



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NASIONALE
SENIOR SERTIFKAAT**

GRAAD 12

WISKUNDE V2

NOVEMBER 2023

PUNTE: 150

TYD: 3 uur

**Hierdie vraestel bestaan uit 13 bladsye, 1 inligtingsblad
en 'n antwoordeboek van 23 bladsye.**

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat jy die vrae beantwoord.

1. Hierdie vraestel bestaan uit 10 vrae.
2. Beantwoord AL die vrae in die SPESIALE ANTWOORDEBOEK wat verskaf word.
3. Dui ALLE berekeninge, diagramme, grafieke, ens. wat jy in die beantwoording van die vrae gebruik, duidelik aan.
4. Slegs antwoorde sal NIE noodwendig volpunte verdien NIE.
5. Jy mag 'n goedgekeurde wetenskaplike sakrekenaar gebruik (nieprogrammeerbaar en niegrafies), tensy anders vermeld.
6. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.
7. Diagramme is NIE noodwendig volgens skaal geteken NIE.
8. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
9. Skryf netjies en leesbaar.

VRAAG 1

Vragmotorbestuurders ry 'n sekere afstand en neem 'n kort ruskans voordat hulle verder reis. 'n Bestuurder het rekord gehou van die afstand (in km) wat hy oor 8 ritte afgelê het en die hoeveelheid rustyd (in minute) voordat hy sy reis voortgesit het. Die inligting word in die tabel hieronder gegee.

Afstand gery (in km) (x)	180	200	400	600	170	350	270	300
Hoeveelheid rustyd (in minutes) (y)	20	25	55	120	15	50	40	45

- 1.1 Bepaal die vergelyking van die kleinste kwadrate-regressielyn vir die data. (3)
- 1.2 Indien 'n vragmotorbestuurder 550 km gereis het, voorspel die hoeveelheid tyd (in minute) wat hy moet rus voordat hy met sy reis voortgaan. (2)
- 1.3 Skryf die korrelasiekoëffisiënt vir die data neer. (1)
- 1.4 Interpreteer jou antwoord op VRAAG 1.3. (1)
- 1.5 By elke stopplek het die vragmotorbestuurder geld aan kos en ander verversings gespandeer. Die bedrag gespandeer (in rand) word in die tabel hieronder gegee.

100	150	130	200	50	180	200	190
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- 1.5.1 Bereken die gemiddelde bedrag geld wat hy by elke stopplek gespandeer het. (2)
 - 1.5.2 Bereken die standaardafwyking van die data. (1)
 - 1.5.3 By hoeveel stopplekke het die vragmotorbestuurder 'n bedrag gespandeer wat minder as een standaardafwyking onder die gemiddelde was? (2)
- [12]**

VRAAG 2

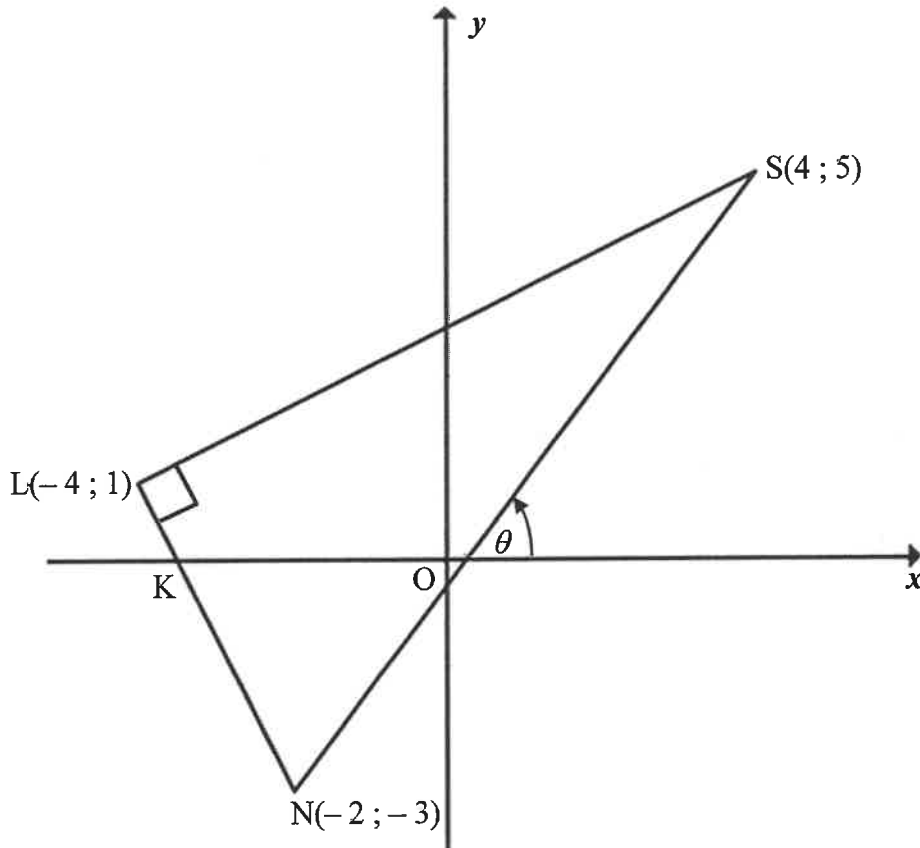
By 'n sekere skool wou die personeelkomitee bepaal hoeveel glase water die personeel gedurende 'n skooldag drink. Alle personeel teenwoordig op 'n spesifieke dag is ondervra. Die inligting word in die tabel hieronder getoon.

AANTAL GLASE WATER PER DAG GEDRINK	GETAL PERSONEELLEDE
$0 \leq x < 2$	5
$2 \leq x < 4$	15
$4 \leq x < 6$	13
$6 \leq x < 8$	5
$8 \leq x < 10$	2

- 2.1 Voltooi die kumulatiewefrekwensie-kolom wat in die tabel in die ANTWOORDEBOEK verskaf word. (2)
- 2.2 Met hoeveel personeellede is onderhoude gevoer? (1)
- 2.3 Hoeveel personeellede het minder as 6 glase water gedurende 'n skooldag gedrink? (1)
- 2.4 Die personeelkomitee het waargeneem dat k personeellede op die dag van die onderhoude afwesig was. Daar is gevind dat die helfte van hierdie k personeellede van 0 tot minder as 2 (dit wil sê $0 \leq x < 2$) glase water per dag gedrink het, terwyl die res van hulle van 4 tot minder as 6 (dit wil sê $4 \leq x < 6$) glase water per dag gedrink het. Wanneer hierdie k onderwysers by die data ingereken word, is die benaderde gemiddelde 4 glase water per personeellid per dag.
- Hoeveel personeellede was op die dag van die onderhoude afwesig? (4)
- [8]**

VRAAG 3

In die figuur is $L(-4 ; 1)$, $S(4 ; 5)$ en $N(-2 ; -3)$ die hoekpunte van 'n driehoek met $\hat{S}LN = 90^\circ$. LN sny die x -as by K .

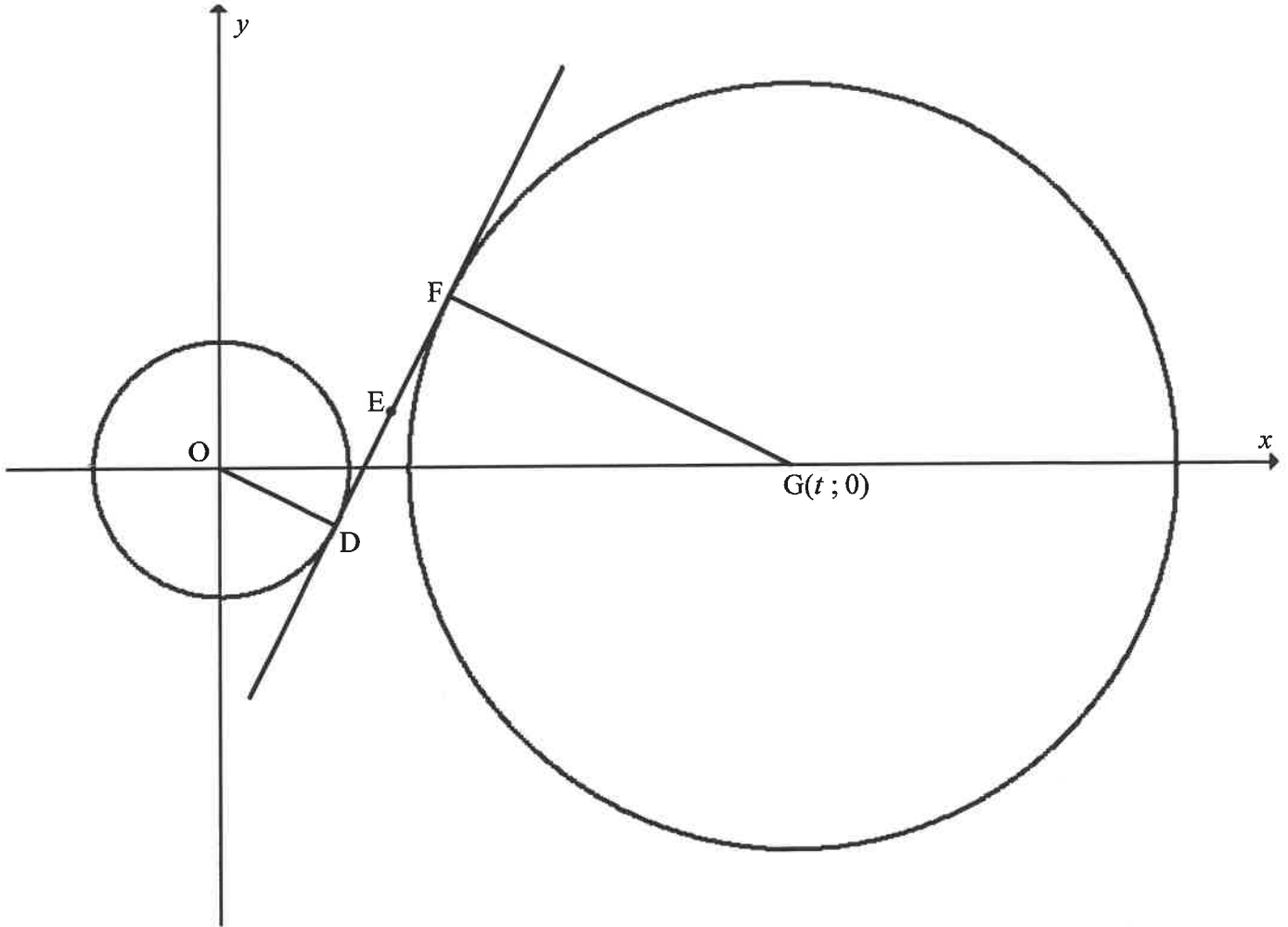


- 3.1 Bereken die lengte van SL . Laat jou antwoord in wortelvorm. (2)
- 3.2 Bereken die gradiënt van SN . (2)
- 3.3 Bereken die grootte van θ , die inklinasiehoek van SN . (2)
- 3.4 Bereken die grootte van $\hat{L}NS$. (3)
- 3.5 Bepaal die vergelyking van die lyn wat deur L gaan en parallel aan SN is. Skryf jou antwoord in die vorm $y = mx + c$. (3)
- 3.6 Bereken die oppervlak van $\triangle LSN$. (3)
- 3.7 Bereken die koördinate van punt P , wat ewe ver van L , S en N is. (3)
- 3.8 Bereken die grootte van $\hat{L}PS$. (2)

[20]

VRAAG 4

In die diagram is O die middelpunt van die sirkel met vergelyking $x^2 + y^2 = 20$. $G(t ; 0)$ is die middelpunt van die groter sirkel. 'n Gemeenskaplike raaklyn raak die sirkels by D en F onderskeidelik, sodanig dat $D(p ; -2)$ in die 4^{de} kwadrant lê.



