))^2 + (5 - 1)^2}$ $SL = \sqrt{80} = 4\sqrt{5} = 8,94 \text{ units}$	✓ substitution of S and L into correct formula ✓ answer (2)
3.2	$m_{SN} = \frac{5 - (-3)}{4 - (-2)}$ $m_{SN} = \frac{4}{3}$	✓ substitution of S and N into correct formula ✓ answer (2)
3.3	$m = \tan \theta = \frac{4}{3}$ $\theta = 53,13^\circ$	✓ $\tan \theta = m_{SN}$ ✓ answer (2)
3.4	$m_{LN} = \frac{1 - (-3)}{-4 - (-2)}$ $m_{LN} = -2$ $\hat{LKO} = 116,565\dots^\circ$ $\hat{LNS} = 116,565\dots^\circ - 53,13^\circ$ $\hat{LNS} = 63,44^\circ$	✓ $m_{LN} = -2$ ✓ size of \hat{LKO} ✓ answer (3)

	<p>OR</p> <p>SN = 10 units</p> $\sin \hat{LNS} = \frac{4\sqrt{5}}{10}$ $\hat{LNS} = 63,44^\circ$ <p>OR</p> <p>LN = $2\sqrt{5}$ units</p> $\tan \hat{LNS} = \frac{4\sqrt{5}}{2\sqrt{5}}$ $\hat{LNS} = 63,44^\circ$ <p>OR</p> <p>SN = 10 units</p> <p>LN = $2\sqrt{5}$ units</p> $\cos \hat{LNS} = \frac{2\sqrt{5}}{10}$ $\hat{LNS} = 63,44^\circ$	<p>✓ SN = 10 units</p> <p>✓ correct trig ratio</p> <p>✓ answer</p> <p>(3)</p> <p>✓ LN = $2\sqrt{5}$ units</p> <p>✓ correct trig ratio</p> <p>✓ answer</p> <p>(3)</p> <p>✓ SN = 10 units and LN = $2\sqrt{5}$ units</p> <p>✓ correct trig ratio</p> <p>✓ answer</p> <p>(3)</p>
3.5	$m = \frac{4}{3}$ $1 = \frac{4}{3}(-4) + c$ $c = \frac{19}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$ <p>OR</p> $y - 1 = \frac{4}{3}(x - (-4))$ $y - 1 = \frac{4}{3}x + \frac{16}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$	<p>✓ m_{SN}</p> <p>✓ substitution of m_{SN} & L</p> <p>✓ equation</p> <p>(3)</p>
3.6	<p>SL = $4\sqrt{5}$</p> $LN = \sqrt{(-4 - (-2))^2 + (1 - (-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ <p>Area $\triangle LSN = \frac{1}{2}(4\sqrt{5})(2\sqrt{5})$</p> $= 20 \text{ units}^2$ <p>OR</p>	<p>✓ LN = $\sqrt{20} = 2\sqrt{5}$</p> <p>✓ substitution into formula</p> <p>✓ answer</p> <p>(3)</p>

	<p>SN = 10 units</p> $LN = \sqrt{(-4 - (-2))^2 + (1 - (-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ $\text{Area } \triangle LSN = \frac{1}{2}(10)(2\sqrt{5})\sin 63,44^\circ$ $= 20 \text{ units}^2$	<p>✓ $LN = \sqrt{20} = 2\sqrt{5}$</p> <p>✓ substitution into formula</p> <p>✓ answer</p> <p>(3)</p>
3.7	<p>$\hat{L} = 90^\circ$ SN is a diameter of circle S, L, N [chord subtends 90° OR converse \angle in semi-circle]</p> <p>Centre of circle = $P\left(\frac{4+(-2)}{2}; \frac{5+(-3)}{2}\right)$ $= P(1; 1)$</p> <p>OR Let the coordinates of P be $(a; b)$. Then, PL = PN: $(-4-a)^2 + (1-b)^2 = (-2-a)^2 + (-3-b)^2$ $a - 2b = -1$equation 1</p> <p>If PS = PN, then: $4a + 2b = 6$ equation 2 Solving simultaneously yields: $a = 1$ and $b = 1$ and $P(1; 1)$</p> <p>OR If PL = PN, then: $a - 2b = -1$equation 1 If PS = PL, then: $2a + b = 3$equation 2 Solving simultaneously yields: $a = 1$ and $b = 1$ and $P(1; 1)$</p>	<p>✓ SN is a diameter of circle S, L, N</p> <p>✓ x-value ✓ y-value</p> <p>(3)</p> <p>✓ 2 correct linear equations ✓ x-value ✓ y-value</p> <p>(3)</p> <p>✓ 2 correct linear equations ✓ x-value ✓ y-value</p> <p>(3)</p>
3.8	<p>$\hat{LPN} = \theta = 53,13^\circ$ [alt \angles; LP \parallel x-axis] $\therefore \hat{LPS} = 126,87^\circ$</p> <p>OR $\hat{LNS} = 63,44^\circ$ $\therefore \hat{LPS} = 126,88^\circ$ [\angle at centre = $2 \times \angle$ at circumference]</p> <p>OR $\hat{LSN} = 26,56^\circ$ [sum of \angles in Δ] $\hat{SLP} = 26,56^\circ$ [\angles opp equal radii] $\therefore \hat{LPS} = 126,88^\circ$ [sum of \angles in Δ]</p> <p>OR $(4\sqrt{5})^2 = 5^2 + 5^2 - 2(5)(5)\cos \hat{LPS}$ $\cos \hat{LPS} = -\frac{3}{5}$ $\therefore \hat{LPS} = 126,87^\circ$</p>	<p>✓ \hat{LPN}</p> <p>✓ answer</p> <p>(2)</p> <p>✓ \hat{LNS}</p> <p>✓ answer</p> <p>(2)</p> <p>✓ \hat{LSN}</p> <p>✓ answer</p> <p>(2)</p> <p>✓ correct substitution into cosine formula</p> <p>✓ answer</p> <p>(2)</p>
		[20]

