basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 23 pages./
Hierdie nasienriglyne bestaan uit 23 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement (A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	**A mark for the correct reason (A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

1.1	$a = -23,846...$ $b = 0,227...$ $\hat{y} = -23,85 + 0,23x$	✓ $a = -23,846...$ ✓ $b = 0,227...$ ✓ equation (3)
1.2	$\hat{y} = -23,85 + 0,23(550)$ $y = 102,65$ **OR** $y = 101,02$	✓ substitution of 550 ✓ answer (2) ✓✓ $y = 101,02$ (calculator) (2)
1.3	$r = 0,98$	✓ $r = 0,98$ (1)
1.4	Very strong positive correlation	✓ strong positive (1)

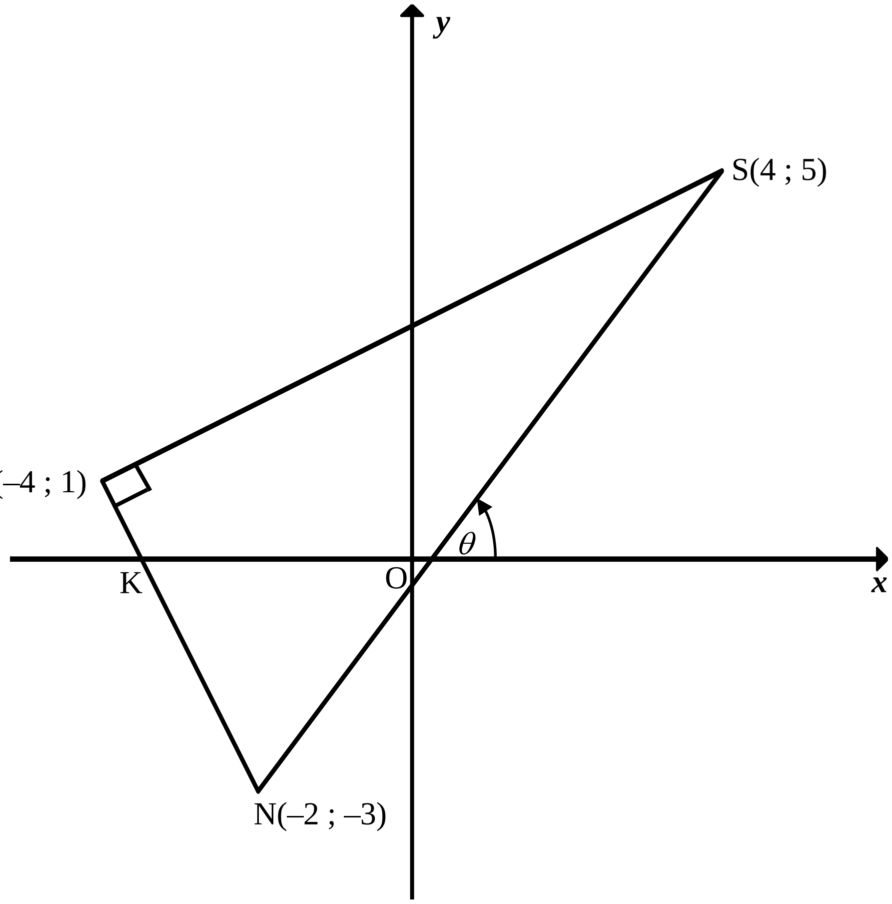
50	100	130	150	180	190	200	200

1.5.1	$\bar{x} = \frac{1200}{8}$ $\bar{x} = 150$ **OR** $\bar{x} = 150$	✓ 1200 ✓ answer (2) ✓✓ $\bar{x} = 150$ (2)
1.5.2	$\sigma = 50,50$	✓ $\sigma = 50,50$ (1)
1.5.3	$\bar{x} - \sigma$ $= 150 - 50,50$ $= 99,50$ $\therefore$ 1 stop	✓ calculation of $\bar{x} - \sigma$ ✓ answer (2)
		[12]

QUESTION/*VRAAG* 2

2.1	<table><tr><th>Number of glasses of water per day</th><th>Number of staff members</th><th>Cumulative frequency</th></tr><tr><td>$0 \le x < 2$</td><td>5</td><td>5</td></tr><tr><td>$2 \le x < 4$</td><td>15</td><td>20</td></tr><tr><td>$4 \le x < 6$</td><td>13</td><td>33</td></tr><tr><td>$6 \le x < 8$</td><td>5</td><td>38</td></tr><tr><td>$8 \le x < 10$</td><td>2</td><td>40</td></tr></table>	✓ 5; 20 ✓ 40 (2)
2.2	40 staff members	✓ answer (1)
2.3	33 staff members	✓ answer (1)
2.4	$\bar{x} = \dfrac{\left(1\times\left(5+\frac{k}{2}\right)\right)+(3\times15)+\left(5\times\left(13+\frac{k}{2}\right)\right)+(7\times5)+(9\times2)}{40+k}=4$ $5+\frac{k}{2}+45+65+\frac{5k}{2}+35+18=160+4k$ $3k+168=160+4k$ $k=8$	✓ answer from Q2.2 + k ✓ $\left(1\times\left(5+\frac{k}{2}\right)\right)$ ✓ $\left(5\times\left(13+\frac{k}{2}\right)\right)$ ✓ answer (4)
	OR $\bar{x} = \dfrac{(1\times5)+(15\times3)+(13\times5)+(5\times7)+(2\times9)}{40}$ $= 4{,}2$ $\bar{x}_{\text{old}} - \bar{x}_{\text{current}} = 4{,}2-4$ $= 0{,}2$ $\therefore 0{,}2\times40$ $= 8$ teachers	✓ 4,2 ✓ $\bar{x}_{\text{old}}-4$ ✓ difference ✓ answer (4)
		[8]