- 4.1 Dit word gegee dat $D(p ; -2)$ op die kleiner sirkel lê. Toon dat $p = 4$. (2)
- 4.2 $E(6 ; 2)$ is die middelpunt van DF . Bepaal die koördinate van F . (3)
- 4.3 Bepaal die vergelyking van die gemeenskaplike raaklyn, DF , in die vorm $y = mx + c$. (4)
- 4.4 Bereken die waarde van t . Toon ALLE berekeninge. (3)
- 4.5 Bepaal die vergelyking van die groter sirkel in die vorm $ax^2 + by^2 + cx + dy + e = 0$. (4)
- 4.6 Die kleiner sirkel moet k eenhede langs die x -as getransleer word sodat dit die groter sirkel inwendig raak. Bereken die moontlike waardes van k . (4)

[20]

VRAAG 5

5.1 Gegee: $\sin \beta = \frac{1}{3}$, waar $\beta \in (90^\circ ; 270^\circ)$

Sonder die gebruik van 'n sakrekenaar, bepaal elk van die volgende:

5.1.1 $\cos \beta$ (3)

5.1.2 $\sin 2\beta$ (3)

5.1.3 $\cos(450^\circ - \beta)$ (3)

5.2 Gegee: $\frac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$

5.2.1 Bewys dat $\frac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x} = 1 - \sin x$ (4)

5.2.2 Vir watter waarde(s) van x in die interval $x \in [0^\circ ; 360^\circ]$ is $\frac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$ ongedefinieerd? (2)

5.2.3 Skryf die minimum waarde neer van die funksie gedefinieer deur $y = \frac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$ (2)

5.3 Gegee: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

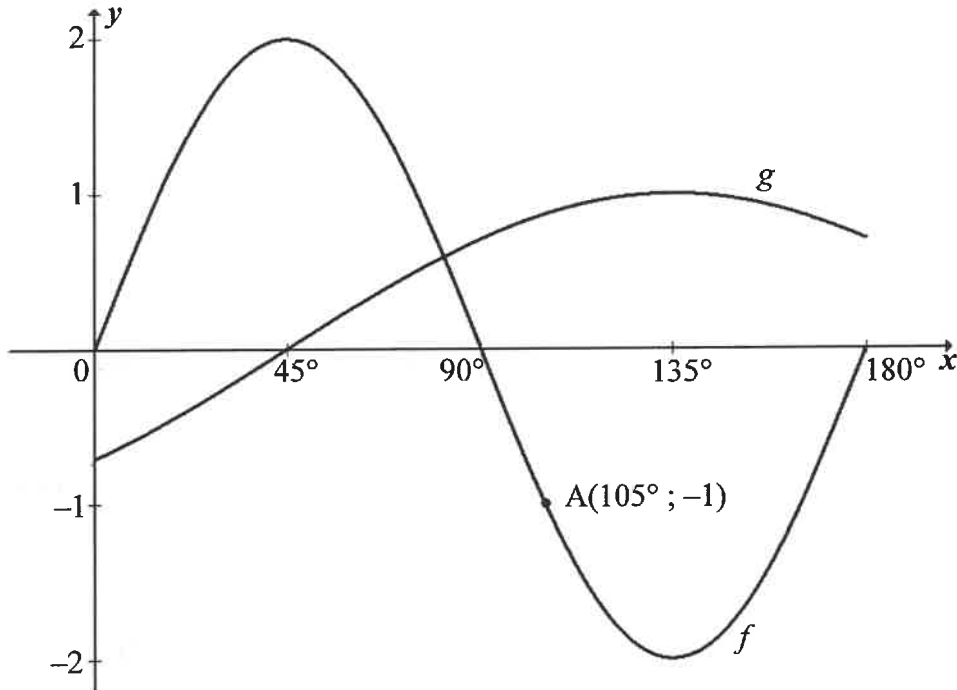
5.3.1 Gebruik die identiteit hierbo om af te lei dat $\sin(A - B) = \sin A \cos B - \cos A \sin B$ (3)

5.3.2 Bepaal vervolgens of andersins die algemene oplossing van die vergelyking $\sin 48^\circ \cos x - \cos 48^\circ \sin x = \cos 2x$ (5)

5.4 Vereenvoudig $\frac{\sin 3x + \sin x}{\cos 2x + 1}$ tot 'n enkele trigonometriese verhouding. (6)
[31]

VRAAG 6

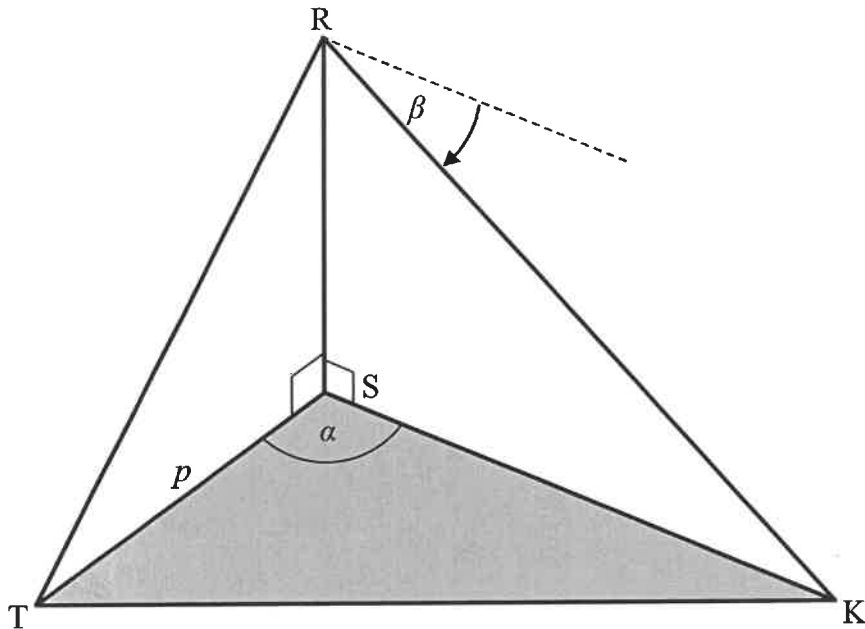
In die diagram is die grafieke van $f(x) = 2\sin 2x$ en $g(x) = -\cos(x + 45^\circ)$ vir die interval $x \in [0^\circ; 180^\circ]$ geskets. $A(105^\circ; -1)$ is 'n punt op f .



- 6.1 Skryf die periode van f neer. (1)
- 6.2 Bepaal die waardeversameling van g in die interval $x \in [0^\circ; 180^\circ]$. (2)
- 6.3 Bepaal die waardes van x , in die interval $x \in [0^\circ; 180^\circ]$, waarvoor:
- 6.3.1 $f(x) \cdot g(x) > 0$ (2)
- 6.3.2 $f(x) + 1 \leq 0$ (2)
- 6.4 'n Ander grafiek p word as $p(x) = -f(x)$ gedefinieer. $D(k; -1)$ lê op p . Bepaal die waarde(s) van k in die interval $x \in [0^\circ; 180^\circ]$. (3)
- 6.5 Grafiek h word verkry wanneer g 45° na links getransleer word. Bepaal die vergelyking van h . Skryf jou antwoord in sy eenvoudigste vorm. (2)
- [12]**

VRAAG 7

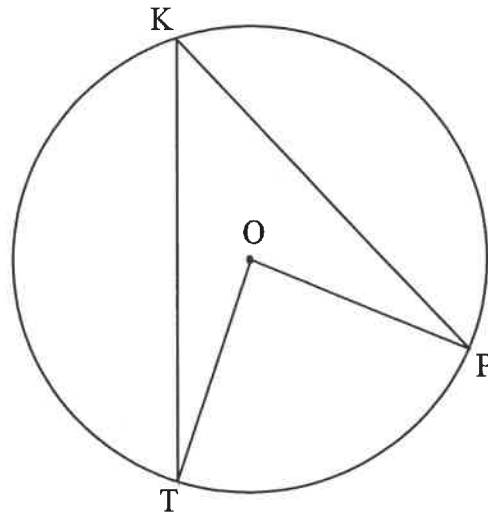
In die diagram lê S, T en K in dieselfde horisontale vlak. RS is 'n vertikale toring. Die dieptehoek van R na K is β . $\hat{T\hat{S}K} = \alpha$, $TS = p$ meter en die oppervlak van ΔSTK is $q \text{ m}^2$.



- 7.1 Bepaal die lengte van SK in terme van p , q en α . (2)
- 7.2 Toon dat $RS = \frac{2q \tan \beta}{p \sin \alpha}$ (2)
- 7.3 Bereken die grootte van α as $\alpha < 90^\circ$ en $RS = 70 \text{ m}$, $p = 80 \text{ m}$, $q = 2\,500 \text{ m}^2$ en $\beta = 42^\circ$. (3)
- [7]

VRAAG 8

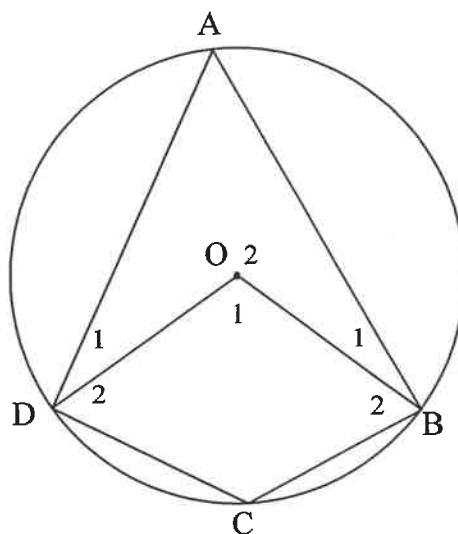
8.1 In die diagram is O die middelpunt van die sirkel.



Gebruik die diagram hierbo om die stelling te bewys wat sê dat die hoek wat onderspan word deur 'n koord by die middelpunt van die sirkel, gelyk is aan twee keer die hoek wat deur dieselfde koord op die omtrek van die sirkel onderspan word, met ander woorde, bewys dat $\hat{TÔP} = 2\hat{TÔP}$.

(5)

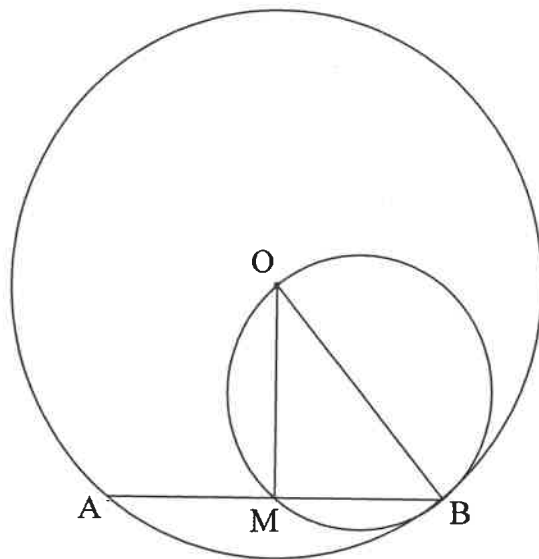
8.2 In die diagram is O die middelpunt van die sirkel en $ABCD$ is 'n koordevierhoek. OB en OD word getrek.



Indien $\hat{O}_1 = 4x + 100^\circ$ en $\hat{C} = x + 34^\circ$, bereken, met redes, die grootte van x .

(5)

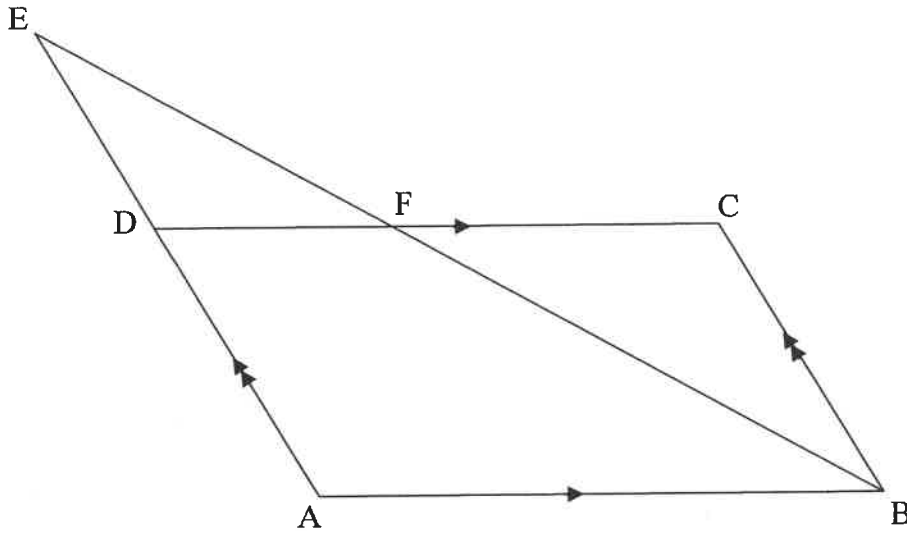
- 8.3 In die diagram is O die middelpunt van die groter sirkel. OB is 'n middellyn van die kleiner sirkel. Koord AB van die groter sirkel sny die kleiner sirkel by M en B .



