basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V1

NOVEMBER 2023

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 9 bladsye en 1 inligtingsblad.

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies noukeurig deur voordat jy die vrae beantwoord.

1. Hierdie vraestel bestaan uit 10 vrae.
2. Beantwoord AL die vrae.
3. Nommer die antwoorde korrek volgens die nommeringstelsel wat in hierdie vraestel gebruik is.
4. Dui ALLE berekeninge, diagramme, grafieke, ens. wat jy gebruik het om jou antwoorde te bepaal, duidelik aan.
5. Volpunte sal NIE noodwendig aan slegs antwoorde toegeken word NIE.
6. Jy mag 'n goedgekeurde, wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders vermeld.
7. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.
8. Diagramme is NIE noodwendig volgens skaal geteken NIE.
9. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
10. Skryf netjies en leesbaar.

VRAAG 1

1.1 Los op vir x:

1.1.1 $x^2 + x - 12 = 0$ (3)

1.1.2 $3x^2 - 2x = 6$ (antwoorde korrek tot TWEE desimale plekke) (4)

1.1.3 $\sqrt{2x+1} = x - 1$ (4)

1.1.4 $x^2 - 3 > 2x$ (4)

1.2 Los gelyktydig vir x en y op:

$x + 2 = 2y$ en $\frac{1}{x} + \frac{1}{y} = 1$ (5)

1.3 Gegee: $2^{m+1} + 2^m = 3^{n+2} - 3^n$ waar m en n heelgetalle is.

Bepaal die waarde van $m + n$. (4)

[24]

VRAAG 2

2.1 Gegee die rekenkundige reeks: $7 + 12 + 17 + ...$

2.1.1 Bepaal die waarde van T_{91} (3)

2.1.2 Bereken S_{91} (2)

2.1.3 Bereken die waarde van n waarvoor $T_n = 517$ (3)

2.2 Die volgende inligting oor 'n kwadratiese getalpatroon word gegee:

$T_1 = 3$, $T_2 - T_1 = 9$ en $T_3 - T_2 = 21$

2.2.1 Toon dat $T_5 = 111$ (2)

2.2.2 Toon dat die algemene term van die kwadratiese patroon $T_n = 6n^2 - 9n + 6$ is. (3)

2.2.3 Toon dat die patroon stygend is vir alle $n \in N$. (3)
[16]

VRAAG 3

3.1 Gegee die meetkundige reeks: $3 + 6 + 12 + ...$ tot n terme.

3.1.1 Skryf die algemene term van hierdie reeks neer. (1)

3.1.2 Bereken die waarde van k sodanig dat: $\sum_{p=1}^{k} \frac{3}{2}(2)^p = 98\,301$ (4)

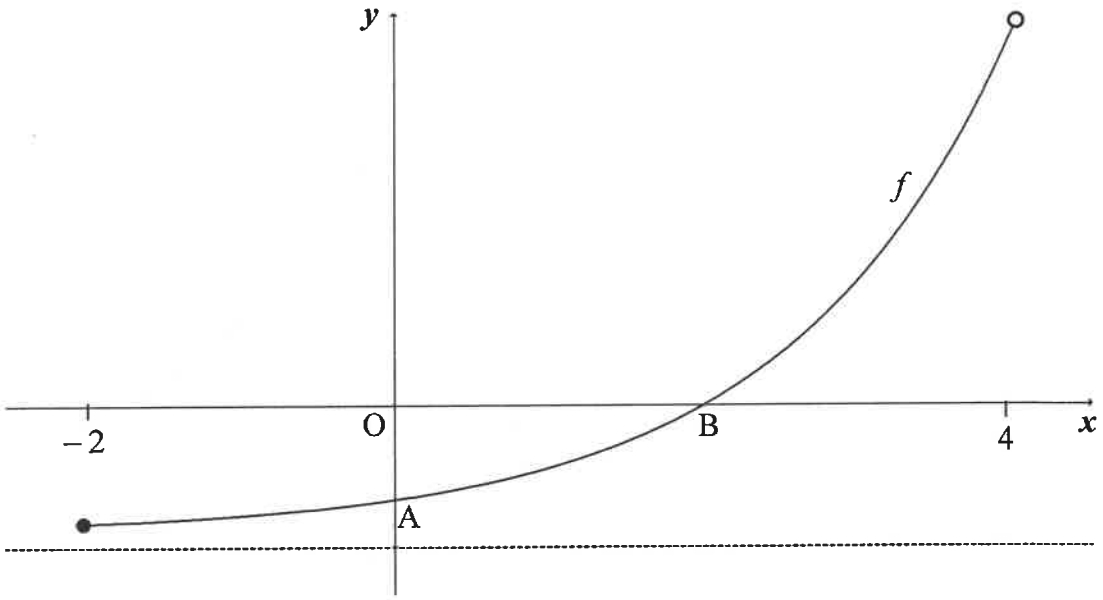
3.2 'n Meetkundige ry en 'n rekenkundige ry het dieselfde eerste term.