QUESTION/*VRAAG* 3

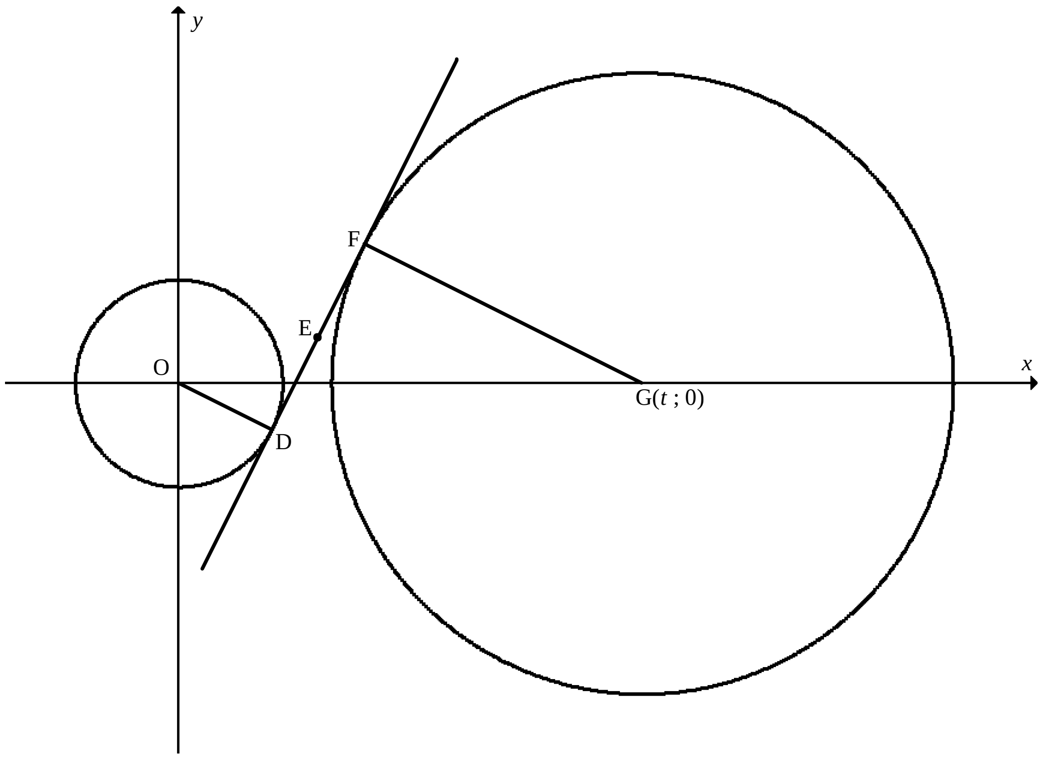


3.1	$SL = \sqrt{(x_S - x_L)^2 + (y_S - y_L)^2}$ $SL = \sqrt{(4-(-4))^2 + (5-1)^2}$ $SL = \sqrt{80} = 4\sqrt{5} = 8,94$ units	✓ substitution of S and L into correct formula ✓ answer (2)
3.2	$m_{SN} = \dfrac{5-(-3)}{4-(-2)}$ $m_{SN} = \dfrac{4}{3}$	✓ substitution of S and N into correct formula ✓ answer (2)
3.3	$m = \tan\theta = \dfrac{4}{3}$ $\theta = 53,13°$	✓ $\tan\theta = m_{SN}$ ✓ answer (2)
3.4	$m_{LN} = \dfrac{1-(-3)}{-4-(-2)}$ $m_{LN} = -2$ $L\hat{K}O = 116,565...°$ $L\hat{N}S = 116,565...° - 53,13°$ $L\hat{N}S = 63,44°$	✓ $m_{LN} = -2$ ✓ size of $L\hat{K}O$ ✓ answer (3)

	OR SN = 10 units $\sin L\hat{N}S = \frac{4\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$	✓ SN = 10 units ✓ correct trig ratio ✓ answer (3)
	OR $LN = 2\sqrt{5}$ units $\tan L\hat{N}S = \frac{4\sqrt{5}}{2\sqrt{5}}$ $L\hat{N}S = 63{,}44°$	✓ $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
	OR SN = 10 units $LN = 2\sqrt{5}$ units $\cos L\hat{N}S = \frac{2\sqrt{5}}{10}$ $L\hat{N}S = 63{,}44°$	✓ SN = 10 units and $LN = 2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)
3.5	$m = \frac{4}{3}$ $1 = \frac{4}{3}(-4) + c$ **OR** $y - 1 = \frac{4}{3}(x - (-4))$ $c = \frac{19}{3}$ $y - 1 = \frac{4}{3}x + \frac{16}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$	✓ m_{SN} ✓ substitution of m_{SN} & L ✓ equation (3)
3.6	$SL = 4\sqrt{5}$ $LN = \sqrt{(-4-(-2))^2 + (1-(-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ Area $\Delta LSN = \frac{1}{2}(4\sqrt{5})(2\sqrt{5})$ $= 20$ units^2 **OR**	✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)