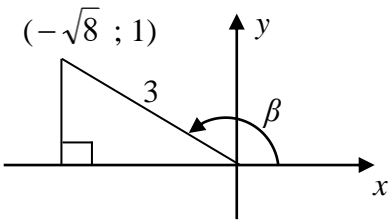
QUESTION/VRAAG 4

4.1	$D(p; -2)$ $x^2 + y^2 = 20$ $p^2 + (-2)^2 = 20$ $p^2 = 16$ $p = \pm 4$ $p = 4$	✓ substitution of point $D(p; -2)$ ✓ $p^2 = 16$ (2)
4.2	$\frac{4 + x_F}{2} = 6$ $x_F = 8$ $F(8; 6)$ OR $x_E - x_D = 6 - 4$ $= 2$ $x_F = 6 + 2 = 8$ $F(8; 6)$	$\frac{-2 + y_F}{2} = 2$ $y_F = 6$ ✓ method ✓ x-value ✓ y-value (3) ✓ method ✓ x-value ✓ y-value (3)

4.3	$m_{DE} = \frac{-2-2}{4-6}$ $m_{DE} = 2$ $-2 = 2(4) + c \quad \text{OR} \quad y - (-2) = 2(x - 4)$ $c = -10 \quad y + 2 = 2x - 8$ $y = 2x - 10 \quad y = 2x - 10$ <p>OR</p> $m_{OD} = -\frac{2}{4} = -\frac{1}{2}$ $\therefore m_{DE} = 2 \quad [\tan \perp \text{radius}]$ $-2 = 2(4) + c \quad \text{OR} \quad y - (-2) = 2(x - 4)$ $c = -10 \quad y + 2 = 2x - 8$ $y = 2x - 10 \quad y = 2x - 10$	✓ correct substitution ✓ gradient of DE, DF or EF ✓ substitution of point D(4 ; -2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4) ✓ correct gradient of OD ✓ gradient of DE ✓ substitution of point D(4 ; -2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
4.4	$m_{DE} = 2$ $\therefore m_{GF} = -\frac{1}{2} \quad [\tan \perp \text{radius}]$ $\frac{0-6}{t-8} = -\frac{1}{2}$ $-(t-8) = 2(-6)$ $t = 20$ <p>OR</p> $y = 2x - 10$ $0 = 2x - 10$ $x = 5$ $A(5 ; 0)$ <p>In $\triangle AFG$: $FA \perp FG$</p> $FA^2 = (6-0)^2 + (8-5)^2 = 45$ $FG^2 = (t-8)^2 + (0-6)^2$ $= t^2 - 16t + 100$ $GA^2 = (t-5)^2$ $= t^2 - 10t + 25$ $\therefore GA^2 = GF^2 + FA^2$ $t^2 - 10t + 25 = t^2 - 16t + 100 + 45$ $6t = 120$ $t = 20$	✓ correct gradient of GF ✓ substitution of F ✓ answer (3) ✓ x-intercept of DF ✓ substitution into Pythagoras ✓ answer (3)

4.5	$F(8;6)$ $G(20;0)$ $(8-20)^2 + (6-0)^2 = r^2$ $r^2 = 180$ $(x-20)^2 + y^2 = 180$ $x^2 + y^2 - 40x + 220 = 0$	✓ substitution of F and G ✓ value of r^2 ✓ equation of circle ✓ answer (4)
4.6	Smaller circle $r = 2\sqrt{5}$ Larger circle $r = 6\sqrt{5}$ $G(20;0)$ $k = 20 - (6\sqrt{5} - 2\sqrt{5})$ or $k = 20 + (6\sqrt{5} - 2\sqrt{5})$ $= 20 - 4\sqrt{5}$ $= 20 + 4\sqrt{5}$ $= 11,06$ units $= 28,94$ units OR Smaller circle $r = 2\sqrt{5}$ $k = 2(2\sqrt{5}) + 20 - 8\sqrt{5}$ or $k = 2(6\sqrt{5}) + 20 - 8\sqrt{5}$ $= 20 - 4\sqrt{5}$ $= 20 + 4\sqrt{5}$ $= 11,06$ units $= 28,94$ units OR $x^2 + y^2 - 40x + 220 = 0$ $y = 0$ $\therefore x^2 - 40x + 220 = 0$ $\therefore x = 20 + 6\sqrt{5}$ or $x = 20 - 6\sqrt{5}$ $\therefore k = 20 + 6\sqrt{5} - \sqrt{20}$ or $k = 20 - 6\sqrt{5} + \sqrt{20}$ $\therefore k = 20 + 4\sqrt{5}$ $\therefore k = 20 - 4\sqrt{5}$ $= 11,06$ units $= 28,94$ units	✓ $r = 2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ $r = 2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4) ✓ x-intercepts ✓ method ✓ answer ✓ answer (4)
		[20]