- 8.3.1 Skryf die grootte van \hat{OMB} neer. Verskaf 'n rede. (2)
- 8.3.2 Indien $AB = \sqrt{300}$ eenhede en $OM = 5$ eenhede, bereken, met redes, die lengte van OB . (4)
- [16]

VRAAG 9

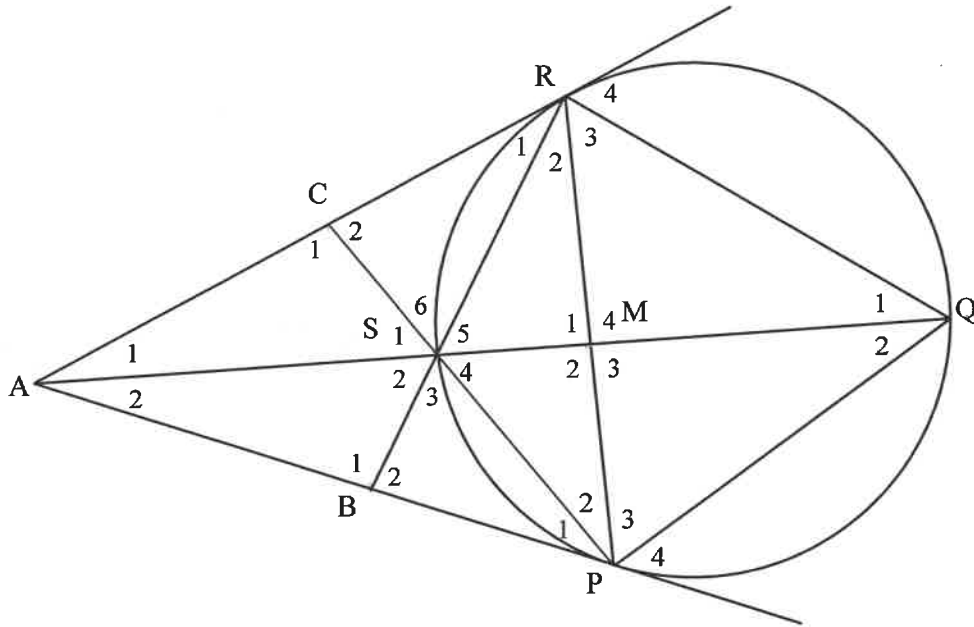
In die diagram is ABCD 'n parallelogram met $AB = 14$ eenhede. AD word na E verleng sodat $AD : DE = 4 : 3$. EB sny DC by F. $EB = 21$ eenhede.



- 9.1 Bereken, met redes, die lengte van FB. (3)
 - 9.2 Bewys, met redes, dat $\triangle EDF \parallel \triangle EAB$. (3)
 - 9.3 Bereken, met redes, die lengte van FC. (3)
- [9]**

VRAAG 10

In die diagram is PQRS 'n koordevierhoek sodat $PQ = PR$. Die raaklyne aan die sirkel deur P en R sny QS verleng by A. RS is verleng om raaklyn AP by B te sny. PS is verleng om raaklyn AR by C te sny. PR en QS sny by M.



Bewys, met redes, dat:

- 10.1 $\hat{S}_3 = \hat{S}_4$ (5)
- 10.2 SMRC 'n koordevierhoek is (4)
- 10.3 RP 'n raaklyn van die sirkel is wat deur P, S en A by P gaan (6)

[15]

TOTAAL: 150

INLIGTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG
1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.	1. <i>Skryf jou eksamennommer en sentrumnummer duidelik in die ruimtes soos verskaf en plak jou stafieskodeplakker in die ruimte soos verskaf.</i>
2. Remember that your own name (or the name of your school) may not appear anywhere on or in this answer book.	2. <i>Onthou dat jou eie naam (of die naam van jou skool) nie op of in hierdie antwoordeboek mag voorkom nie.</i>
3. Answer ALL questions in the spaces provided.	3. <i>Beantwoord ALLE vrae in die ruimtes wat voorsien is.</i>
4. No pages may be torn from this answer book.	4. <i>Geen bladsye mag uit hierdie antwoordeboek geskeur word nie.</i>
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	5. <i>Lees die instruksies wat op jou eksamenrooster gedruk is, sorgvuldig deur, asook enige ander instruksies wat op elke eksamenvraestel gegee word.</i>
6. Candidates may not retain an answer book or remove it from the examination room.	6. <i>Geen antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word nie.</i>
7. Answers must be written in black/blue ink as distinctly as possible. Do not write in the margins.	7. <i>Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.</i>
8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.	8. <i>Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.</i>
9. If you require additional space for your answers: 9.1 Use the additional space provided at the end of the answer book. 9.2 When answering a question in the additional space, indicate clearly the question number in the column on the LHS. 9.3 Rule off after each answer.	9. <i>In geval jy bykomende ruimte benodig vir jou antwoorde:</i> 9.1 <i>Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek voorsien word.</i> 9.2 <i>As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan.</i> 9.3 <i>Trek 'n lyn na elke antwoord.</i>
10. Draw a neat line through any work/rough work that must not be marked.	10. <i>Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.</i>

QUESTION/VRAAG 1

Distance travelled (in km)/ Afstand gery (in km) (x)	180	200	400	600	170	350	270	300
Amount of rest time (in minutes)/ Hoeveelheid rustyd (in minute) (y)	20	25	55	120	15	50	40	45

	Solution/Oplossing	Marks Punte
1.1		(3)
1.2		(2)
1.3		(1)
1.4		(1)

1.5

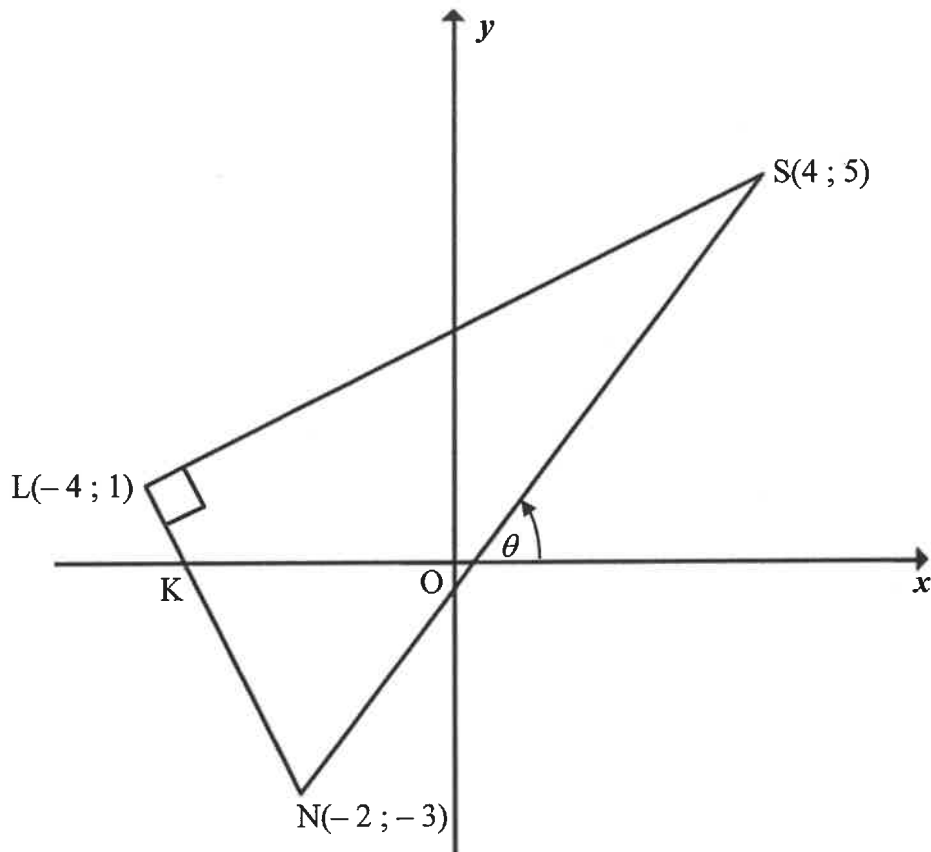
100	150	130	200	50	180	200	190
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	Solution/Oplissing	Marks Punte
1.5.1		(2)
1.5.2		(1)
1.5.3		(2)
		[12]

QUESTION/VRAAG 2

	Solution/Oplissing	Marks Punte																		
2.1	<table border="1"> <thead> <tr> <th>Number of glasses of water drank per day/ Aantal glase water per dag gedrink</th> <th>Number of staff members/ Getal personeellede</th> <th>Cumulative frequency/ Kumulatiewe frekwensie</th> </tr> </thead> <tbody> <tr> <td>$0 \leq x < 2$</td> <td>5</td> <td></td> </tr> <tr> <td>$2 \leq x < 4$</td> <td>15</td> <td></td> </tr> <tr> <td>$4 \leq x < 6$</td> <td>13</td> <td></td> </tr> <tr> <td>$6 \leq x < 8$</td> <td>5</td> <td></td> </tr> <tr> <td>$8 \leq x < 10$</td> <td>2</td> <td></td> </tr> </tbody> </table>	Number of glasses of water drank per day/ Aantal glase water per dag gedrink	Number of staff members/ Getal personeellede	Cumulative frequency/ Kumulatiewe frekwensie	$0 \leq x < 2$	5		$2 \leq x < 4$	15		$4 \leq x < 6$	13		$6 \leq x < 8$	5		$8 \leq x < 10$	2		(2)
Number of glasses of water drank per day/ Aantal glase water per dag gedrink	Number of staff members/ Getal personeellede	Cumulative frequency/ Kumulatiewe frekwensie																		
$0 \leq x < 2$	5																			
$2 \leq x < 4$	15																			
$4 \leq x < 6$	13																			
$6 \leq x < 8$	5																			
$8 \leq x < 10$	2																			
2.2		(1)																		
2.3		(1)																		
2.4		(4)																		
		[8]																		

QUESTION/VRAAG 3

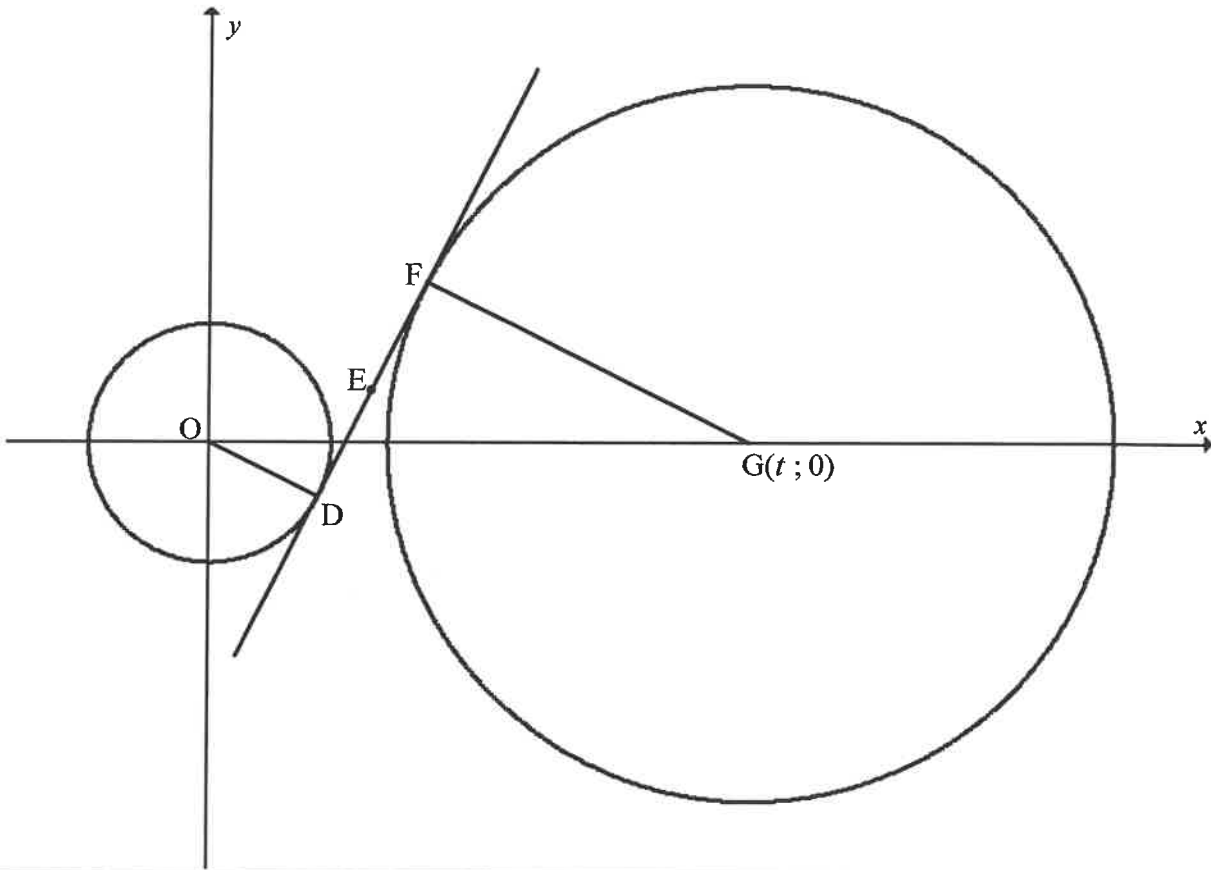


	Solution/Oplissing	Marks Punte
3.1		(2)
3.2		(2)