- Die konstante verhouding van die meetkundige ry is $\frac{1}{3}$
- Die konstante verskil van die rekenkundige ry is 3
- Die som van 22 terme van die rekenkundige ry is 734 meer as die som tot oneindigheid van die meetkundige ry.

Bereken die waarde van die eerste term. (5)
[10]

VRAAG 4

Die grafiek van $f(x) = 2^x - 4$ vir $x \in [-2 ; 4)$ is hieronder geskets.
A en B is onderskeidelik die y- en x-afsnitte van f.



4.1 Skryf die vergelyking van die asimptoot van f neer. (1)

4.2 Bepaal die koördinate van B. (2)

4.3 Bepaal die vergelyking van k, 'n reguitlyn wat deur A en B gaan in die vorm $k(x) = \ldots$ (3)

4.4 Bereken die vertikale afstand tussen k en f by $x = 1$ (3)

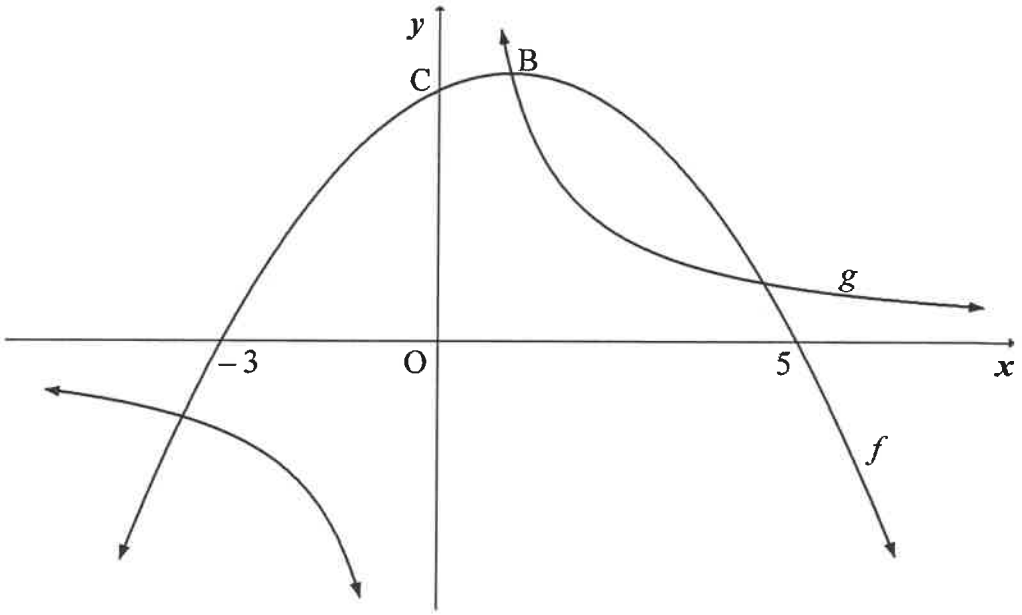
4.5 Skryf die vergelyking van g neer indien dit gegee word dat $g(x) = f(x) + 4$ (1)

4.6 Skryf die definisieversameling van g^{-1} neer. (2)

4.7 Skryf die vergelyking van g^{-1} in die vorm $y = \ldots$ neer. (2)
[14]

VRAAG 5

Die grafieke van $f(x)=-\frac{1}{2}(x-1)^2+8$ en $g(x)=\frac{d}{x}$ is hieronder geskets. 'n Snypunt van f en g is B, die draaipunt van f. Die grafiek f het x-afsnitte by $(-3\ ;\ 0)$ en $(5\ ;\ 0)$ en 'n y-afsnit by C.