QUESTION/VRAAG 5

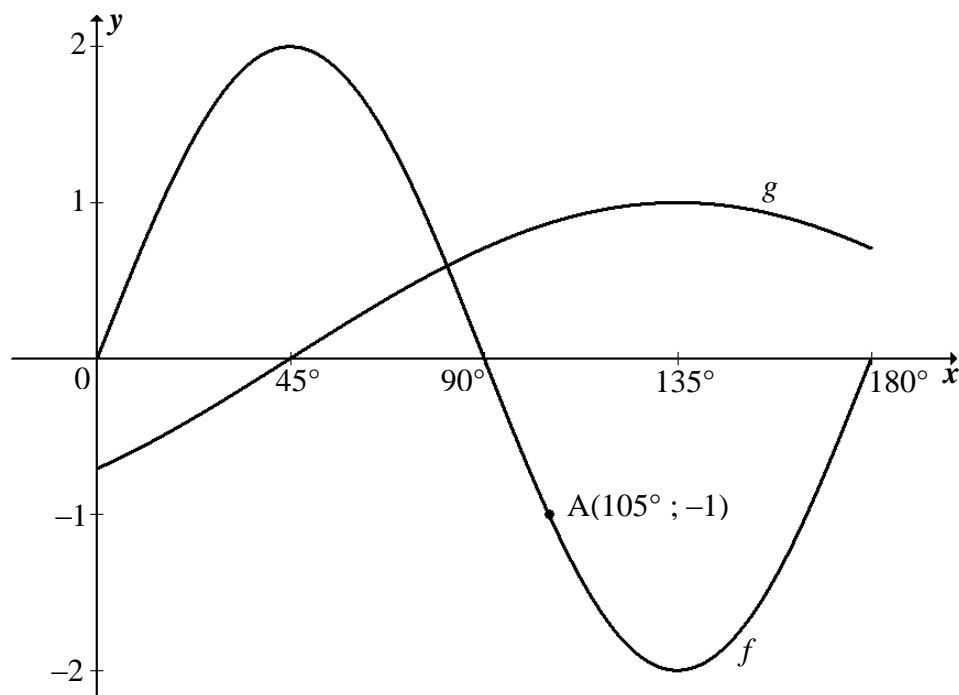
<p>5.1.1</p>	<p>$\sin \beta = \frac{1}{3} \quad \beta \in (90^\circ; 270^\circ)$</p>  <p>$x = -\sqrt{8} = -2\sqrt{2}$</p> <p>$\cos \beta$ $= \frac{-2\sqrt{2}}{3}$</p> <p>OR</p> <p>$\sin \beta = \frac{1}{3} \quad \beta \in (90^\circ; 270^\circ)$</p> <p>$\cos^2 \beta = 1 - \sin^2 \beta$</p> <p>$\cos^2 \beta = 1 - \left(\frac{1}{3}\right)^2$</p> <p>$\cos^2 \beta = \frac{8}{9}$</p> <p>$\cos \beta = \frac{-\sqrt{8}}{3}$ $= \frac{-2\sqrt{2}}{3}$</p>	<p>✓ $x^2 + y^2 = r^2$</p> <p>✓ $x = -2\sqrt{2}$</p> <p>✓ answer (3)</p> <p>✓ square identity</p> <p>✓ $\cos^2 \beta$</p> <p>✓ answer (3)</p>
<p>5.1.2</p>	<p>$\sin 2\beta$ $= 2 \sin \beta \cos \beta$ $= 2 \left(\frac{1}{3}\right) \left(\frac{-\sqrt{8}}{3}\right)$ $= \frac{-2\sqrt{8}}{9} \quad \text{OR} \quad 2 \left(\frac{-2\sqrt{2}}{9}\right)$ $= \frac{-4\sqrt{2}}{9}$</p>	<p>✓ double angle</p> <p>✓ substitution</p> <p>✓ answer (3)</p>
<p>5.1.3</p>	<p>$\cos (450^\circ - \beta)$ $= \cos (90^\circ - \beta)$ $= \sin \beta$ $= \frac{1}{3}$</p> <p>OR</p>	<p>✓ $\cos (90^\circ - \beta)$</p> <p>✓ co-ratio</p> <p>✓ answer (3)</p>