	Solution/Oplissing	Marks Punte
3.3		(2)
3.4		(3)
3.5		(3)
3.6		(3)

	Solution/<i>Oplossing</i>	Marks <i>Punte</i>
3.7		
3.8		
		[20]

QUESTION/VRAAG 4



	Solution/Oplissing	Marks Punte
4.1		(2)
4.2		(3)

	Solution/Oplossing	Marks Punte
4.3		(4)
4.4		(3)
4.5		(4)
4.6		(4)
		[20]

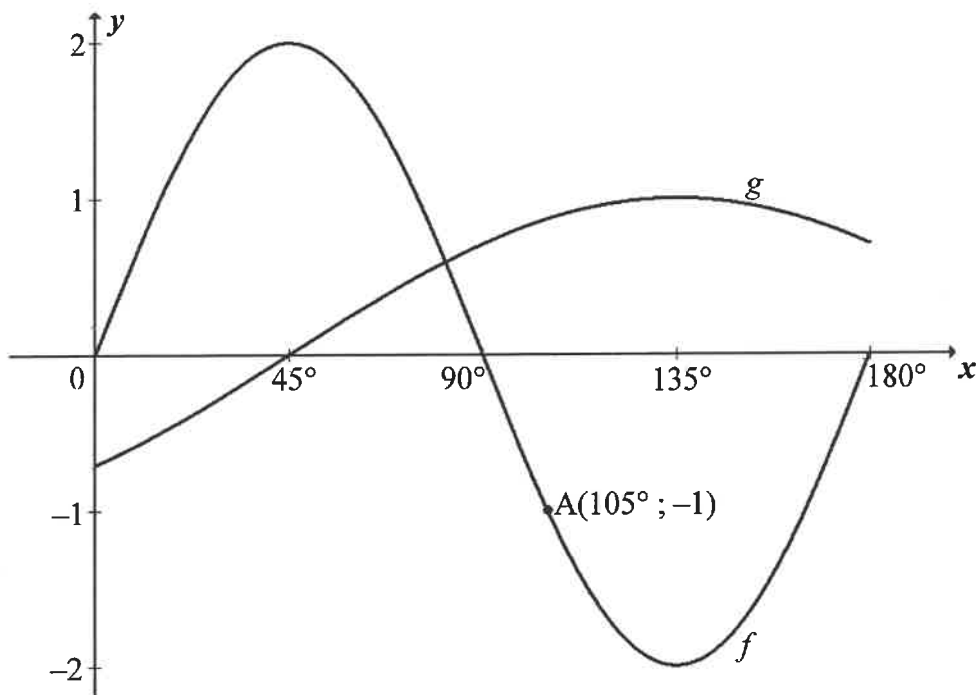
QUESTION/VRAAG 5

	Solution/Oplissing	Marks Punte
5.1.1		(3)
5.1.2		(3)
5.1.3		(3)

	Solution/Oplossing	Marks Punte
5.2.1		(4)
5.2.2		(2)
5.2.3		(2)
5.3.1		(3)

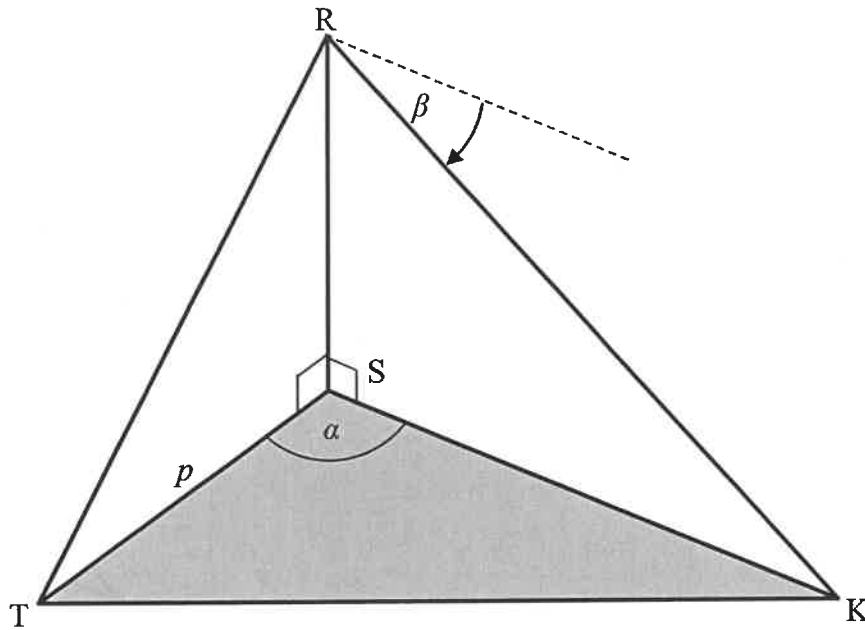
	Solution/Oplissing	Marks Punte
5.3.2		(5)
5.4		(6)
		[31]

QUESTION/VRAAG 6



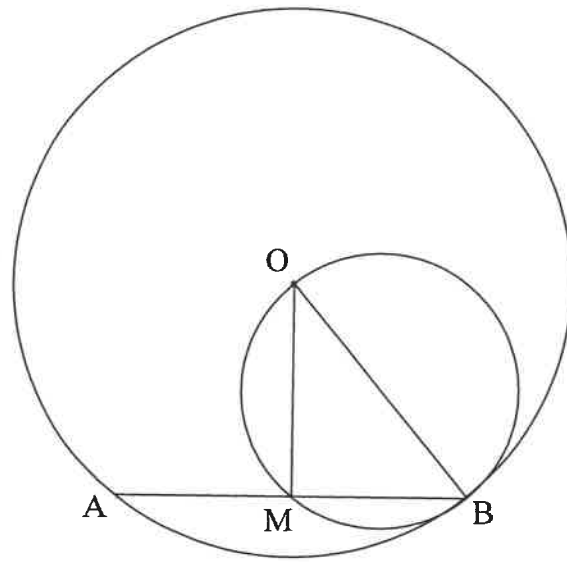
	Solution/Oplissing	Marks Punte
6.1		(1)
6.2		(2)
6.3.1		(2)
6.3.2		(2)
6.4		(3)
6.5		(2)
		[12]

QUESTION/VRAAG 7



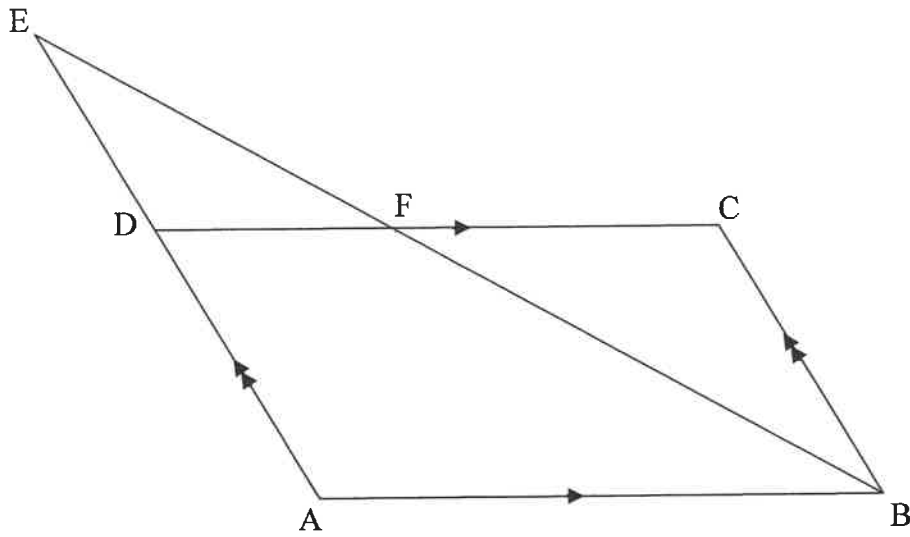
	Solution/Oplissing	Marks Punte
7.1		(2)
7.2		(2)
7.3		(3)
		[7]

8.3



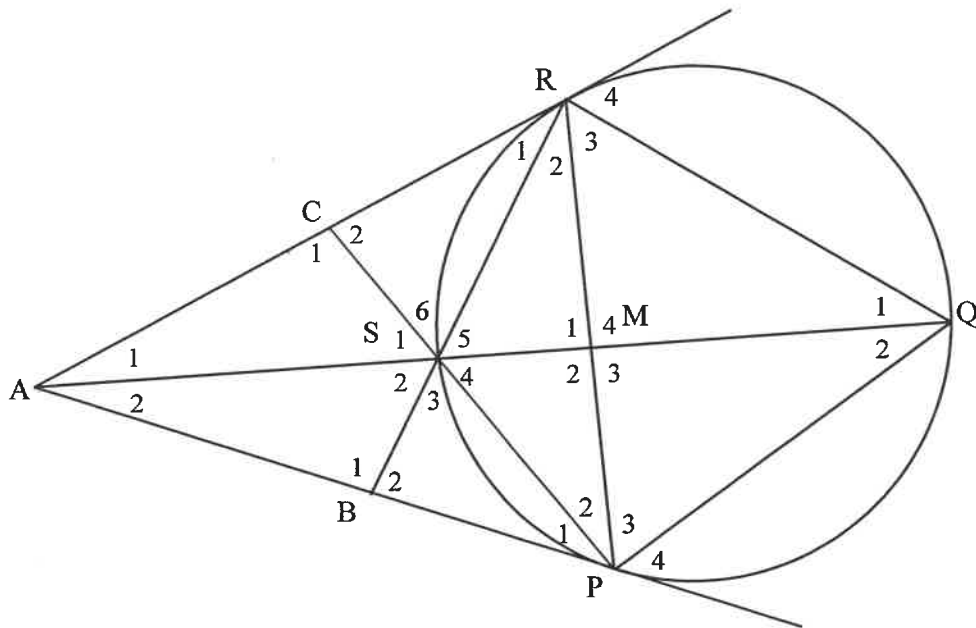
	Solution/Oplissing	Marks Punte
8.3.1		(2)
8.3.2		(4)
		[16]

QUESTION/VRAAG 9



	Solution/Oplissing	Marks Punte
9.1		(3)
9.2		(3)
9.3		(3)
		(3)
		[9]

QUESTION/VRAAG 10



	Solution/Oplissing	Marks Punte
10.1		(5)
10.2		(4)

	Solution/Oplissing	Marks Punte
10.3		
(6) [15]		



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE 12/GRAAD 12

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2023

MARKING GUIDELINES/NASIENRIGLYNE

MARKS/PUNTE: 150

**These marking guidelines consist of 23 pages./
*Hierdie nasienriglyne bestaan uit 23 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason)
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	Award a mark if statement AND reason are both correct
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/VRAAG 1

1.1	$a = -23,846\dots$ $b = 0,227\dots$ $\hat{y} = -23,85 + 0,23x$	✓ $a = -23,846\dots$ ✓ $b = 0,227\dots$ ✓ equation (3)
1.2	$\hat{y} = -23,85 + 0,23(550)$ $y = 102,65$ OR $y = 101,02$	✓ substitution of 550 ✓ answer (2) ✓✓ $y = 101,02$ (calculator) (2)
1.3	$r = 0,98$	✓ $r = 0,98$ (1)
1.4	Very strong positive correlation	✓ strong positive (1)