	SN = 10 units LN = $\sqrt{(-4-(-2))^2+(1-(-3))^2}$ LN = $\sqrt{20} = 2\sqrt{5}$ Area ΔLSN = $\frac{1}{2}(10)(2\sqrt{5})\sin 63,44°$ = 20 units^2	✓ LN = $\sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)
3.7	$\hat{L} = 90°$ SN is a diameter of circle S, L, N [chord subtends 90° **OR** converse ∠ in semi-circle] Centre of circle = $P\left(\frac{4+(-2)}{2};\frac{5+(-3)}{2}\right)$ $= P(1;1)$ **OR** Let the coordinates of P be (a ; b). Then, PL = PN: $(-4-a)^2+(1-b)^2=(-2-a)^2+(-3-b)^2$ $a-2b=-1$ …..equation 1 If PS = PN, then: $4a+2b=6$ ………….. equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1;1)$ **OR** If PL = PN, then: $a-2b=-1$ …..equation 1 If PS = PL, then: $2a+b=3$.....equation 2 Solving simultaneously yields: $a=1$ and $b=1$ and $P(1;1)$	✓ SN is a diameter of circle S, L, N ✓ x-value ✓ y-value (3) ✓ 2 correct linear equations ✓ x-value ✓ y-value (3) ✓ 2 correct linear equations ✓ x-value ✓ y-value (3)
3.8	$L\hat{P}N = \theta = 53,13°$ [alt ∠s; LP ‖ x –axis] ∴ $L\hat{P}S$ = 126,87° **OR** $L\hat{N}S$ = 63,44° ∴ $L\hat{P}S$=126,88° [∠ at centre = 2 ×∠ at circumference] **OR** $L\hat{S}N$ = 26,56° [sum of ∠s in Δ] $S\hat{L}P$ = 26,56° [∠s opp equal radii] ∴ $L\hat{P}S$ = 126,88° [sum of ∠s in Δ] **OR** $(4\sqrt{5})^2 = 5^2+5^2-2(5)(5)\cos L\hat{P}S$ $\cos L\hat{P}S = -\frac{3}{5}$ ∴ $L\hat{P}S$ = 126,87°	✓ $L\hat{P}N$ ✓ answer (2) ✓ $L\hat{N}S$ ✓ answer (2) ✓ $L\hat{S}N$ ✓ answer (2) ✓ correct substitution into cosine formula ✓ answer (2)
		[20]

QUESTION/*VRAAG* 4

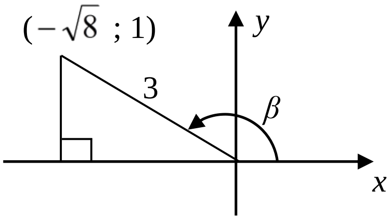


4.1	D$(p\,;\,-2)$ $x^2+y^2=20$ $p^2+(-2)^2=20$ $p^2=16$ $p=\pm4$ $p=4$	✓ substitution of point D$(p\,;-2)$ ✓ $p^2=16$ (2)
4.2	$\frac{4+x_F}{2}=6$ $\frac{-2+y_F}{2}=2$ $x_F=8$ $y_F=6$ $F(8;6)$ **OR** $x_E-x_D=6-4$ $y_E-y_D=2-(-2)$ $=2$ $=4$ $x_F=6+2=8$ $y_F=2+4=6$ $F(8;6)$	✓ method ✓ x-value ✓ y-value (3) ✓ method ✓ x-value ✓ y-value (3)

4.3	$m_{DE} = \frac{-2-2}{4-6}$ $m_{DE} = 2$ $-2 = 2(4)+c$ **OR** $y-(-2) = 2(x-4)$ $c = -10$ $y+2 = 2x-8$ $y = 2x-10$ $y = 2x-10$ **OR** $m_{OD} = -\frac{2}{4} = -\frac{1}{2}$ $\therefore m_{DE} = 2$ [tan ⊥ radius] $-2 = 2(4)+c$ **OR** $y-(-2) = 2(x-4)$ $c = -10$ $y+2 = 2x-8$ $y = 2x-10$ $y = 2x-10$	✓ correct substitution ✓ gradient of DE, DF or EF ✓ substitution of point D(4 ; –2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4) ✓ correct gradient of OD ✓ gradient of DE ✓ substitution of point D(4 ; –2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
4.4	$m_{DE} = 2$ $\therefore m_{GF} = -\frac{1}{2}$ [tan ⊥ radius] $\frac{0-6}{t-8} = -\frac{1}{2}$ $-(t-8) = 2(-6)$ $t = 20$ **OR** $y = 2x-10$ $0 = 2x-10$ $x = 5$ A(5 ; 0) In ΔAFG: FA ⊥ FG $FA^2 = (6-0)^2 + (8-5)^2 = 45$ $FG^2 = (t-8)^2 + (0-6)^2$ $= t^2 - 16t + 100$ $GA^2 = (t-5)^2$ $= t^2 - 10t + 25$ $\therefore GA^2 = GF^2 + FA^2$ $t^2 - 10t + 25 = t^2 - 16t + 100 + 45$ $6t = 120$ $t = 20$	✓ correct gradient of GF ✓ substitution of F ✓ answer (3) ✓ x-intercept of DF ✓ substitution into Pythagoras ✓ answer (3)

4.5	$F(8;6)$ $G(20\ ;\ 0)$ $(8-20)^2+(6-0)^2=r^2$ $r^2=180$ $(x-20)^2+y^2=180$ $x^2+y^2-40x+220=0$	✓ substitution of F and G ✓ value of r^2 ✓ equation of circle ✓ answer (4)
4.6	Smaller circle $r=2\sqrt{5}$ Larger circle $r=6\sqrt{5}$ $G(20\ ;\ 0)$ $k=20-(6\sqrt{5}-2\sqrt{5})$ or $k=20+(6\sqrt{5}-2\sqrt{5})$ $=20-4\sqrt{5}$ $\quad$ $=20+4\sqrt{5}$ $= 11{,}06$ units $\quad$ $= 28{,}94$ units	✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4)
	OR Smaller circle $r=2\sqrt{5}$ $k=2\left(2\sqrt{5}\right)+20-8\sqrt{5}$ or $k=2\left(6\sqrt{5}\right)+20-8\sqrt{5}$ $=20-4\sqrt{5}$ $\quad$ $=20+4\sqrt{5}$ $= 11{,}06$ units $\quad$ $= 28{,}94$ units	✓ $r=2\sqrt{5}$ ✓ method ✓ answer ✓ answer (4)
	OR $x^2+y^2-40x+220=0$ $y=0$ $\therefore x^2-40x+220=0$ $\therefore x=20+6\sqrt{5}$ or $x=20-6\sqrt{5}$ $\therefore k=20+6\sqrt{5}-\sqrt{20}$ or $k=20-6\sqrt{5}+\sqrt{20}$ $\therefore k=20+4\sqrt{5}$ $\quad$ $\therefore k=20-4\sqrt{5}$ $= 11{,}06$ units $\quad$ $= 28{,}94$ units	✓ x-intercepts ✓ method ✓ answer ✓ answer (4)
		[20]