	$\cos(450^\circ - \beta)$ $= \cos 450^\circ \cos \beta + \sin 450^\circ \sin \beta$ $= \cos 90^\circ \cos \beta + \sin 90^\circ \sin \beta$ $= \sin \beta$ $= \frac{1}{3}$	✓ expansion ✓ reduction ✓ answer (3)
5.2.1	$\text{LHS} = \frac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$ $= \frac{\cos^2 x (\cos^2 x + \sin^2 x)}{1 + \sin x}$ $= \frac{1 - \sin^2 x}{1 + \sin x}$ $= \frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x}$ $= 1 - \sin x$ $= \text{RHS}$ <p>OR</p> $\text{LHS} = \frac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$ $= \frac{\cos^4 x + (1 - \cos^2 x) \cos^2 x}{1 + \sin x}$ $= \frac{\cos^4 x + \cos^2 x - \cos^4 x}{1 + \sin x}$ $= \frac{1 - \sin^2 x}{1 + \sin x}$ $= \frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x}$ $= 1 - \sin x$ $= \text{RHS}$ <p>OR</p> $\text{RHS} = 1 - \sin x$ $= (1 - \sin x) \times \frac{1 + \sin x}{1 + \sin x}$ $= \frac{1 - \sin^2 x}{1 + \sin x}$ $= \frac{\cos^2 x}{1 + \sin x}$ $= \frac{\cos^2 x (\sin^2 x + \cos^2 x)}{1 + \sin x}$ $= \frac{\cos^4 x + \cos^2 x \cdot \sin^2 x}{1 + \sin x}$ $= \text{LHS}$	✓ factors ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $\cos^2 x = 1 - \sin^2 x$ ✓ factors (4)
	$\text{RHS} = 1 - \sin x$ $= (1 - \sin x) \times \frac{1 + \sin x}{1 + \sin x}$ $= \frac{1 - \sin^2 x}{1 + \sin x}$ $= \frac{\cos^2 x}{1 + \sin x}$ $= \frac{\cos^2 x (\sin^2 x + \cos^2 x)}{1 + \sin x}$ $= \frac{\cos^4 x + \cos^2 x \cdot \sin^2 x}{1 + \sin x}$ $= \text{LHS}$	✓ $\sin^2 x = 1 - \cos^2 x$ ✓ expansion ✓ $\cos^2 x = 1 - \sin^2 x$ ✓ factors (4)
	$\text{RHS} = 1 - \sin x$ $= (1 - \sin x) \times \frac{1 + \sin x}{1 + \sin x}$ $= \frac{1 - \sin^2 x}{1 + \sin x}$ $= \frac{\cos^2 x}{1 + \sin x}$ $= \frac{\cos^2 x (\sin^2 x + \cos^2 x)}{1 + \sin x}$ $= \frac{\cos^4 x + \cos^2 x \cdot \sin^2 x}{1 + \sin x}$ $= \text{LHS}$	✓ $\times \frac{1 + \sin x}{1 + \sin x}$ ✓ product ✓ $1 - \sin^2 x = \cos^2 x$ ✓ $1 = \cos^2 x + \sin^2 x$ (4)

5.2.2	$\sin x + 1 = 0$ $\sin x = -1$ ref. $\angle = 90^\circ$ $x = 270^\circ$	✓ $\sin x + 1 = 0$ ✓ $x = 270^\circ$ (2)
5.2.3	$y = \frac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$ $y = 1 - \sin x$ $\therefore \text{Minimum} = 0$	✓✓ Minimum = 0 (2)
5.3.1	$\sin(A - B)$ $= \cos[90^\circ - (A - B)]$ $= \cos[(90^\circ - A) - (-B)]$ $= \cos(90^\circ - A)\cos(-B) + \sin(90^\circ - A)\sin(-B)$ $= \sin A \cos B + \cos A(-\sin B)$ $= \sin A \cos B - \cos A \sin B$ OR $\sin(A - B)$ $= \cos[90^\circ - (A - B)]$ $= \cos[(90^\circ + B) - A]$ $= \cos(90^\circ + B)\cos A + \sin(90^\circ + B)\sin A$ $= -\sin B \cos A + \cos B \sin A$ $= \sin A \cos B - \cos A \sin B$	✓ co-ratio ✓ compound angle ✓ reduction (3) ✓ co-ratio ✓ compound angle ✓ reduction (3)
5.3.2	$\sin 48^\circ \cos x - \cos 48^\circ \sin x = \cos 2x$ $\sin(48^\circ - x) = \cos 2x$ $\sin(48^\circ - x) = \sin(90^\circ - 2x)$ $48^\circ - x = 90^\circ - 2x + k \cdot 360^\circ \quad \text{or}$ $48^\circ - x = 180^\circ - (90^\circ - 2x) + k \cdot 360^\circ$ $x = 42^\circ + k \cdot 360^\circ$ $-3x = 42^\circ + k \cdot 360^\circ$ $x = -14^\circ - k \cdot 120^\circ ; k \in \mathbb{Z}$ OR $\sin 48^\circ \cos x - \cos 48^\circ \sin x = \cos 2x$ $\sin(48^\circ - x) = \cos 2x$ $\cos(90^\circ - 48^\circ + x) = \cos 2x$ $\cos(42^\circ + x) = \cos 2x$ $42^\circ + x = 2x + k \cdot 360^\circ \quad \text{or} \quad 42^\circ + x = 360^\circ - 2x + k \cdot 360^\circ$ $-x = -42^\circ + k \cdot 360^\circ$ $x = 42^\circ - k \cdot 360^\circ$ $3x = 318^\circ + k \cdot 360^\circ$ $x = 106^\circ + k \cdot 120^\circ ; k \in \mathbb{Z}$	✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k \in \mathbb{Z}$ (5) ✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k \in \mathbb{Z}$ (5)