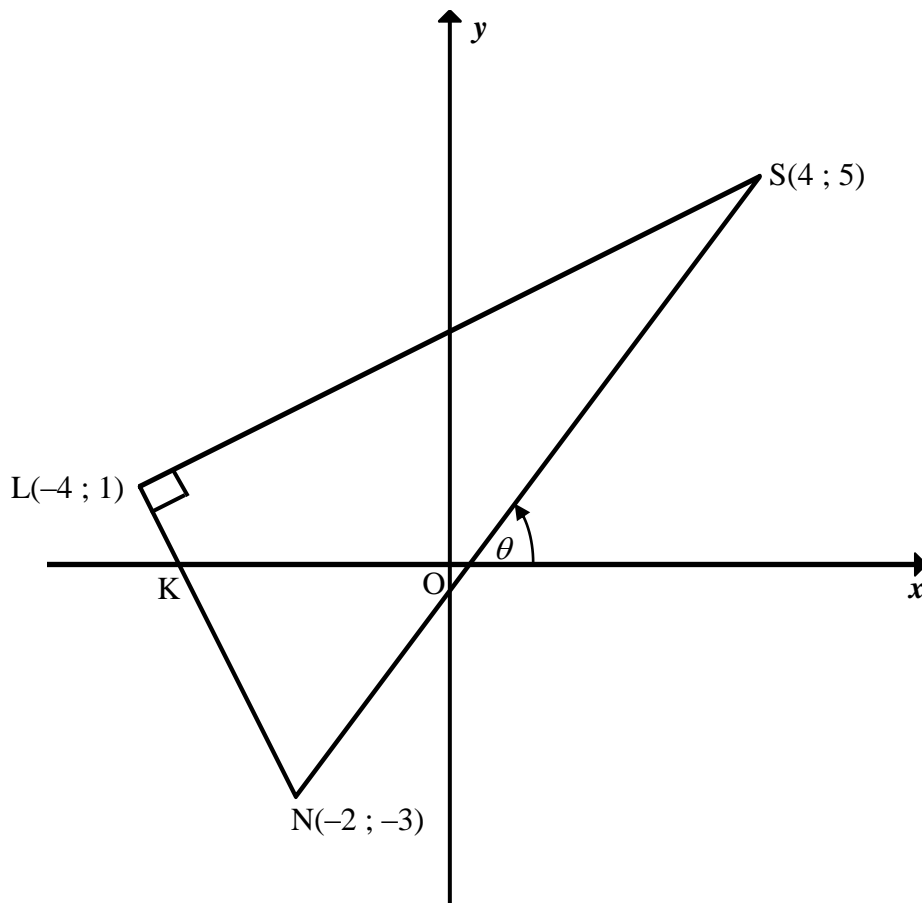
50	100	130	150	180	190	200	200
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1.5.1	$\bar{x} = \frac{1200}{8}$ $\bar{x} = 150$ OR $\bar{x} = 150$	✓ 1200 ✓ answer (2) ✓✓ $\bar{x} = 150$ (2)
1.5.2	$\sigma = 50,50$	✓ $\sigma = 50,50$ (1)
1.5.3	$\bar{x} - \sigma$ $= 150 - 50,50$ $= 99,50$ \therefore 1 stop	✓ calculation of $\bar{x} - \sigma$ ✓ answer (2)
		[12]

QUESTION/VRAAG 2

2.1	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Number of glasses of water per day</th> <th>Number of staff members</th> <th>Cumulative frequency</th> </tr> </thead> <tbody> <tr> <td>$0 \leq x < 2$</td> <td>5</td> <td>5</td> </tr> <tr> <td>$2 \leq x < 4$</td> <td>15</td> <td>20</td> </tr> <tr> <td>$4 \leq x < 6$</td> <td>13</td> <td>33</td> </tr> <tr> <td>$6 \leq x < 8$</td> <td>5</td> <td>38</td> </tr> <tr> <td>$8 \leq x < 10$</td> <td>2</td> <td>40</td> </tr> </tbody> </table>	Number of glasses of water per day	Number of staff members	Cumulative frequency	$0 \leq x < 2$	5	5	$2 \leq x < 4$	15	20	$4 \leq x < 6$	13	33	$6 \leq x < 8$	5	38	$8 \leq x < 10$	2	40	<p>✓ 5; 20</p> <p>✓ 40</p> <p style="text-align: right;">(2)</p>
Number of glasses of water per day	Number of staff members	Cumulative frequency																		
$0 \leq x < 2$	5	5																		
$2 \leq x < 4$	15	20																		
$4 \leq x < 6$	13	33																		
$6 \leq x < 8$	5	38																		
$8 \leq x < 10$	2	40																		
2.2	40 staff members	<p>✓ answer</p> <p style="text-align: right;">(1)</p>																		
2.3	33 staff members	<p>✓ answer</p> <p style="text-align: right;">(1)</p>																		
2.4	$\bar{x} = \frac{\left(1 \times \left(5 + \frac{k}{2}\right)\right) + (3 \times 15) + \left(5 \times \left(13 + \frac{k}{2}\right)\right) + (7 \times 5) + (9 \times 2)}{40 + k} = 4$ $5 + \frac{k}{2} + 45 + 65 + \frac{5k}{2} + 35 + 18 = 160 + 4k$ $3k + 168 = 160 + 4k$ $k = 8$ <p>OR</p> $\bar{x} = \frac{(1 \times 5) + (15 \times 3) + (13 \times 5) + (5 \times 7) + (2 \times 9)}{40}$ $= 4,2$ $\bar{x}_{\text{old}} - \bar{x}_{\text{current}} = 4,2 - 4$ $= 0,2$ $\therefore 0,2 \times 40$ $= 8 \text{ teachers}$	<p>✓ answer from Q2.2 + k</p> <p>✓ $\left(1 \times \left(5 + \frac{k}{2}\right)\right)$</p> <p>✓ $\left(5 \times \left(13 + \frac{k}{2}\right)\right)$</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p> <p>✓ 4,2</p> <p>✓ $\bar{x}_{\text{old}} - 4$</p> <p>✓ difference</p> <p>✓ answer</p> <p style="text-align: right;">(4)</p>																		
		[8]																		

QUESTION/VRAAG 3

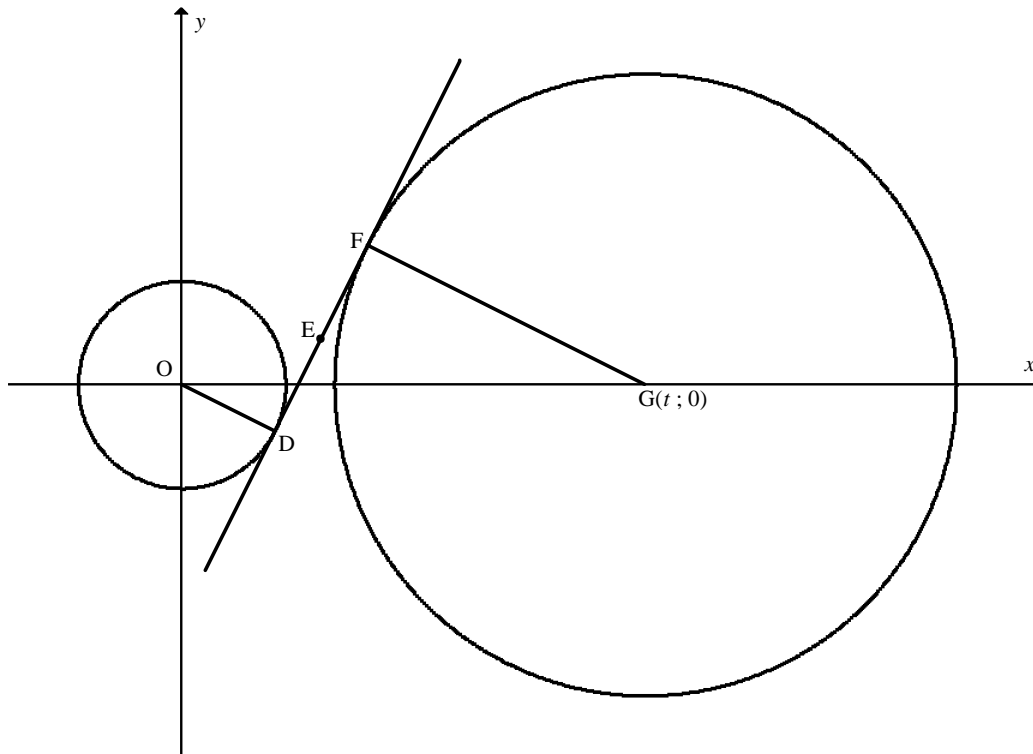


3.1	$SL = \sqrt{(x_S - x_L)^2 + (y_S - y_L)^2}$ $SL = \sqrt{(4 - (-4))^2 + (5 - 1)^2}$ $SL = \sqrt{80} = 4\sqrt{5} = 8,94 \text{ units}$	✓ substitution of S and L into correct formula ✓ answer (2)
3.2	$m_{SN} = \frac{5 - (-3)}{4 - (-2)}$ $m_{SN} = \frac{4}{3}$	✓ substitution of S and N into correct formula ✓ answer (2)
3.3	$m = \tan \theta = \frac{4}{3}$ $\theta = 53,13^\circ$	✓ $\tan \theta = m_{SN}$ ✓ answer (2)
3.4	$m_{LN} = \frac{1 - (-3)}{-4 - (-2)}$ $m_{LN} = -2$ $\widehat{L\hat{K}O} = 116,565\dots^\circ$ $\widehat{L\hat{N}S} = 116,565\dots^\circ - 53,13^\circ$ $\widehat{L\hat{N}S} = 63,44^\circ$	✓ $m_{LN} = -2$ ✓ size of $\widehat{L\hat{K}O}$ ✓ answer (3)

	<p>OR</p> <p>SN = 10 units $\sin \hat{LNS} = \frac{4\sqrt{5}}{10}$ $\hat{LNS} = 63,44^\circ$</p> <p>OR</p> <p>LN = $2\sqrt{5}$ units $\tan \hat{LNS} = \frac{4\sqrt{5}}{2\sqrt{5}}$ $\hat{LNS} = 63,44^\circ$</p> <p>OR</p> <p>SN = 10 units LN = $2\sqrt{5}$ units</p> <p>$\cos \hat{LNS} = \frac{2\sqrt{5}}{10}$ $\hat{LNS} = 63,44^\circ$</p>	<p>✓ SN = 10 units ✓ correct trig ratio ✓ answer (3)</p> <p>✓ LN = $2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)</p> <p>✓ SN = 10 units and LN = $2\sqrt{5}$ units ✓ correct trig ratio ✓ answer (3)</p>
<p>3.5</p>	<p>$m = \frac{4}{3}$ $1 = \frac{4}{3}(-4) + c$ $c = \frac{19}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$</p> <p>OR</p> <p>$y - 1 = \frac{4}{3}(x - (-4))$ $y - 1 = \frac{4}{3}x + \frac{16}{3}$ $y = \frac{4}{3}x + \frac{19}{3}$</p>	<p>✓ m_{SN} ✓ substitution of m_{SN} & L ✓ equation (3)</p>
<p>3.6</p>	<p>SL = $4\sqrt{5}$ $LN = \sqrt{(-4 - (-2))^2 + (1 - (-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$</p> <p>Area $\triangle LSN = \frac{1}{2}(4\sqrt{5})(2\sqrt{5})$ = 20 units²</p> <p>OR</p>	<p>✓ $LN = \sqrt{20} = 2\sqrt{5}$ ✓ substitution into formula ✓ answer (3)</p>