5.1 Skryf die koördinate van die draaipunt van f neer. (2)

5.2 Bereken die koördinate van C. (2)

5.3 Bereken die waarde van d. (1)

5.4 Skryf die waardeversameling van g neer. (1)

5.5 Vir watter waardes van x sal $f(x).g(x) \leq 0$? (3)

5.6 Bereken die waardes van k sodanig dat $h(x) = -2x + k$ nie die grafiek van g sal sny nie. (5)

5.7 h is 'n raaklyn aan g by R, 'n punt in die eerste kwadrant. Bereken t sodanig dat $y=f(x)+t$ vir g by R sny. (4)

[18]

VRAAG 6

6.1 Patrick het 'n bedrag van R18 500 gedeponeer in 'n rekening wat r% rente p.j. verdien, maandeliks saamgestel. Na 6 maande was sy balans R19 319,48.

6.1.1 Bereken die waarde van r. (3)

6.1.2 Bereken die effektiewe rentekoers. (2)

6.2 Kuda het op 31 Januarie 2019 'n skootrekenaar ('laptop') vir R10 000 gekoop. Hy gaan dit oor 5 jaar, op 31 Januarie 2024, met 'n nuwe een vervang.

6.2.1 Die waarde van die ou skootrekenaar verminder jaarliks in waarde teen 'n koers van 20% p.j. volgens die reguitlynmetode. Na hoeveel jaar sal die waarde van die skootrekenaar R0 wees? (2)

6.2.2 Kuda gaan 'n skootrekenaar, wat R20 000 kos, koop. Om die koopprys te kan betaal, het hy sy eerste maandelikse deposito op 28 Februarie 2019 in 'n spaarrekening gemaak. Hy sal op 31 Januarie 2024 sy 60ste maandelikse deposito maak. Die spaarrekening betaal rente teen 8,7% p.j., maandeliks saamgestel. Bereken Kuda se maandelikse deposito in hierdie rekening. (4)

6.3 Tino wen 'n boerpot van R1 600 000. Hy belê al sy wengeld in 'n fonds wat rente verdien van 11,2% p.j., maandeliks saamgestel. Hy onttrek aan die einde van elke maand R20 000 uit die fonds. Sy eerste onttrekking is presies 1 maand na sy oorspronklike belegging. Hoeveel onttrekkings van R20 000 sal Tino uit hierdie fonds kan maak? (5)
[16]

VRAAG 7

7.1 Bepaal $f'(x)$ vanuit eerste beginsels indien $f(x) = -4x^2$ (5)

7.2 Bepaal:

7.2.1 $f'(x)$ indien $f(x) = 2x^3 - 3x$ (2)

7.2.2 $D_x\left(7\sqrt[3]{x^2} + 2x^{-5}\right)$ (3)

7.3 Vir watter waardes van x sal die raaklyn aan $f(x) = -2x^3 + 8x$ 'n positiewe helling hê? (3)
[13]

VRAAG 8

Gegee: $f(x) = -x^3 + 6x^2 - 9x + 4 = (x-1)^2(-x+4)$

8.1 Bepaal die koördinate van die draaipunte van f. (4)

8.2 Teken 'n sketsgrafiek van f. Dui duidelik alle snypunte met die asse en enige draaipunte aan. (4)

8.3 Gebruik die grafiek om die waarde(s) van k te bepaal waarvoor $-x^3 + 6x^2 - 9x + 4 = k$ drie reële en ongelyke wortels sal hê. (2)

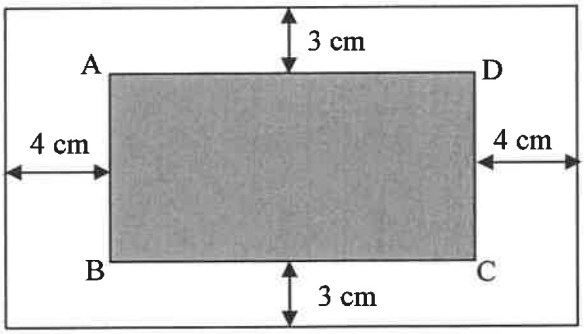
8.4 Die lyn $g(x) = ax + b$ is die raaklyn aan f by f se infleksiepunt. Bepaal die vergelyking van g. (6)

8.5 Bereken die waarde van θ, die skerphoek gevorm tussen g en die x-as in die eerste kwadrant. (2)

[18]

VRAAG 9

Die diagram hieronder stel 'n gedrukte plakkaat voor. Reghoek ABCD is die deel waarop die teks gedruk word. Hierdie ingekleurde oppervlakte ABCD is $432\ \text{cm}^2$ en $AD = x$ cm. ABCD is 4 cm vanaf die linkerkant en regterkant van die bladsy en 3 cm vanaf die bokant en onderkant van die bladsy.