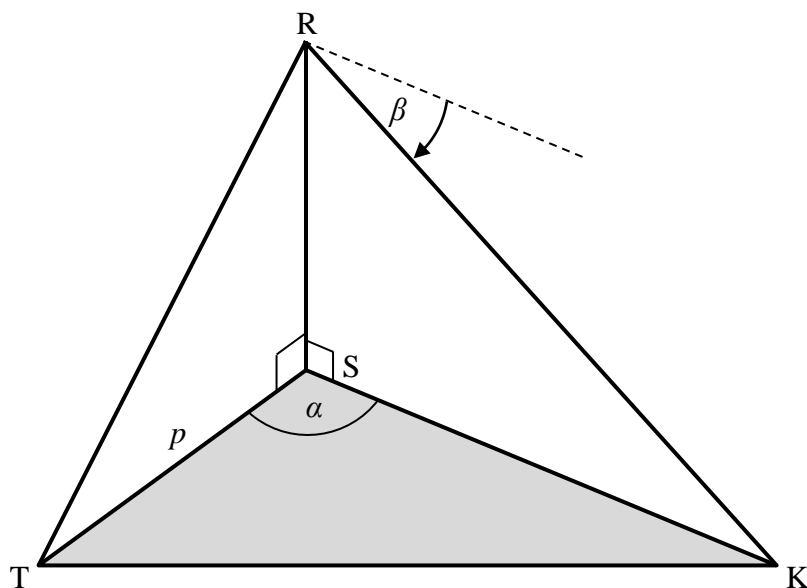
5.4	$\frac{\sin 3x + \sin x}{\cos 2x + 1}$ $= \frac{\sin(2x + x) + \sin(2x - x)}{\cos 2x + 1}$ $= \frac{\sin 2x \cos x + \cos 2x \sin x + \sin 2x \cos x - \cos 2x \sin x}{2\cos^2 x - 1 + 1}$ $= \frac{2\sin 2x \cos x}{2\cos^2 x}$ $= \frac{2(2\sin x \cos x)\cos x}{2\cos^2 x}$ $= \frac{4\sin x \cos^2 x}{2\cos^2 x}$ $= 2\sin x$ <p>OR</p> $\frac{\sin 3x + \sin x}{\cos 2x + 1}$ $= \frac{\sin(2x + x) + \sin x}{2\cos^2 x - 1 + 1}$ $= \frac{\sin 2x \cos x + \cos 2x \sin x + \sin x}{2\cos^2 x}$ $= \frac{2\sin x \cos x \cos x + \cos 2x \sin x + \sin x}{2\cos^2 x}$ $= \frac{\sin x(2\cos^2 x + \cos 2x + 1)}{2\cos^2 x}$ $= \frac{\sin x(2\cos^2 x + 2\cos^2 x - 1 + 1)}{2\cos^2 x}$ $= 2\sin x$	<p>✓ $3x = (2x + x)$</p> <p>✓ expansion</p> <p>✓ double angle of $\cos 2x$</p> <p>✓ simplification</p> <p>✓ $\sin 2x = 2\sin x \cos x$</p> <p>✓ answer (6)</p> <p>✓ $3x = (2x + x)$</p> <p>✓ double angle of $\cos 2x$</p> <p>✓ expansion</p> <p>✓ $\sin 2x = 2\sin x \cos x$</p> <p>✓ common factor</p> <p>✓ answer (6)</p>
		[31]

QUESTION/VRAAG 6



6.1	Period = 180°	✓ 180° (1)
6.2	$y \in \left[-\frac{\sqrt{2}}{2}; 1\right]$ OR $y \in [-0,71; 1]$ OR $-\frac{\sqrt{2}}{2} \leq y \leq 1$	✓ $-\frac{\sqrt{2}}{2}$ ✓ $y \in \left[-\frac{\sqrt{2}}{2}; 1\right]$ (2)
6.3.1	$x \in (45^\circ; 90^\circ)$ OR $45^\circ < x < 90^\circ$	✓✓ $x \in (45^\circ; 90^\circ)$ (2)
6.3.2	$f(x) + 1 \leq 0$ $f(x) \leq -1$ $x \in [105^\circ; 165^\circ]$ OR $105^\circ \leq x \leq 165^\circ$	✓✓ $x \in [105^\circ; 165^\circ]$ (2)
6.4	$p(x) = -2 \sin 2x$ $-2 \sin 2x = -1$ OR $2 \sin 2x = 1$ $k = 15^\circ$ or $k = 75^\circ$	✓ reading off $f(x) = 1$ or $-f(x) = -1$ ✓ 15° ✓ 75° (3)
6.5	$g(x) = -\cos(x + 45^\circ)$ $h(x) = -\cos(x + 90^\circ)$ $h(x) = \sin x$	✓ $-\cos(x + 90^\circ)$ ✓ answer (2)
		[12]