	<p>SN = 10 units</p> $LN = \sqrt{(-4 - (-2))^2 + (1 - (-3))^2}$ $LN = \sqrt{20} = 2\sqrt{5}$ $\text{Area } \triangle LSN = \frac{1}{2}(10)(2\sqrt{5})\sin 63,44^\circ$ $= 20 \text{ units}^2$	<p>✓ $LN = \sqrt{20} = 2\sqrt{5}$</p> <p>✓ substitution into formula</p> <p>✓ answer</p> <p>(3)</p>
<p>3.7</p>	<p>$\hat{L} = 90^\circ$ SN is a diameter of circle S, L, N [chord subtends 90° OR converse \angle in semi-circle]</p> <p>Centre of circle = $P\left(\frac{4+(-2)}{2}; \frac{5+(-3)}{2}\right)$ $= P(1; 1)$</p> <p>OR</p> <p>Let the coordinates of P be (a ; b). Then, PL = PN: $(-4 - a)^2 + (1 - b)^2 = (-2 - a)^2 + (-3 - b)^2$ $a - 2b = -1$equation 1</p> <p>If PS = PN, then: $4a + 2b = 6$ equation 2 Solving simultaneously yields: $a = 1$ and $b = 1$ and P(1 ; 1)</p> <p>OR</p> <p>If PL = PN, then: $a - 2b = -1$equation 1 If PS = PL, then: $2a + b = 3$equation 2 Solving simultaneously yields: $a = 1$ and $b = 1$ and P(1 ; 1)</p>	<p>✓ SN is a diameter of circle S, L, N</p> <p>✓ x-value ✓ y-value</p> <p>(3)</p> <p>✓ 2 correct linear equations ✓ x-value ✓ y-value</p> <p>(3)</p> <p>✓ 2 correct linear equations ✓ x-value ✓ y-value</p> <p>(3)</p>
<p>3.8</p>	<p>$\hat{LPN} = \theta = 53,13^\circ$ [alt \angles; LP x-axis] $\therefore \hat{LPS} = 126,87^\circ$</p> <p>OR</p> <p>$\hat{LNS} = 63,44^\circ$ $\therefore \hat{LPS} = 126,88^\circ$ [\angle at centre = $2 \times \angle$ at circumference]</p> <p>OR</p> <p>$\hat{LSN} = 26,56^\circ$ [sum of \angles in Δ] $\hat{SLP} = 26,56^\circ$ [\angles opp equal radii] $\therefore \hat{LPS} = 126,88^\circ$ [sum of \angles in Δ]</p> <p>OR</p> <p>$(4\sqrt{5})^2 = 5^2 + 5^2 - 2(5)(5)\cos \hat{LPS}$ $\cos \hat{LPS} = -\frac{3}{5}$ $\therefore \hat{LPS} = 126,87^\circ$</p>	<p>✓ \hat{LPN}</p> <p>✓ answer</p> <p>(2)</p> <p>✓ \hat{LNS}</p> <p>✓ answer</p> <p>(2)</p> <p>✓ \hat{LSN}</p> <p>✓ answer</p> <p>(2)</p> <p>✓ correct substitution into cosine formula</p> <p>✓ answer</p> <p>(2)</p>
<p>[20]</p>		

QUESTION/VRAAG 4

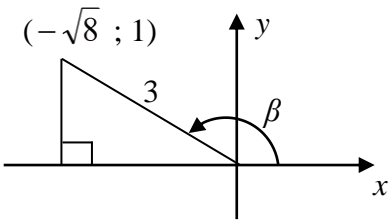


<p>4.1</p>	$D(p ; -2)$ $x^2 + y^2 = 20$ $p^2 + (-2)^2 = 20$ $p^2 = 16$ $p = \pm 4$ $p = 4$	<p>✓ substitution of point $D(p ; -2)$</p> <p>✓ $p^2 = 16$</p> <p style="text-align: right;">(2)</p>																
<p>4.2</p>	<table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;">$\frac{4 + x_F}{2} = 6$</td> <td style="width: 50%; border: none;">$\frac{-2 + y_F}{2} = 2$</td> </tr> <tr> <td style="border: none;">$x_F = 8$</td> <td style="border: none;">$y_F = 6$</td> </tr> <tr> <td colspan="2" style="border: none;">F(8;6)</td> </tr> <tr> <td colspan="2" style="border: none;">OR</td> </tr> <tr> <td style="border: none;">$x_E - x_D = 6 - 4$</td> <td style="border: none;">$y_E - y_D = 2 - (-2)$</td> </tr> <tr> <td style="border: none;">$= 2$</td> <td style="border: none;">$= 4$</td> </tr> <tr> <td style="border: none;">$x_F = 6 + 2 = 8$</td> <td style="border: none;">$y_F = 2 + 4 = 6$</td> </tr> <tr> <td colspan="2" style="border: none;">F(8;6)</td> </tr> </table>	$\frac{4 + x_F}{2} = 6$	$\frac{-2 + y_F}{2} = 2$	$x_F = 8$	$y_F = 6$	F(8;6)		OR		$x_E - x_D = 6 - 4$	$y_E - y_D = 2 - (-2)$	$= 2$	$= 4$	$x_F = 6 + 2 = 8$	$y_F = 2 + 4 = 6$	F(8;6)		<p>✓ method</p> <p>✓ x-value ✓ y-value</p> <p style="text-align: right;">(3)</p> <p>✓ method</p> <p>✓ x-value ✓ y-value</p> <p style="text-align: right;">(3)</p>
$\frac{4 + x_F}{2} = 6$	$\frac{-2 + y_F}{2} = 2$																	
$x_F = 8$	$y_F = 6$																	
F(8;6)																		
OR																		
$x_E - x_D = 6 - 4$	$y_E - y_D = 2 - (-2)$																	
$= 2$	$= 4$																	
$x_F = 6 + 2 = 8$	$y_F = 2 + 4 = 6$																	
F(8;6)																		

<p>4.3</p>	$m_{DE} = \frac{-2-2}{4-6}$ $m_{DE} = 2$ $-2 = 2(4) + c \quad \text{OR} \quad y - (-2) = 2(x-4)$ $c = -10 \quad \quad \quad y + 2 = 2x - 8$ $y = 2x - 10 \quad \quad \quad y = 2x - 10$ <p>OR</p> $m_{OD} = -\frac{2}{4} = -\frac{1}{2}$ $\therefore m_{DE} = 2 \quad \quad \quad [\tan \perp \text{radius}]$ $-2 = 2(4) + c \quad \text{OR} \quad y - (-2) = 2(x-4)$ $c = -10 \quad \quad \quad y + 2 = 2x - 8$ $y = 2x - 10 \quad \quad \quad y = 2x - 10$	<ul style="list-style-type: none"> ✓ correct substitution ✓ gradient of DE, DF or EF ✓ substitution of point D(4 ; -2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4) ✓ correct gradient of OD ✓ gradient of DE ✓ substitution of point D(4 ; -2) or E(6 ; 2) or F(8 ; 6) ✓ answer (4)
<p>4.4</p>	$m_{DE} = 2$ $\therefore m_{GF} = -\frac{1}{2} \quad \quad \quad [\tan \perp \text{radius}]$ $\frac{0-6}{t-8} = -\frac{1}{2}$ $-(t-8) = 2(-6)$ $t = 20$ <p>OR</p> $y = 2x - 10$ $0 = 2x - 10$ $x = 5$ $A(5 ; 0)$ <p>In $\triangle AFG$: $FA \perp FG$</p> $FA^2 = (6-0)^2 + (8-5)^2 = 45$ $FG^2 = (t-8)^2 + (0-6)^2$ $= t^2 - 16t + 100$ $GA^2 = (t-5)^2$ $= t^2 - 10t + 25$ $\therefore GA^2 = GF^2 + FA^2$ $t^2 - 10t + 25 = t^2 - 16t + 100 + 45$ $6t = 120$ $t = 20$	<ul style="list-style-type: none"> ✓ correct gradient of GF ✓ substitution of F ✓ answer (3) ✓ x-intercept of DF ✓ substitution into Pythagoras ✓ answer (3)

<p>4.5</p>	<p>F(8;6) G(20 ; 0)</p> $(8-20)^2 + (6-0)^2 = r^2$ $r^2 = 180$ $(x-20)^2 + y^2 = 180$ $x^2 + y^2 - 40x + 220 = 0$	<p>✓ substitution of F and G ✓ value of r^2</p> <p>✓ equation of circle ✓ answer</p> <p>(4)</p>
<p>4.6</p>	<p>Smaller circle $r = 2\sqrt{5}$ Larger circle $r = 6\sqrt{5}$</p> <p>G(20 ; 0)</p> $k = 20 - (6\sqrt{5} - 2\sqrt{5}) \quad \text{or} \quad k = 20 + (6\sqrt{5} - 2\sqrt{5})$ $= 20 - 4\sqrt{5} \quad \quad \quad = 20 + 4\sqrt{5}$ $= 11,06 \text{ units} \quad \quad \quad = 28,94 \text{ units}$ <p>OR</p> <p>Smaller circle $r = 2\sqrt{5}$</p> $k = 2(2\sqrt{5}) + 20 - 8\sqrt{5} \quad \text{or} \quad k = 2(6\sqrt{5}) + 20 - 8\sqrt{5}$ $= 20 - 4\sqrt{5} \quad \quad \quad = 20 + 4\sqrt{5}$ $= 11,06 \text{ units} \quad \quad \quad = 28,94 \text{ units}$ <p>OR</p> $x^2 + y^2 - 40x + 220 = 0$ $y = 0$ $\therefore x^2 - 40x + 220 = 0$ $\therefore x = 20 + 6\sqrt{5} \quad \text{or} \quad x = 20 - 6\sqrt{5}$ $\therefore k = 20 + 6\sqrt{5} - \sqrt{20} \quad \text{or} \quad k = 20 - 6\sqrt{5} + \sqrt{20}$ $\therefore k = 20 + 4\sqrt{5} \quad \quad \quad \therefore k = 20 - 4\sqrt{5}$ $= 11,06 \text{ units} \quad \quad \quad = 28,94 \text{ units}$	<p>✓ $r = 2\sqrt{5}$</p> <p>✓ method</p> <p>✓ answer ✓ answer</p> <p>(4)</p> <p>✓ $r = 2\sqrt{5}$</p> <p>✓ method</p> <p>✓ answer ✓ answer</p> <p>(4)</p> <p>✓ x-intercepts ✓ method ✓ answer ✓ answer</p> <p>(4)</p>
		<p>[20]</p>

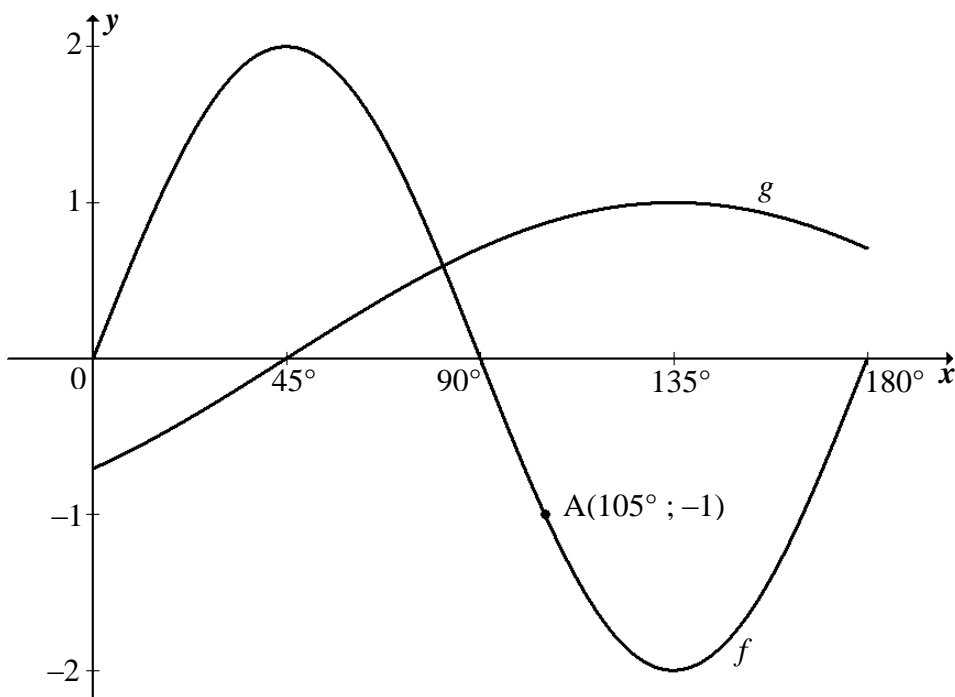
QUESTION/VRAAG 5

<p>5.1.1</p>	<p>$\sin \beta = \frac{1}{3}$ $\beta \in (90^\circ; 270^\circ)$</p>  <p>$x = -\sqrt{8} = -2\sqrt{2}$</p> <p>$\cos \beta$ $= \frac{-2\sqrt{2}}{3}$</p> <p>OR</p> <p>$\sin \beta = \frac{1}{3}$ $\beta \in (90^\circ; 270^\circ)$</p> <p>$\cos^2 \beta = 1 - \sin^2 \beta$</p> <p>$\cos^2 \beta = 1 - \left(\frac{1}{3}\right)^2$</p> <p>$\cos^2 \beta = \frac{8}{9}$</p> <p>$\cos \beta = \frac{-\sqrt{8}}{3}$ $= \frac{-2\sqrt{2}}{3}$</p>	<p>✓ $x^2 + y^2 = r^2$</p> <p>✓ $x = -2\sqrt{2}$</p> <p>✓ answer (3)</p> <p>✓ square identity</p> <p>✓ $\cos^2 \beta$</p> <p>✓ answer (3)</p>
<p>5.1.2</p>	<p>$\sin 2\beta$ $= 2 \sin \beta \cos \beta$ $= 2 \left(\frac{1}{3}\right) \left(\frac{-\sqrt{8}}{3}\right)$ $= \frac{-2\sqrt{8}}{9}$ OR $2 \left(\frac{-2\sqrt{2}}{9}\right)$ $= \frac{-4\sqrt{2}}{9}$</p>	<p>✓ double angle</p> <p>✓ substitution</p> <p>✓ answer (3)</p>
<p>5.1.3</p>	<p>$\cos (450^\circ - \beta)$ $= \cos (90^\circ - \beta)$ $= \sin \beta$ $= \frac{1}{3}$</p> <p>OR</p>	<p>✓ $\cos (90^\circ - \beta)$</p> <p>✓ co-ratio</p> <p>✓ answer (3)</p>