QUESTION/VRAAG 5

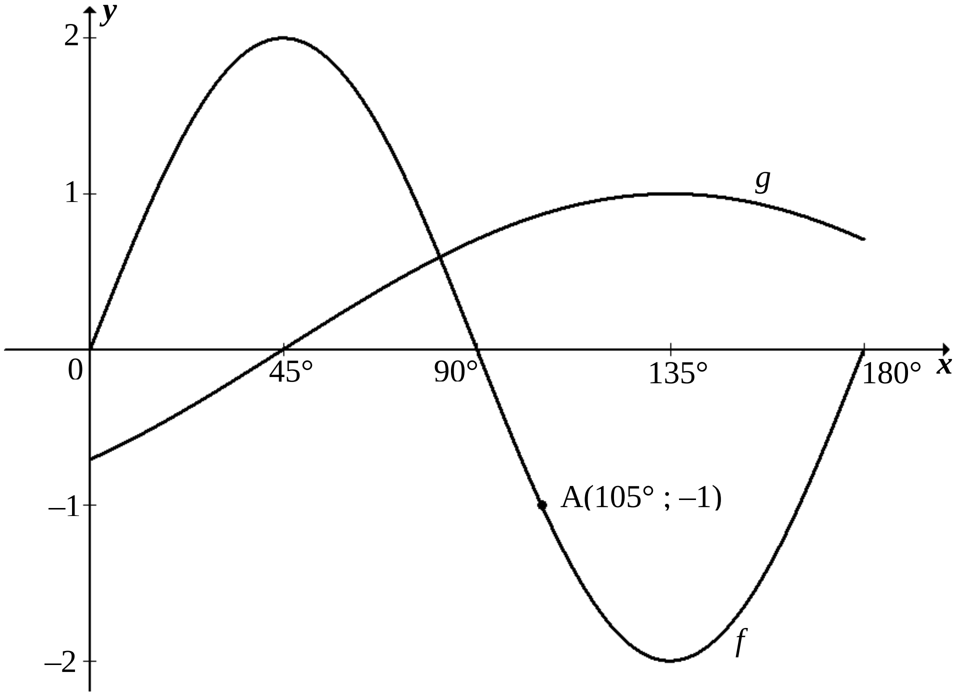
5.1.1	$\sin\beta = \frac{1}{3}$ $\quad \beta \in (90°;270°)$ 	✓ $x^2 + y^2 = r^2$
	$x = -\sqrt{8} = -2\sqrt{2}$	✓ $x = -2\sqrt{2}$
	$\cos\beta$ $= \frac{-2\sqrt{2}}{3}$	✓ answer (3)
	OR	
	$\sin\beta = \frac{1}{3}$ $\quad \beta \in (90°;270°)$ $\cos^2\beta = 1 - \sin^2\beta$ $\cos^2\beta = 1 - \left(\frac{1}{3}\right)^2$	✓ square identity
	$\cos^2\beta = \frac{8}{9}$	✓ $\cos^2\beta$
	$\cos\beta = \frac{-\sqrt{8}}{3}$ $= \frac{-2\sqrt{2}}{3}$	✓ answer (3)
5.1.2	$\sin 2\beta$ $= 2\sin\beta\cos\beta$ $= 2\left(\frac{1}{3}\right)\left(\frac{-\sqrt{8}}{3}\right)$ $= \frac{-2\sqrt{8}}{9}$ **OR** $2\left(\frac{-2\sqrt{2}}{9}\right)$ $= \frac{-4\sqrt{2}}{9}$	✓ double angle ✓ substitution ✓ answer (3)
5.1.3	$\cos(450° - \beta)$ $= \cos(90° - \beta)$ $= \sin\beta$ $= \frac{1}{3}$ **OR**	✓ $\cos(90° - \beta)$ ✓ co-ratio ✓ answer (3)

	$\cos(450° - \beta)$ $= \cos 450° \cos \beta + \sin 450° \sin \beta$ $= \cos 90° \cos \beta + \sin 90° \sin \beta$ $= \sin \beta$ $= \frac{1}{3}$	✓ expansion ✓ reduction ✓ answer (3)
5.2.1	LHS $= \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \frac{\cos^2 x(\cos^2 x + \sin^2 x)}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ = RHS **OR** LHS $= \frac{\cos^4 x + \sin^2 x.\cos^2 x}{1+\sin x}$ $= \frac{\cos^4 x + (1-\cos^2 x)\cos^2 x}{1+\sin x}$ $= \frac{\cos^4 x + \cos^2 x - \cos^4 x}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{(1-\sin x)(1+\sin x)}{1+\sin x}$ $= 1-\sin x$ = RHS **OR** RHS $= 1-\sin x$ $= (1-\sin x) \times \frac{1+\sin x}{1+\sin x}$ $= \frac{1-\sin^2 x}{1+\sin x}$ $= \frac{\cos^2 x}{1+\sin x}$ $= \frac{\cos^2 x(\sin^2 x + \cos^2 x)}{1+\sin x}$ $= \frac{\cos^4 x + \cos^2 x.\sin^2 x}{1+\sin x}$ = LHS	✓ factors ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $\cos^2 x = 1-\sin^2 x$ ✓ factors (4) ✓ $\sin^2 x = 1-\cos^2 x$ ✓ expansion ✓ $\cos^2 x = 1-\sin^2 x$ ✓ factors (4) ✓ $\times \frac{1+\sin x}{1+\sin x}$ ✓ product ✓ $1-\sin^2 x = \cos^2 x$ ✓ $1 = \cos^2 x + \sin^2 x$ (4)