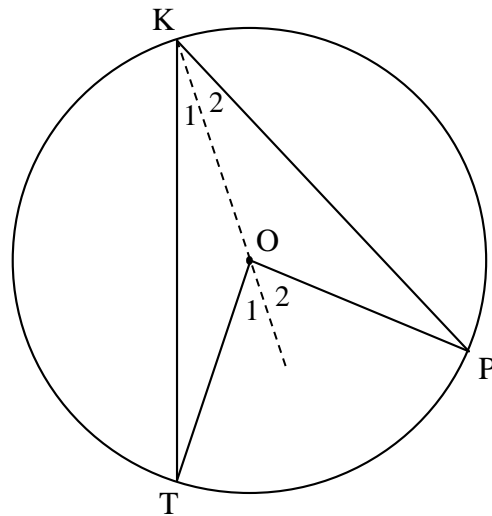
QUESTION/VRAAG 7



7.1	$\text{Area } \Delta STK = \frac{1}{2} p(SK) \sin \alpha$ $q = \frac{1}{2} p(SK) \sin \alpha$ $SK = \frac{q}{\frac{1}{2} p \sin \alpha}$ $= \frac{2q}{p \sin \alpha}$	✓ substitution into the correct formula ✓ answer (2)
7.2	$\hat{RKS} = \beta$ $\frac{RS}{SK} = \tan \beta$ $RS = \frac{2q \tan \beta}{p \sin \alpha}$ <p>OR</p> $\frac{RS}{\sin \beta} = \frac{SK}{\sin (90^\circ - \beta)}$ $RS \cos \beta = SK \sin \beta$ $RS = SK \tan \beta$ $RS = \frac{2q \tan \beta}{p \sin \alpha}$	✓ $\hat{RKS} = \beta$ ✓ correct trig ratio (2) ✓ $\hat{RKS} = \beta$ ✓ $\tan \beta = \frac{\sin \beta}{\cos \beta}$ (2)
7.3	$70 = \frac{2(2500) \tan 42^\circ}{80 \sin \alpha}$ $\sin \alpha = \frac{25}{28} \tan 42^\circ \quad \text{OR} \quad \sin \alpha = 0,80\dots$ $\alpha = 53,51^\circ$	✓ correct substitution of values into RS ✓ value of $\sin \alpha$ ✓ answer (3)
		[7]

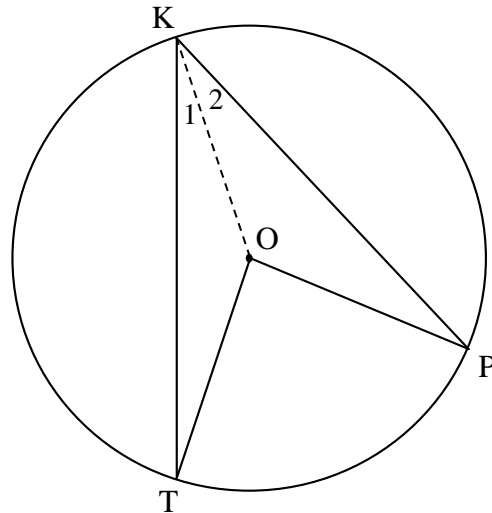
QUESTION/VRAAG 8

8.1



8.1	<p>Construction: Draw KO produced</p> $\hat{O}_1 = \hat{K}_1 + \hat{T} \quad [\text{ext } \angle \text{ of } \Delta]$ <p>But $\hat{K}_1 = \hat{T}$ [\angles opp equal sides]</p> $\therefore \hat{O}_1 = 2\hat{K}_1$ $\hat{O}_2 = \hat{K}_2 + P \quad [\text{ext } \angle \text{ of } \Delta]$ <p>But $\hat{K}_2 = P$ [\angles opp equal sides]</p> $\therefore \hat{O}_2 = 2\hat{K}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$ $= 2(\hat{K}_1 + \hat{K}_2)$ $\therefore \hat{TOP} = 2\hat{TKP}$ <p>OR</p>	<p>✓ construction</p> <p>✓ S / R</p> <p>✓ S</p> <p>✓ S</p> <p>✓ S</p> <p>(5)</p>
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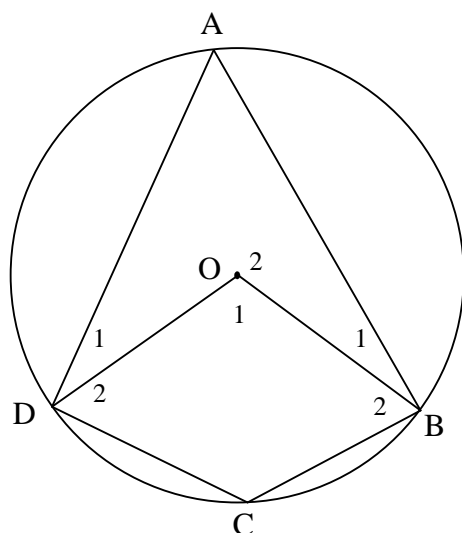
8.1