	$\begin{aligned} &\cos(450^\circ - \beta) \\ &= \cos 450^\circ \cos \beta + \sin 450^\circ \sin \beta \\ &= \cos 90^\circ \cos \beta + \sin 90^\circ \sin \beta \\ &= \sin \beta \\ &= \frac{1}{3} \end{aligned}$	<p>✓ expansion ✓ reduction ✓ answer</p> <p>(3)</p>
<p>5.2.1</p>	<p>LHS = $\frac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$ $= \frac{\cos^2 x (\cos^2 x + \sin^2 x)}{1 + \sin x}$ $= \frac{1 - \sin^2 x}{1 + \sin x}$ $= \frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x}$ $= 1 - \sin x$ $= \text{RHS}$</p> <p>OR</p> <p>LHS = $\frac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$ $= \frac{\cos^4 x + (1 - \cos^2 x) \cos^2 x}{1 + \sin x}$ $= \frac{\cos^4 x + \cos^2 x - \cos^4 x}{1 + \sin x}$ $= \frac{1 - \sin^2 x}{1 + \sin x}$ $= \frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x}$ $= 1 - \sin x$ $= \text{RHS}$</p> <p>OR</p> <p>RHS = $1 - \sin x$ $= (1 - \sin x) \times \frac{1 + \sin x}{1 + \sin x}$ $= \frac{1 - \sin^2 x}{1 + \sin x}$ $= \frac{\cos^2 x}{1 + \sin x}$ $= \frac{\cos^2 x (\sin^2 x + \cos^2 x)}{1 + \sin x}$ $= \frac{\cos^4 x + \cos^2 x \cdot \sin^2 x}{1 + \sin x}$ $= \text{LHS}$</p>	<p>✓ factors ✓ $\sin^2 x + \cos^2 x = 1$ ✓ $\cos^2 x = 1 - \sin^2 x$ ✓ factors</p> <p>(4)</p> <p>✓ $\sin^2 x = 1 - \cos^2 x$ ✓ expansion ✓ $\cos^2 x = 1 - \sin^2 x$ ✓ factors</p> <p>(4)</p> <p>✓ $\times \frac{1 + \sin x}{1 + \sin x}$ ✓ product ✓ $1 - \sin^2 x = \cos^2 x$ ✓ $1 = \cos^2 x + \sin^2 x$</p> <p>(4)</p>

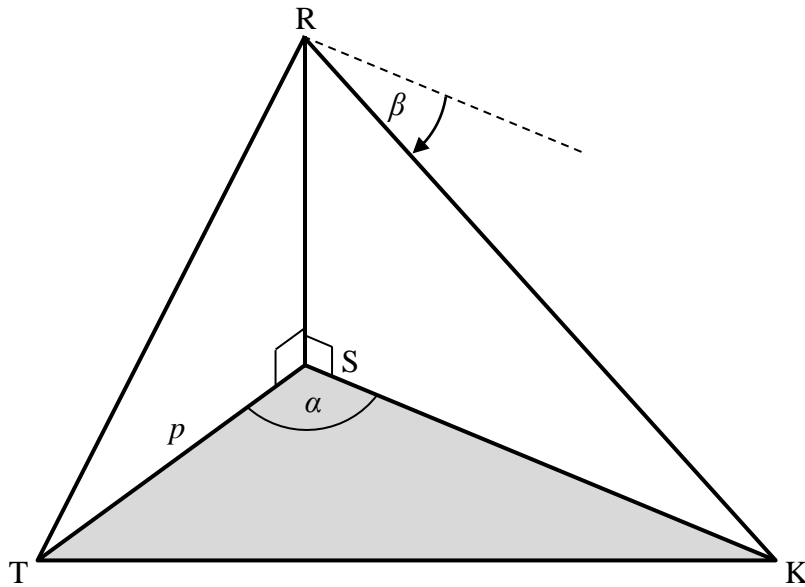
<p>5.4</p>	$\frac{\sin 3x + \sin x}{\cos 2x + 1}$ $= \frac{\sin(2x + x) + \sin(2x - x)}{\cos 2x + 1}$ $= \frac{\sin 2x \cos x + \cos 2x \sin x + \sin 2x \cos x - \cos 2x \sin x}{2 \cos^2 x - 1 + 1}$ $= \frac{2 \sin 2x \cos x}{2 \cos^2 x}$ $= \frac{2(2 \sin x \cos x) \cos x}{2 \cos^2 x}$ $= \frac{4 \sin x \cos^2 x}{2 \cos^2 x}$ $= 2 \sin x$ <p>OR</p> $\frac{\sin 3x + \sin x}{\cos 2x + 1}$ $= \frac{\sin(2x + x) + \sin x}{2 \cos^2 x - 1 + 1}$ $= \frac{\sin 2x \cos x + \cos 2x \sin x + \sin x}{2 \cos^2 x}$ $= \frac{2 \sin x \cos x \cos x + \cos 2x \sin x + \sin x}{2 \cos^2 x}$ $= \frac{\sin x(2 \cos^2 x + \cos 2x + 1)}{2 \cos^2 x}$ $= \frac{\sin x(2 \cos^2 x + 2 \cos^2 x - 1 + 1)}{2 \cos^2 x}$ $= 2 \sin x$	<p>✓ $3x = (2x + x)$</p> <p>✓ expansion</p> <p>✓ double angle of $\cos 2x$</p> <p>✓ simplification</p> <p>✓ $\sin 2x = 2 \sin x \cos x$</p> <p>✓ answer (6)</p> <p>✓ $3x = (2x + x)$</p> <p>✓ double angle of $\cos 2x$</p> <p>✓ expansion</p> <p>✓ $\sin 2x = 2 \sin x \cos x$</p> <p>✓ common factor</p> <p>✓ answer (6)</p>
<p>[31]</p>		

QUESTION/VRAAG 6



6.1	Period = 180°	✓ 180° (1)
6.2	$y \in \left[-\frac{\sqrt{2}}{2}; 1\right]$ OR $y \in [-0,71; 1]$ OR $-\frac{\sqrt{2}}{2} \leq y \leq 1$	✓ $-\frac{\sqrt{2}}{2}$ ✓ $y \in \left[-\frac{\sqrt{2}}{2}; 1\right]$ (2)
6.3.1	$x \in (45^\circ; 90^\circ)$ OR $45^\circ < x < 90^\circ$	✓✓ $x \in (45^\circ; 90^\circ)$ (2)
6.3.2	$f(x) + 1 \leq 0$ $f(x) \leq -1$ $x \in [105^\circ; 165^\circ]$ OR $105^\circ \leq x \leq 165^\circ$	✓✓ $x \in [105^\circ; 165^\circ]$ (2)
6.4	$p(x) = -2 \sin 2x$ $-2 \sin 2x = -1$ OR $2 \sin 2x = 1$ $k = 15^\circ$ or $k = 75^\circ$	✓ reading off $f(x) = 1$ or $-f(x) = -1$ ✓ 15° ✓ 75° (3)
6.5	$g(x) = -\cos(x + 45^\circ)$ $h(x) = -\cos(x + 90^\circ)$ $h(x) = \sin x$	✓ $-\cos(x + 90^\circ)$ ✓ answer (2)
		[12]

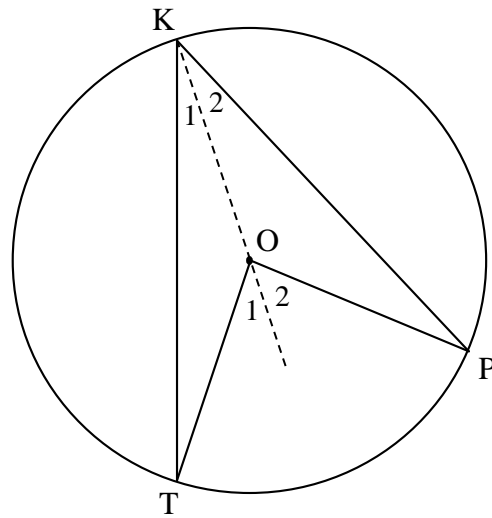
QUESTION/VRAAG 7



<p>7.1</p>	$\text{Area } \Delta\text{STK} = \frac{1}{2}p(\text{SK})\sin\alpha$ $q = \frac{1}{2}p(\text{SK})\sin\alpha$ $\text{SK} = \frac{q}{\frac{1}{2}p\sin\alpha}$ $= \frac{2q}{p\sin\alpha}$	<p>✓ substitution into the correct formula ✓ answer</p> <p style="text-align: right;">(2)</p>
<p>7.2</p>	$\hat{\text{RKS}} = \beta$ $\frac{\text{RS}}{\text{SK}} = \tan\beta$ $\text{RS} = \frac{2q \tan\beta}{p\sin\alpha}$ <p>OR</p> $\frac{\text{RS}}{\sin\beta} = \frac{\text{SK}}{\sin(90^\circ - \beta)}$ $\text{RS}\cos\beta = \text{SK}\sin\beta$ $\text{RS} = \text{SK}\tan\beta$ $\text{RS} = \frac{2q \tan\beta}{p\sin\alpha}$	<p>✓ $\hat{\text{RKS}} = \beta$ ✓ correct trig ratio</p> <p style="text-align: right;">(2)</p> <p>✓ $\hat{\text{RKS}} = \beta$ ✓ $\tan\beta = \frac{\sin\beta}{\cos\beta}$</p> <p style="text-align: right;">(2)</p>
<p>7.3</p>	$70 = \frac{2(2500)\tan 42^\circ}{80\sin\alpha}$ $\sin\alpha = \frac{25}{28}\tan 42^\circ \quad \text{OR} \quad \sin\alpha = 0,80\dots$ $\alpha = 53,51^\circ$	<p>✓ correct substitution of values into RS ✓ value of $\sin\alpha$ ✓ answer</p> <p style="text-align: right;">(3)</p>
[7]		

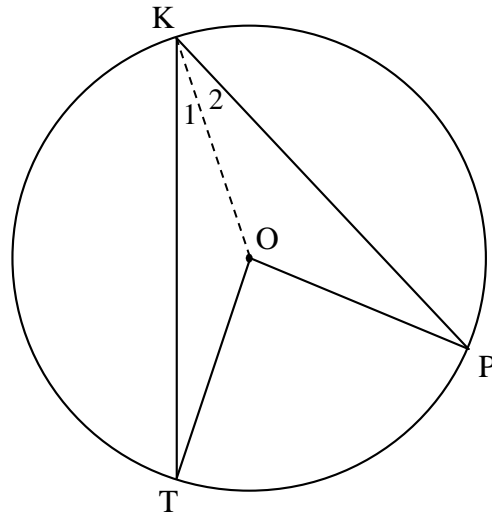
QUESTION/VRAAG 8

8.1



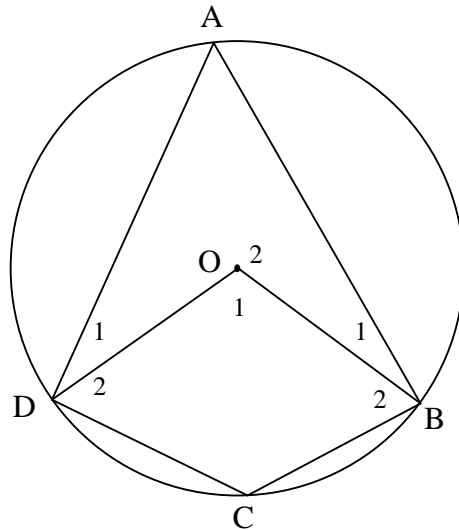
<p>8.1</p>	<p>Construction: Draw KO produced</p> <p>$\hat{O}_1 = \hat{K}_1 + \hat{T}$ [ext \angle of Δ]</p> <p>But $\hat{K}_1 = \hat{T}$ [\angles opp equal sides]</p> <p>$\therefore \hat{O}_1 = 2\hat{K}_1$</p> <p>$\hat{O}_2 = \hat{K}_2 + P$ [ext \angle of Δ]</p> <p>But $\hat{K}_2 = P$ [\angles opp equal sides]</p> <p>$\therefore \hat{O}_2 = 2\hat{K}_2$</p> <p>$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{K}_1 + 2\hat{K}_2$</p> <p style="padding-left: 40px;">$= 2(\hat{K}_1 + \hat{K}_2)$</p> <p>$\therefore \hat{TOP} = 2\hat{TKP}$</p> <p>OR</p>	<p>✓ construction</p> <p>✓ S / R</p> <p>✓ S</p> <p>✓ S</p> <p>✓ S</p> <p>(5)</p>
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8.1