9.1 Toon dat die totale oppervlakte van die bladsy gegee word deur:

$$A(x) = \frac{3\,456}{x} + 6x + 480$$ (3)

9.2 Bepaal die waarde van x sodanig dat die totale oppervlakte van die bladsy 'n minimum is. (3)

[6]

VRAAG 10

10.1 A en B is onafhanklike gebeurtenisse. $P(A) = \frac{1}{3}$ en $P(B) = \frac{3}{4}$

Bepaal:

10.1.1 P(A en B) (2)

10.1.2 P(ten minste EEN gebeurtenis plaasvind) (2)

10.2 Die waarskynlikheid dat dit in Junie op die Drakensberge sal sneeu, is 5%.

- Wanneer dit op die berge sneeu, is die waarskynlikheid dat die minimum temperatuur in Sentraal-Suid-Afrika tot onder 0 °C sal daal, 72%.
- As dit nie op die berge sneeu nie, is die waarskynlikheid dat die minimum temperatuur in Sentraal-Suid-Afrika tot onder 0 °C sal daal, 35%.

10.2.1 Stel die gegewe inligting op 'n boomdiagram voor. Dui die waarskynlikhede wat met ELKE tak geassosieer word, duidelik aan. (3)

10.2.2 Bereken die waarskynlikheid dat die temperatuur in Sentraal-Suid-Afrika in Junie 2024 NIE tot onder 0 °C sal daal NIE. (3)

10.3 Tien leerders staan willekeurig agter mekaar in 'n ry.

10.3.1 Op hoeveel verskillende maniere kan die tien leerders in die ry staan? (1)

10.3.2 Bereken die waarskynlikheid dat daar 5 leerders tussen die 2 jongste leerders in die ry sal staan. (4)
[15]

TOTAAL: 150

INLIGTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

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NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

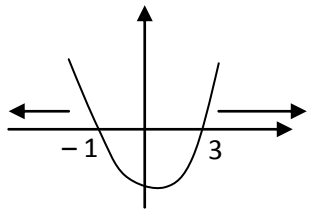
MARKS/*PUNTE*: 150

These marking guidelines consist of 17 pages.
Hierdie nasienriglyne bestaan uit 17 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

LET WEL:
- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die memorandum van toepassing.*

QUESTION 1/*VRAAG* 1

1.1.1	$x^2+x-12=0$ $(x-3)(x+4)=0$ $x=3$ or $x=-4$	✓factors/formula ✓answer ✓answer (3)
1.1.2	$3x^2-2x=6$ $3x^2-2x-6=0$ $x=\frac{2\pm\sqrt{(-2)^2-4(3)(-6)}}{2(3)}$ $x=1{,}79$ or $x=-1{,}12$	✓ standard form ✓ substitution into correct formula ✓ answer ✓ answer (4)
1.1.3	$\sqrt{2x+1}=x-1$ $2x+1=(x-1)^2$ $2x+1=x^2-2x+1$ $x^2-4x=0$ $x(x-4)=0$ $x=0$ or $x=4$ $x\neq 0$ or $x=4$	✓ squaring both sides ✓ standard form ✓ both x-values ✓ valid answer (4)
1.1.4	$x^2-2x>3$ $x^2-2x-3>0$ $(x-3)(x+1)>0$ CV's: $x=-1$; $x=3$  $x<-1$ or $x>3$	✓ standard form ✓ critical values/factors ✓✓answer (4)

1.2	$\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$ $x+2=2y \quad \ldots \quad (2)$ $x=2y-2$ $\frac{1}{2y-2}+\frac{1}{y}=1$ $y+2y-2=2y^2-2y$ $2y^2-5y+2=0$ $(2y-1)(y-2)=0$ $y=\frac{1}{2}$ or $y=2$ $x=-1$ or $x=2$ **OR/*OF*** $\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$ $x+2=2y \quad \ldots \quad (2)$ $y=\frac{x}{2}+1$ $\frac{1}{x}+\frac{1}{\frac{x}{2}+1}=1$ $\frac{1}{x}+\frac{2}{x+2}=1$ $x+2+2x=x^2+2x$ $x^2-x-2=0$ $(x+1)(x-2)=0$ $x=-1$ or $x=2$ $y=\frac{1}{2}$ or $y=2$	✓ $x=2y-2$ ✓substitution ✓standard form ✓*y*-values ✓*x*-values (5) **OR/*OF*** ✓ $y=\frac{x}{2}+1$ ✓substitution ✓standard form ✓*x*-values ✓*y*-values (5)