8.1	<p>Construction: Draw KO</p> $\hat{T} = \hat{K}_1$ [\angle s opp. equal sides] $\therefore \hat{KOT} = 180^\circ - 2\hat{K}_1$ [sum of \angle s of ΔKOT] $\hat{P} = \hat{K}_2$ [\angle s opp. equal sides] $\therefore \hat{KOP} = 180^\circ - 2\hat{K}_2$ [sum of \angle s of ΔKOP] $\hat{TOP} = 360^\circ - (\hat{KOT} + \hat{KOP})$ [\angle s around a point] $= 360^\circ - (180^\circ - 2\hat{K}_1 + 180^\circ - 2\hat{K}_2)$ $= 2\hat{K}_1 + 2\hat{K}_2$ $= 2(\hat{K}_1 + \hat{K}_2)$ $\therefore \hat{TOP} = 2\hat{TKP}$	<p>✓ construction</p> <p>✓ S / R</p> <p>✓ S</p> <p>✓ S</p> <p>✓ S</p>
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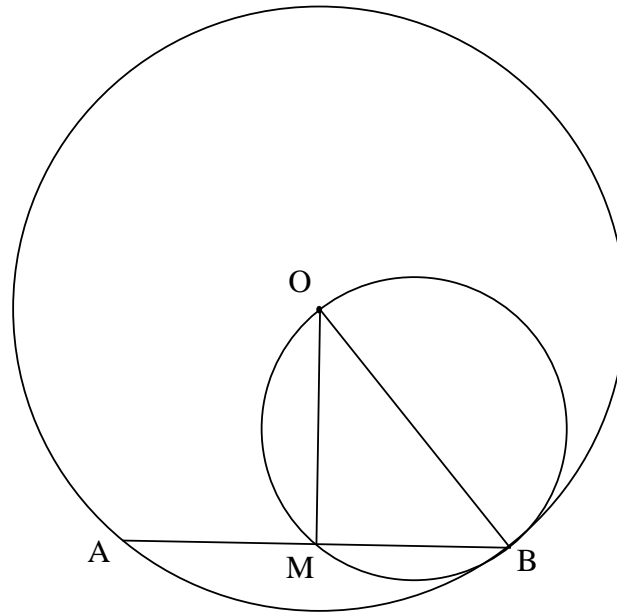
(5)

8.2

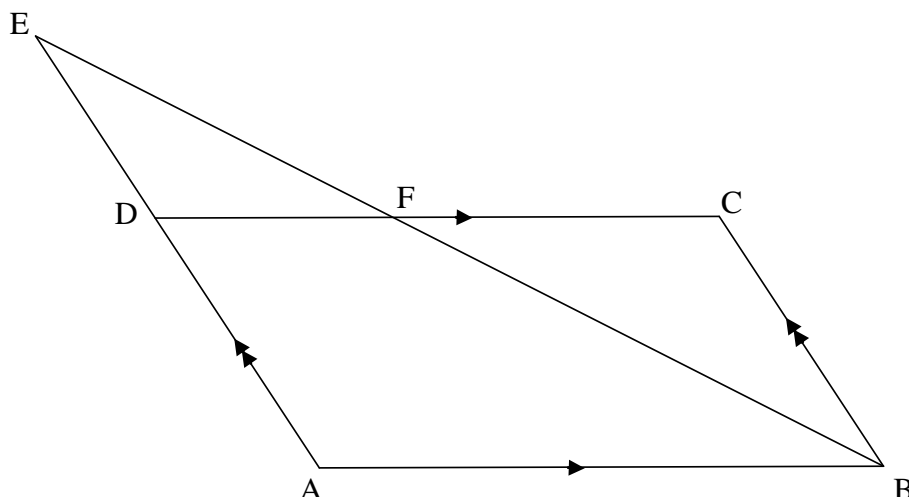


8.2	$\hat{O}_1 = 4x + 100^\circ$ [given] $\therefore \hat{A} = 2x + 50^\circ$ [\angle at centre = $2 \times \angle$ at circumference] $x + 34^\circ + 2x + 50^\circ = 180^\circ$ [opp \angle s of cyclic quad] $3x = 96^\circ$ $x = 32^\circ$ OR $\hat{O}_2 = 2x + 68^\circ$ [\angle at centre = $2 \times \angle$ at circumference] $4x + 100^\circ + 2x + 68^\circ = 360^\circ$ [\angle s round a pt] $6x = 192^\circ$ $x = 32^\circ$ OR $\hat{O}_2 = -4x + 260^\circ$ [\angle s round a pt] $2\hat{C} = -4x + 260^\circ$ [\angle at centre = $2 \times \angle$ at circumference] $\hat{C} = -2x + 130^\circ$ $x + 34^\circ = -2x + 130^\circ$ $3x = 96^\circ$ $x = 32^\circ$	<div>✓ S ✓ R</div> <div>✓ S ✓ R</div> <div>✓ answer</div> <div>(5)</div> <div>✓ S ✓ R</div> <div>✓ S ✓ R</div> <div>✓ answer</div> <div>(5)</div> <div>✓ S ✓ R</div> <div>✓ S ✓ R</div> <div>✓ answer</div> <div>(5)</div>
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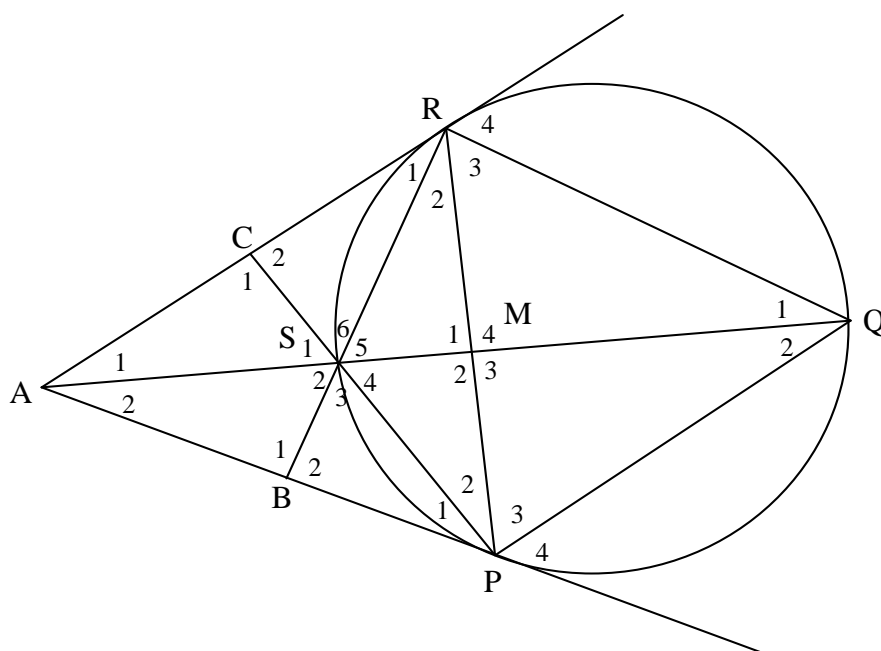
8.3