5.2.2	$\sin x+1=0$ $\sin x=-1$ ref. $\angle = 90°$ $x = 270°$	✓ $\sin x+1=0$ ✓ $x = 270°$ (2)
5.2.3	$y=\dfrac{\cos^4 x+\sin^2 x.\cos^2 x}{1+\sin x}$ $y=1-\sin x$ $\therefore$ Minimum = 0	✓✓ Minimum = 0 (2)
5.3.1	sin (A – B) $=\cos\left[90°-\left(\text{A}-\text{B}\right)\right]$ $=\cos\left[(90°-\text{A})-(-\text{B})\right]$ $=\cos\left(90°-\text{A}\right)\cos\left(-\text{B}\right)+\sin\left(90°-\text{A}\right)\sin\left(-\text{B}\right)$ $=\sin\text{A}\cos\text{B}+\cos\text{A}\left(-\sin\text{B}\right)$ $=\sin\text{A}\cos\text{B}-\cos\text{A}\sin\text{B}$ **OR** sin (A – B) $=\cos\left[90°-\left(\text{A}-\text{B}\right)\right]$ $=\cos\left[(90°+\text{B})-\text{A}\right]$ $=\cos\left(90°+\text{B}\right)\cos\text{A}+\sin\left(90°+\text{B}\right)\sin\text{A}$ $=-\sin\text{B}\cos\text{A}+\cos\text{B}\sin\text{A}$ $=\sin\text{A}\cos\text{B}-\cos\text{A}\sin\text{B}$	✓ co-ratio ✓ compound angle ✓ reduction (3) ✓ co-ratio ✓ compound angle ✓ reduction (3)
5.3.2	$\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin\left(48°-x\right)=\cos 2x$ $\sin\left(48°-x\right)=\sin\left(90°-2x\right)$ $48°-x=90°-2x+k.360°$ or $48°-x=180°-\left(90°-2x\right)+k.360°$ $x=42°+k.360°$ $\quad -3x=42°+k.360°$ $x=-14°-k.120°$; $k\in\text{Z}$ **OR** $\sin 48°\cos x-\cos 48°\sin x=\cos 2x$ $\sin\left(48°-x\right)=\cos 2x$ $\cos\left(90°-48°+x\right)=\cos 2x$ $\cos\left(42°+x\right)=\cos 2x$ $42°+x=2x+k.360°$ or $42°+x=360°-2x+k.360°$ $-x=-42°+k.360°$ $\quad 3x=318°+k.360°$ $x=42°-k.360°$ $\quad x=106°+k.120°$; $k\in\text{Z}$	✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in$ Z (5) ✓ compound angle ✓ co-ratio ✓ both equations ✓ general solution ✓ general solution; $k\in$ Z (5)

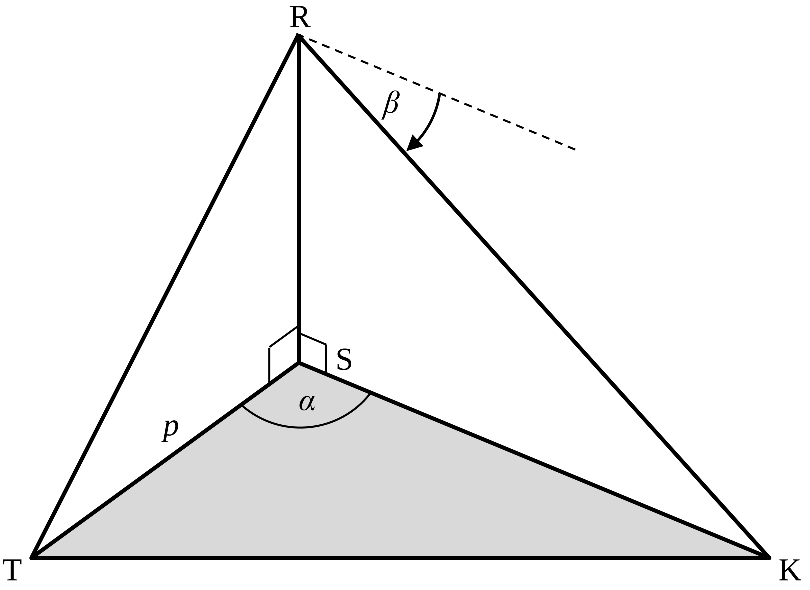
5.4	$\dfrac{\sin 3x+\sin x}{\cos 2x+1}$	
	$=\dfrac{\sin(2x+x)+\sin(2x-x)}{\cos 2x+1}$	✓ $3x=(2x+x)$
	$=\dfrac{\sin 2x\cos x+\cos 2x\sin x+\sin 2x\cos x-\cos 2x\sin x}{2\cos^2 x-1+1}$	✓ expansion ✓ double angle of cos2x
	$=\dfrac{2\sin 2x\cos x}{2\cos^2 x}$	✓ simplification
	$=\dfrac{2(2\sin x\cos x)\cos x}{2\cos^2 x}$	✓ $\sin 2x=2\sin x\cos x$
	$=\dfrac{4\sin x\cos^2 x}{2\cos^2 x}$	
	$=2\sin x$	✓ answer (6)
	OR	
	$\dfrac{\sin 3x+\sin x}{\cos 2x+1}$	
	$=\dfrac{\sin(2x+x)+\sin x}{2\cos^2 x-1+1}$	✓ $3x=(2x+x)$ ✓ double angle of cos2x
	$=\dfrac{\sin 2x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$	✓ expansion
	$=\dfrac{2\sin x\cos x\cos x+\cos 2x\sin x+\sin x}{2\cos^2 x}$	✓ $\sin 2x=2\sin x\cos x$
	$=\dfrac{\sin x\left(2\cos^2 x+\cos 2x+1\right)}{2\cos^2 x}$	✓ common factor
	$=\dfrac{\sin x\left(2\cos^2 x+2\cos^2 x-1+1\right)}{2\cos^2 x}$	
	$=2\sin x$	✓ answer (6)
		[31]