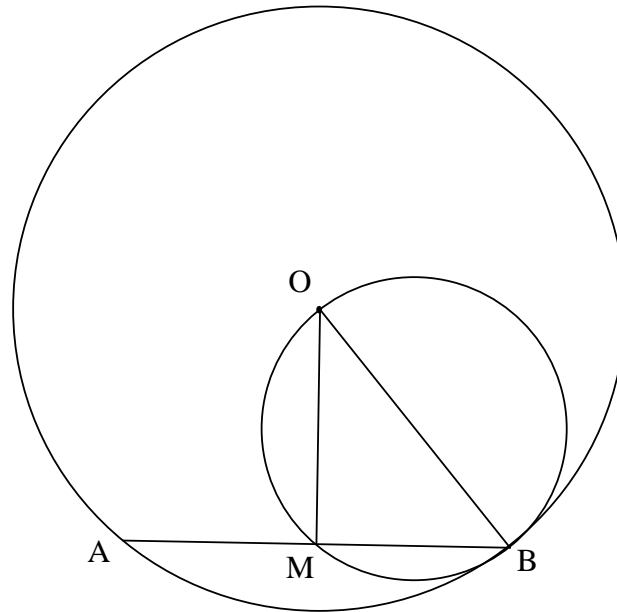
<p>8.1</p>	<p>Construction: Draw KO</p> $\hat{T} = \hat{K}_1$ [∠s opp. equal sides] $\therefore \hat{KOT} = 180^\circ - 2\hat{K}_1$ [sum of ∠s of ΔKOT] $\hat{P} = \hat{K}_2$ [∠s opp. equal sides] $\therefore \hat{KOP} = 180^\circ - 2\hat{K}_2$ [sum of ∠s of ΔKOP] $\hat{TOP} = 360^\circ - (\hat{KOT} + \hat{KOP})$ [∠s around a point] $= 360^\circ - (180^\circ - 2\hat{K}_1 + 180^\circ - 2\hat{K}_2)$ $= 2\hat{K}_1 + 2\hat{K}_2$ $= 2(\hat{K}_1 + \hat{K}_2)$ $\therefore \hat{TOP} = 2\hat{TKP}$	<p>✓ construction</p> <p>✓ S / R</p> <p>✓ S</p> <p>✓ S</p> <p>✓ S</p> <p>(5)</p>
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8.2



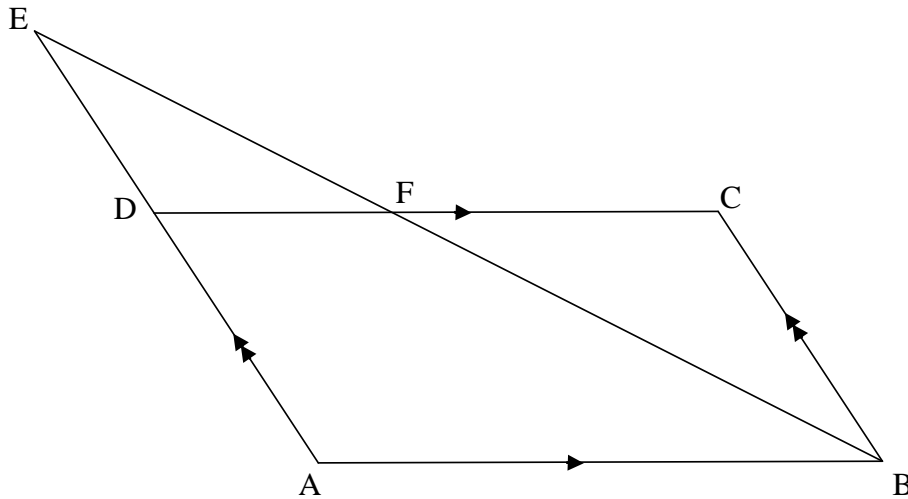
8.2	$\hat{O}_1 = 4x + 100^\circ$ [given]	
	$\therefore \hat{A} = 2x + 50^\circ$ [∠ at centre = 2 × ∠ at circumference]	✓ S ✓ R
	$x + 34^\circ + 2x + 50^\circ = 180^\circ$ [opp ∠s of cyclic quad]	✓ S ✓ R
	$3x = 96^\circ$	
	$x = 32^\circ$	✓ answer
	OR	
	$\hat{O}_2 = 2x + 68^\circ$ [∠ at centre = 2 × ∠ at circumference]	✓ S ✓ R
	$4x + 100^\circ + 2x + 68^\circ = 360^\circ$ [∠s round a pt]	✓ S ✓ R
	$6x = 192^\circ$	
	$x = 32^\circ$	✓ answer
	OR	
	$\hat{O}_2 = -4x + 260^\circ$ [∠s round a pt]	✓ S ✓ R
	$2\hat{C} = -4x + 260^\circ$ [∠ at centre = 2 × ∠ at circumference]	✓ S ✓ R
	$\hat{C} = -2x + 130^\circ$	
	$x + 34^\circ = -2x + 130^\circ$	
	$3x = 96^\circ$	
	$x = 32^\circ$	✓ answer

8.3



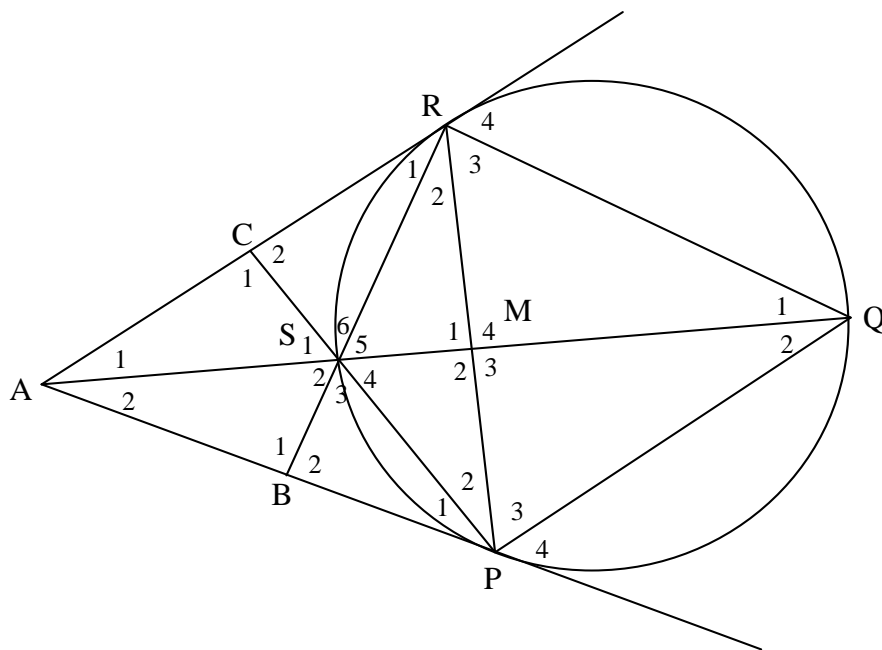
8.3.1	$\hat{O}MB = 90^\circ$ [∠ in semi circle]	✓ S ✓ R (2)
8.3.2	$AB = \sqrt{300} = 10\sqrt{3}$ $\therefore MB = 5\sqrt{3}$ [line from centre \perp to chord] $OB^2 = OM^2 + MB^2$ [Pythagoras] $OB^2 = 5^2 + (5\sqrt{3})^2$ $OB = 10$ units	✓ S ✓ R ✓ S ✓ answer (4)
		[16]

QUESTION/VRAAG 9



<p>9.1</p>	$\frac{FB}{EB} = \frac{DA}{EA}$ <p>[prop theorem; DC AB] OR [line one side of Δ]</p> $FB = \frac{4p \times 21}{7p}$ <p>FB = 12 units</p>	<p>✓ S ✓ R</p> <p>✓ answer</p> <p>(3)</p>
<p>9.2</p>	<p>In ΔEDF and ΔEAB:</p> <p>\hat{E} is common</p> <p>$\hat{EDF} = \hat{A}$ [corresp ∠s; EA CB]</p> <p>$\hat{EFD} = \hat{EBA}$ [corresp ∠s; DC AB]</p> <p>ΔEDF ΔEAB [∠;∠;∠]</p>	<p>✓ S</p> <p>✓ S/R</p> <p>✓ S OR R</p> <p>(3)</p>
<p>9.3</p>	$\frac{DF}{AB} = \frac{ED}{EA}$ <p>[Δs]</p> $DF = \frac{3p \times 14}{7p}$ <p>DF = 6 units</p> <p>FC = 8 units [DC = AB = 14 units; opp sides of ^m]</p> <p>OR</p> <p>ΔEDF ΔBCF [∠;∠;∠]</p> $\frac{ED}{BC} = \frac{DF}{CF}$ <p>[Δs]</p> $\frac{3}{4} = \frac{14 - FC}{FC}$ <p>[BC = AD; opp sides of ^m]</p> <p>3FC = 56 – 4FC</p> <p>FC = 8</p>	<p>✓ S</p> <p>✓ DF = 6</p> <p>✓ FC = 14 – DF</p> <p>(3)</p> <p>✓ ΔEDF ΔBCF</p> <p>✓ $\frac{3}{4} = \frac{14 - FC}{FC}$</p> <p>✓ answer</p> <p>(3)</p>
		<p>[9]</p>

QUESTION/VRAAG 10



<p>10.1</p>	<p>$\hat{S}_3 = \hat{PQR}$ [ext \angle of cyclic quad] $\hat{R}_3 = \hat{PQR}$ [\angles opp equal sides] $\therefore \hat{S}_3 = \hat{R}_3$ But $\hat{S}_4 = \hat{R}_3$ [\angles in the same seg] $\therefore \hat{S}_3 = \hat{S}_4$</p>	<p>✓ S ✓ R ✓ S / R ✓ S ✓ R (5)</p>
<p>10.2</p>	<p>$\hat{R}_1 + \hat{R}_2 = \hat{PQR}$ [tan chord theorem] $\hat{S}_4 = \hat{PQR}$ [proved in 10.1] $\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$ SMRC is a cyclic quad [converse ext \angle of cyclic quad]</p>	<p>✓ S ✓ R ✓ S ✓ R (4)</p>
<p>10.3</p>	<p>$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$ [ext \angle of Δ] $\hat{S}_4 = \hat{P}_1 + \hat{A}_2$ [ext \angle of Δ] $\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$ But $\hat{P}_1 = \hat{R}_2$ [tan chord theorem] $\therefore \hat{P}_2 = \hat{A}_2$ RP is a tangent to the circle [converse tan chord theorem] OR [\angle between line and chord] OR [converse alt seg theorem]</p> <p>OR</p>	<p>✓ S ✓ R ✓ S ✓ S ✓ R ✓ R (6)</p>

	<p>In $\triangle MSP$ and $\triangle MPA$</p> <p>\hat{M}_2 is common</p> <p>$AR = AP$ [tans from same point]</p> <p>$\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$ [\angles opp equal sides]</p> <p>$\hat{S}_4 = \hat{R}_1 + \hat{R}_2$ [proved in 10.2]</p> <p>$\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$</p> <p>$\therefore \hat{P}_2 = \hat{A}_2$ [sum of \angles in \triangle]</p> <p>RP is a tangent to the circle [converse tan chord theorem]</p>	<p>✓ S</p> <p>✓ S / R</p> <p>✓ S</p> <p></p> <p>✓ S</p> <p>✓ S</p> <p>✓ R</p> <p>(6)</p>
		[15]

TOTAL/TOTAAL: 150