1.3	$2^{m+1}+2^m=3^{n+2}-3^n$ $2^m(2+1)=3^n(3^2-1)$ $2^m(3)=3^n(8)$ $2^m(3)=3^n(2^3)$ $\therefore m=3$ and $n=1$ $\therefore m+n=4$ **OR/*OF*** $2^{m+1}+2^m=3^{n+2}-3^n$ $2^m(2+1)=3^n(3^2-1)$ $2^m(3)=3^n(8)$ $2^m(3)=3^n(2^3)$ $2^{m-3}=3^{n-1}$ Only true if $m-3=0$ and $n-1=0$ $\therefore m+n=4$	✓ factors ✓ $2^m(3)=3^n(2^3)$ (same bases) ✓ $m=3$ and $n=1$ ✓ $m+n=4$ (4) **OR/*OF*** ✓ factors ✓ $2^m(3)=3^n(2^3)$ (same bases) ✓ $m-3=0$ and $n-1=0$ ✓ $m+n=4$ (4)
		[24]

QUESTION 2/*VRAAG* 2

2.1.1	$7 + 12 + 17 + \ldots..$ $T_n = a + (n-1)d$ $T_{91} = 7 + (91-1)(5)$ $T_{91} = 457$ **OR/*OF*** $d = 5$ $T_n = 5n + 2$ $T_{91} = 5(91) + 2$ $T_{91} = 457$	✓$d = 5$ ✓substitution into correct formula ✓answer (3) **OR/*OF*** ✓$d = 5$ ✓substitution $n = 91$ ✓answer (3)
2.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{91} = \frac{91}{2}[2 \times 7 + (91-1)(5)]$ $S_9 = 21\ 112$ **OR/*OF*** $S_n = \frac{n}{2}(a + l)$ $S_{91} = \frac{91}{2}(7 + 457)$ $S_{91} = 21\ 112$	✓substitution into correct formula ✓answer (2) **OR/*OF*** ✓substitution into correct formula ✓answer (2)
2.1.3	$T_n = 7 + (n-1)(5)$ $5n + 2 = 517$ $5n = 515$ $n = 103$	✓substitution into correct formula ✓equate ✓answer (3)
2.2.1	$T_1 = 3$; $T_2 - T_1 = 9$ and $T_3 - T_2 = 21$ 3 12 33 66 111 9 21 33 45 12 12 12 $\therefore T_5 = 3 + 9 + 21 + 33 + 45 = 111$ **OR/*OF*** $2a = 12$ $a = 6$ $3(6) + b = 9$ $b = -9$ $6 - 9 + c = 3$ $T_5 = 6(5)^2 - 9(5) + 6 = 111$	✓ constant second diff = 12 ✓ first differences : 33 and 45 (2) **OR/*OF*** ✓ constant second diff = 12 ✓substitute 5 (2)

2.2.2	$2a=12$ $a=6$ $3(6)+b=9$ or $5\times6+b=21$ $b=-9$ $6-9+c=3$ $c=6$ $T_n=6n^2-9n+6$	✓ $2a=12$ ✓ $3(6)+b=9/5\times6+b=21$ ✓ $6-9+c=3$ (3)
2.2.3	$T_n^{/}=12n-9>0$ $n>\frac{3}{4}$ $\therefore\ T_n$ is increasing for $n\in N$ **OR/*OF*** $n=-\frac{b}{2a}=-\frac{-9}{2(6)}$ $n=\frac{3}{4}$ $\therefore$ min at $n=1$ for $n\in N$ $\therefore\ T_n$ is increasing for $n\in N$	✓ $T_n^{/}=12n-9$ ✓ $n>\frac{3}{4}$ ✓ increasing for $n\in N$ (3) **OR/*OF*** ✓ $n=-\frac{b}{2a}=\frac{9}{2(6)}$ ✓ $n=\frac{3}{4}$ ✓ increasing for $n\in N$ (3)
		[16]