QUESTION/*VRAAG* 6



6.1	Period = 180°	✓ 180° (1)
6.2	$y \in \left[-\frac{\sqrt{2}}{2}\,;1\right]$ **OR** $y \in [-0,71;1]$ **OR** $-\frac{\sqrt{2}}{2} \le y \le 1$	✓ $-\frac{\sqrt{2}}{2}$ ✓ $y \in \left[-\frac{\sqrt{2}}{2}\,;1\right]$ (2)
6.3.1	$x \in (45°\,;\ 90°)$ **OR** $45° < x < 90°$	✓✓ $x \in (45°\,;\ 90°)$ (2)
6.3.2	$f(x)+1 \le 0$ $f(x) \le -1$ $x \in [105°\,;\ 165°]$ **OR** $105° \le x \le 165°$	✓✓ $x \in [105°\,;\ 165°]$ (2)
6.4	$p(x) = -2\sin 2x$ $-2\sin 2x = -1$ **OR** $2\sin 2x = 1$ k = 15° or k = 75°	✓ reading off $f(x) = 1$ or $-f(x) = -1$ ✓ 15° ✓ 75° (3)
6.5	$g(x) = -\cos(x+45°)$ $h(x) = -\cos(x+90°)$ $h(x) = \sin x$	✓ $-\cos(x+90°)$ ✓ answer (2)
		[12]

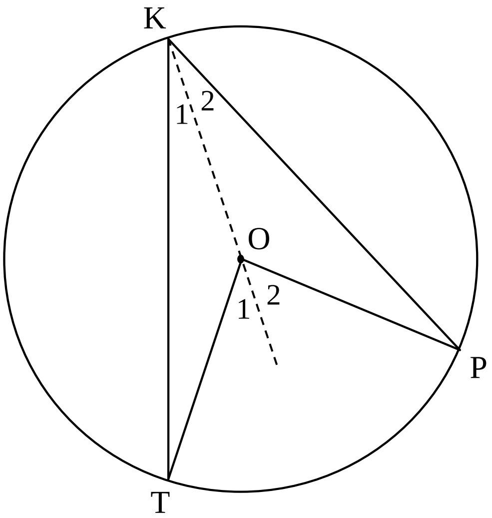
QUESTION/*VRAAG* 7



7.1	Area $\Delta STK = \frac{1}{2}p(SK)\sin\alpha$ $q = \frac{1}{2}p(SK)\sin\alpha$ $SK = \frac{q}{\frac{1}{2}p\sin\alpha}$ $= \frac{2q}{p\sin\alpha}$	✓ substitution into the correct formula ✓ answer (2)
7.2	$R\hat{K}S = \beta$ $\frac{RS}{SK} = \tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$ **OR** $\frac{RS}{\sin\beta} = \frac{SK}{\sin(90° - \beta)}$ $RS\cos\beta = SK\sin\beta$ $RS = SK\tan\beta$ $RS = \frac{2q\tan\beta}{p\sin\alpha}$	✓ $R\hat{K}S = \beta$ ✓ correct trig ratio (2) ✓ $R\hat{K}S = \beta$ ✓ $\tan\beta = \frac{\sin\beta}{\cos\beta}$ (2)
7.3	$70 = \frac{2(2500)\tan 42°}{80\sin\alpha}$ $\sin\alpha = \frac{25}{28}\tan 42°$ **OR** $\sin\alpha = 0,80...$ $\alpha = 53,51°$	✓ correct substitution of values into RS ✓ value of $\sin\alpha$ ✓ answer (3)
		[7]

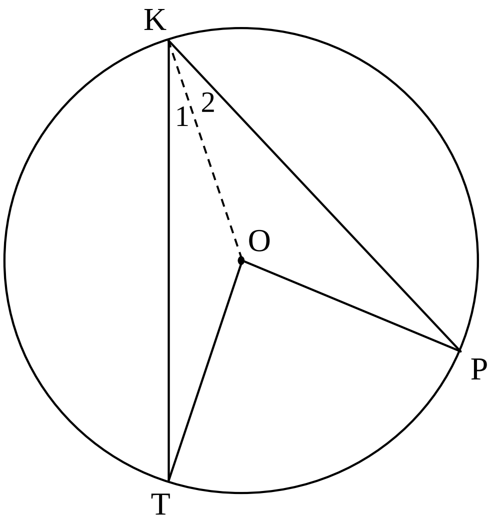
QUESTION/*VRAAG* 8

8.1



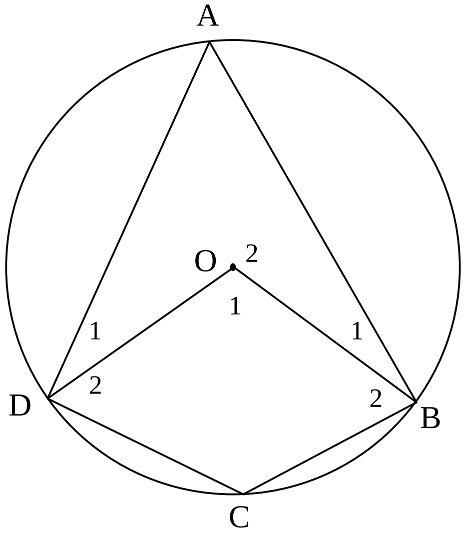
8.1	Construction: Draw KO produced $\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext ∠ of Δ] But $\hat{K}_1 = \hat{T}$ [∠s opp equal sides] $\therefore \hat{O}_1 = 2\hat{K}_1$ $\hat{O}_2 = \hat{K}_2 + P$ [ext ∠ of Δ] But $\hat{K}_2 = P$ [∠s opp equal sides] $\therefore \hat{O}_2 = 2\hat{K}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$ $= 2\left(\hat{K}_1 + \hat{K}_2\right)$ $\therefore T\hat{O}P = 2T\hat{K}P$ **OR**	✓ construction ✓ S / R ✓ S ✓ S ✓ S (5)