8.3.1	$\hat{O}MB = 90^\circ$ [∠ in semi circle]	✓ S ✓ R (2)
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$ $\therefore MB = 5\sqrt{3}$ [line from centre \perp to chord] $OB^2 = OM^2 + MB^2$ [Pythagoras] $OB^2 = 5^2 + (5\sqrt{3})^2$ $OB = 10$ units	✓ S ✓ R ✓ S ✓ answer (4)
		[16]

QUESTION/VRAAG 9

9.1	$\frac{FB}{EB} = \frac{DA}{EA}$ [prop theorem; $DC \parallel AB$] OR [line \parallel one side of Δ] $FB = \frac{4p \times 21}{7p}$ $FB = 12$ units	✓ S ✓ R ✓ answer (3)
9.2	In ΔEDF and ΔEAB : \hat{E} is common $\hat{EDF} = \hat{A}$ [corresp \angle s; $EA \parallel CB$] $\hat{EFD} = \hat{EBA}$ [corresp \angle s; $DC \parallel AB$] $\Delta EDF \parallel \Delta EAB$ [\angle ; \angle ; \angle]	✓ S ✓ S/R ✓ S OR R (3)
9.3	$\frac{DF}{AB} = \frac{ED}{EA}$ [Δ s] $DF = \frac{3p \times 14}{7p}$ $DF = 6$ units $FC = 8$ units [DC = AB = 14 units; opp sides of ^m] OR $\Delta EDF \parallel \Delta BCF$ [\angle ; \angle ; \angle] $\frac{ED}{BC} = \frac{DF}{CF}$ [Δ s] $\frac{3}{4} = \frac{14 - FC}{FC}$ [BC = AD; opp sides of ^m] $3FC = 56 - 4FC$ $FC = 8$	✓ S ✓ DF = 6 ✓ FC = 14 – DF (3) ✓ $\Delta EDF \parallel \Delta BCF$ ✓ $\frac{3}{4} = \frac{14 - FC}{FC}$ ✓ answer (3)
		[9]

QUESTION/VRAAG 10

10.1	$\hat{S}_3 = \hat{PQR}$ [ext \angle of cyclic quad] $\hat{R}_3 = \hat{PQR}$ [\angle s opp equal sides] $\therefore \hat{S}_3 = \hat{R}_3$ But $\hat{S}_4 = \hat{R}_3$ [\angle s in the same seg] $\therefore \hat{S}_3 = \hat{S}_4$	\checkmark S \checkmark R \checkmark S / R \checkmark S \checkmark R (5)
10.2	$\hat{R}_1 + \hat{R}_2 = \hat{PQR}$ [tan chord theorem] $\hat{S}_4 = \hat{PQR}$ [proved in 10.1] $\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$ SMRC is a cyclic quad [converse ext \angle of cyclic quad]	\checkmark S \checkmark R \checkmark S \checkmark R (4)
10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$ [ext \angle of Δ] $\hat{S}_4 = \hat{P}_1 + \hat{A}_2$ [ext \angle of Δ] $\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$ But $\hat{P}_1 = \hat{R}_2$ [tan chord theorem] $\therefore \hat{P}_2 = \hat{A}_2$ RP is a tangent to the circle [converse tan chord theorem] OR [\angle between line and chord] OR [converse alt seg theorem]	\checkmark S \checkmark R \checkmark S \checkmark S \checkmark R \checkmark R (6)

OR

	<p>In $\triangle MSP$ and $\triangle MPA$</p> <p>\hat{M}_2 is common</p> <p>$AR = AP$ [tans from same point]</p> <p>$\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$ [\angles opp equal sides]</p> <p>$\hat{S}_4 = \hat{R}_1 + \hat{R}_2$ [proved in 10.2]</p> <p>$\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$</p> <p>$\therefore \hat{P}_2 = \hat{A}_2$ [sum of \angles in \triangle]</p> <p>RP is a tangent to the circle [converse tan chord theorem]</p>	<p>✓ S</p> <p>✓ S / R</p> <p>✓ S</p> <p></p> <p>✓ S</p> <p>✓ S</p> <p>✓ R</p> <p>(6)</p>
		[15]

TOTAL/TOTAAL: 150