8.1



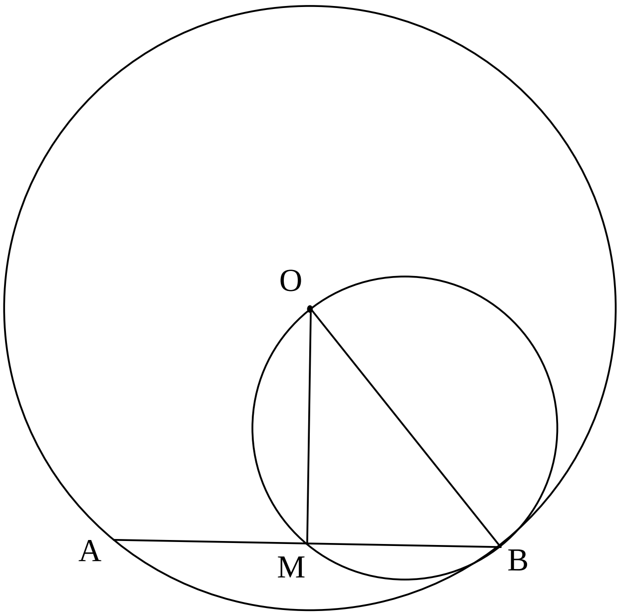
8.1	Construction: Draw KO $\hat{T} = \hat{K}_1$ [$\angle$s opp. equal sides] $\therefore K\hat{O}T = 180° - 2\hat{K}_1$ [sum of $\angle$s of ΔKOT] $\hat{P} = \hat{K}_2$ [$\angle$s opp. equal sides] $\therefore K\hat{O}P = 180° - 2\hat{K}_2$ [sum of $\angle$s of ΔKOP] $T\hat{O}P = 360° - \left(K\hat{O}T + K\hat{O}P\right)$ [$\angle$s around a point] $= 360° - \left(180° - 2\hat{K}_1 + 180° - 2\hat{K}_2\right)$ $= 2\hat{K}_1 + 2\hat{K}_2$ $= 2\left(\hat{K}_1 + \hat{K}_2\right)$ $\therefore T\hat{O}P = 2T\hat{K}P$	✓ construction ✓ S / R ✓ S ✓ S ✓ S (5)

8.2



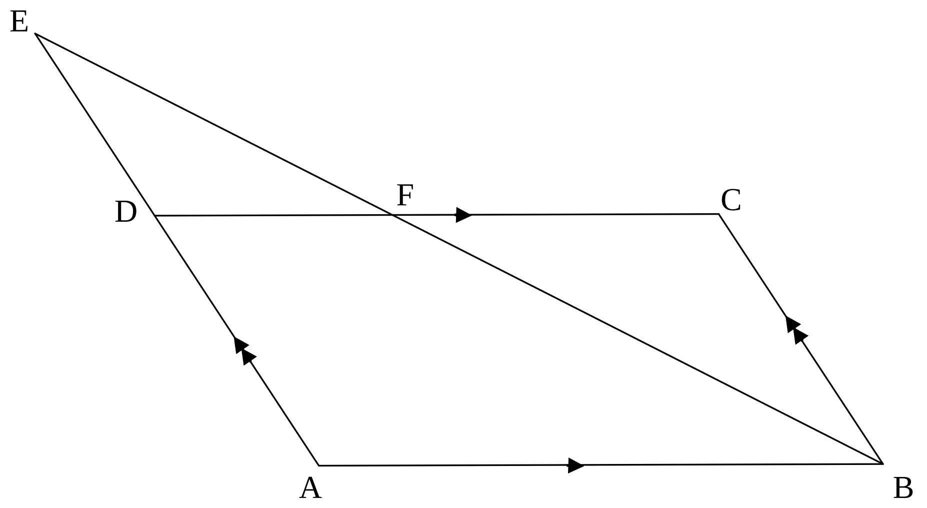
8.2	$\hat{O}_1 = 4x + 100°$ [given] $\therefore \hat{A} = 2x + 50°$ [$\angle$ at centre = 2 ×$\angle$ at circumference] $x + 34° + 2x + 50° = 180°$ [opp $\angle$s of cyclic quad] $3x = 96°$ $x = 32°$ **OR** $\hat{O}_2 = 2x + 68°$ [$\angle$ at centre = 2 ×$\angle$ at circumference] $4x + 100° + 2x + 68° = 360°$ [$\angle$s round a pt] $6x = 192°$ $x = 32°$ **OR** $\hat{O}_2 = -4x + 260°$ [$\angle$s round a pt] $2\hat{C} = -4x + 260°$ [$\angle$ at centre = 2 ×$\angle$ at circumference] $\hat{C} = -2x + 130°$ $x + 34° = -2x + 130°$ $3x = 96°$ $x = 32°$	✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5) ✓ S ✓ R ✓ S ✓ R ✓ answer (5)

8.3



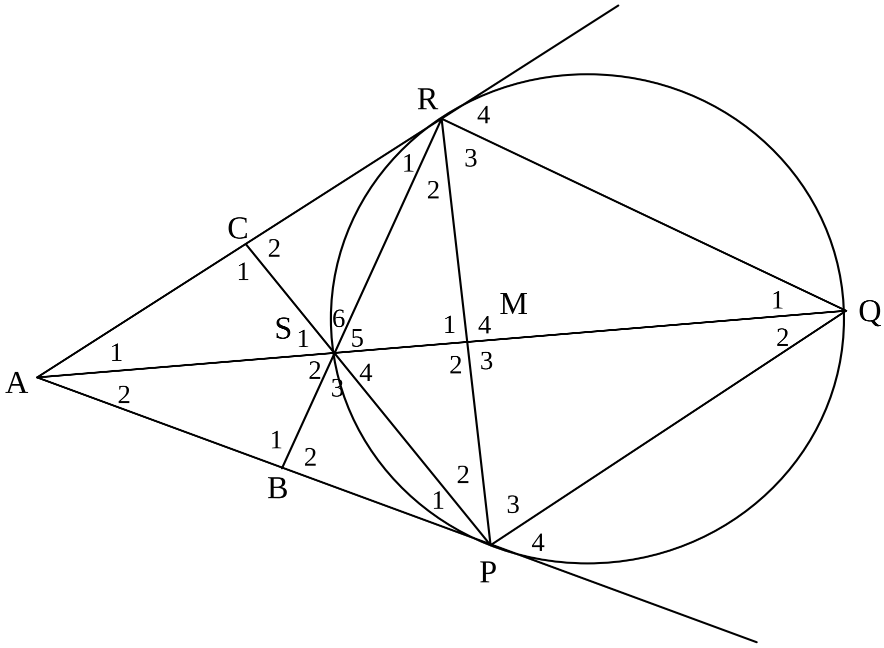
8.3.1	$O\hat{M}B = 90°$ [$\angle$ in semi circle]	✓ S ✓ R (2)
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$ $\therefore MB = 5\sqrt{3}$ [line from centre $\perp$ to chord] $OB^2 = OM^2 + MB^2$ [Pythagoras] $OB^2 = 5^2 + \left(5\sqrt{3}\right)^2$ OB = 10 units	 ✓ S ✓ R ✓ S ✓ answer (4)
		[16]

QUESTION/*VRAAG* 9



9.1	$\frac{FB}{EB}=\frac{DA}{EA}$ [prop theorem; DC ∥ AB] **OR** [line ∥ one side of Δ] $FB=\frac{4p\times 21}{7p}$ FB = 12 units	✓ S ✓ R ✓ answer (3)
9.2	In ΔEDF and ΔEAB: $\hat{E}$ is common $E\hat{D}F=\hat{A}$ [corresp ∠s; EA ∥ CB] $E\hat{F}D=E\hat{B}A$ [corresp ∠s; DC ∥ AB] ΔEDF ∣∣∣ ΔEAB [∠;∠;∠]	✓ S ✓ S/R ✓ S **OR** R (3)
9.3	$\frac{DF}{AB}=\frac{ED}{EA}$ [∣∣∣ Δs] $DF=\frac{3p\times 14}{7p}$ DF = 6 units FC = 8 units [DC = AB = 14 units; opp sides of ∥^{m}] **OR** ΔEDF ∣∣∣ ΔBCF [∠;∠;∠] $\frac{ED}{BC}=\frac{DF}{CF}$ [∣∣∣ Δs] $\frac{3}{4}=\frac{14-FC}{FC}$ [BC = AD; opp sides of ∥^{m}] $3FC=56-4FC$ $FC=8$	✓ S ✓ DF = 6 ✓ FC = 14 − DF (3) ✓ ΔEDF ∣∣∣ ΔBCF ✓ $\frac{3}{4}=\frac{14-FC}{FC}$ ✓ answer (3)
		[9]

QUESTION/*VRAAG* 10



10.1	$\hat{S}_3 = P\hat{Q}R$ $\hat{R}_3 = P\hat{Q}R$ $\therefore \hat{S}_3 = \hat{R}_3$ But $\hat{S}_4 = \hat{R}_3$ $\therefore \hat{S}_3 = \hat{S}_4$	[ext ∠ of cyclic quad] [∠s opp equal sides] [∠s in the same seg]	✓ S ✓ R ✓ S / R ✓ S ✓ R (5)
10.2	$\hat{R}_1 + \hat{R}_2 = P\hat{Q}R$ $\hat{S}_4 = P\hat{Q}R$ $\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$ SMRC is a cyclic quad	[tan chord theorem] [proved in 10.1] [converse ext ∠ of cyclic quad]	✓ S ✓ R ✓ S ✓ R (4)
10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$ $\hat{S}_4 = \hat{P}_1 + \hat{A}_2$ $\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$ But $\hat{P}_1 = \hat{R}_2$ $\therefore \hat{P}_2 = \hat{A}_2$ RP is a tangent to the circle **OR**	[ext ∠ of Δ] [ext ∠ of Δ] [tan chord theorem] [converse tan chord theorem] **OR** [∠ between line and chord] **OR** [converse alt seg theorem]	✓ S ✓ R ✓ S ✓ S ✓ R ✓ R (6)

	In ΔMSP and ΔMPA $\hat{M}_2$ is common AR = AP [tans from same point] $\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$ [$\angle$s opp equal sides] $\hat{S}_4 = \hat{R}_1 + \hat{R}_2$ [proved in 10.2] $\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$ $\therefore \hat{P}_2 = \hat{A}_2$ [sum of $\angle$s in Δ] RP is a tangent to the circle [converse tan chord theorem]	 ✓ S ✓ S / R ✓ S ✓ S ✓ S ✓ R (6)
		[15]

TOTAL/TOTAAL: 150