QUESTION 3/*VRAAG* 3

3.1.1	$T_n = ar^{n-1}$ $T_n = 3(2)^{n-1}$	✓ $T_n = 3(2)^{n-1}$ (1)
3.1.2	$\sum_{p=1}^{k} \frac{3}{2}.2^p = 98\ 301$ $\sum_{p=1}^{k} \frac{3}{2}.2^p = 3+6+12+...$ $n = k$ $\frac{3\left[(2)^k - 1\right]}{2-1} = 98\ 301$ $(2)^k = 32\ 768$ $2^k = 2^{15}$ **OR/*OF*** $k = \log_2 32\ 768$ $\therefore k = 15$	✓expansion ✓ $n = k$ ✓substitution into correct formula ✓$k = 15$ (4)
3.2	$S_{22} = \frac{22}{2}\left[2a + 21(3)\right]$ $S_{22} = 22a + 693$ $S_\infty = \frac{a}{1-\frac{1}{3}}$ $= \frac{3a}{2}$ $\therefore 22a + 693 = \frac{3a}{2} + 734$ $44a + 1386 = 3a + 1468$ $41a = 82$ $a = 2$	✓ substitution into S_n ✓ $S_{22} = 22a + 693$ ✓substitution into S_∞ ✓ $S_{22} = S_\infty + 734$ ✓ answer (5)
		[10]

QUESTION 4/*VRAAG* 4

4.1	$y = -4$	✓ $y = -4$ (1)
4.2	x – intercept: $0 = 2^x - 4$ $4 = 2^x$ $x = 2$ $\therefore B(2;0)$	✓$y = 0$ ✓$x = 2$ (2)
4.3	$y = 2^0 - 4 = -3$ $\therefore A(0;-3)$ $y = mx + c$ $m = \frac{3}{2}$ $k(x) = \frac{3}{2}x - 3$	✓$y = -3$ ✓gradient ✓equation (3)
4.4	$k(1) = \frac{3}{2}(1) - 3 = \frac{-3}{2}$ $f(1) = 2^1 - 4 = -2$ Vertical distance $= -\frac{3}{2} - (-2) = \frac{1}{2}$ units	✓$k(1)$ ✓$f(1) = -2$ ✓answer (3)
4.5	$g(x) = f(x) + 4$ $g(x) = 2^x$; $x \in [-2;4)$	✓ $g(x) = 2^x$ (1)
4.6	Range of g : $y \in \left[\frac{1}{4};16\right)$ Domain of g^{-1} : $x \in \left[\frac{1}{4};16\right)$ or/*of* $\frac{1}{4} \le x < 16$	✓✓$x \in \left[\frac{1}{4};16\right)$ (2)
4.7	$g: y = 2^x$ $g^{-1}: x = 2^y$ $g^{-1}(x) = \log_2 x$, $x \in \left[\frac{1}{4};16\right)$	✓swop x and y ✓ equation (2)
		[14]

QUESTION 5/*VRAAG* 5

5.1	(1 ; 8)	✓ $x = 1$ ✓ $y = 8$ (2)
5.2	$y = -\frac{1}{2}(0-1)^2 + 8$ $= 7\frac{1}{2}$ $C\left(0 ; \frac{15}{2}\right)$	✓$x = 0$ ✓ answer (2)
5.3	$8 = \frac{d}{1}$ $\therefore d = 8$	✓substitution (1 ; 8) (1)
5.4	$y \in R$; $y \neq 0$	✓ $y \neq 0$ (1)
5.5	$-3 \leq x < 0$ or $x \geq 5$ **OR/*OF*** $x \in [-3 ; 0) \cup [5 ; \infty)$	✓✓ $-3 \leq x < 0$ ✓ $x \geq 5$ (3)
5.6	$-2x + k = \frac{8}{x}$ $-2x^2 + kx - 8 = 0$ $\Delta = (k)^2 - 4(-2)(-8)$ $k^2 - 64 < 0$ $CV : k = 8$; $k = -8$ $\therefore -8 < k < 8$ **or/of** $k \in (-8 ; 8)$ **OR/*OF*** $g'(x) = h'(x)$ $-\frac{8}{x^2} = -2$ $-8 = -2x^2$ $x = \pm 2$ $y = \pm 4$ $\therefore$ B(2 ; 4) and A(– 2 ; – 4) For tangents: $h(x) = -2x + k$ or $h(x) = -2x + k$ $4 = -2(2) + k$ $-4 = -2(-2) + k$ $k = 8$ $k = -8$ $\therefore -8 < k < 8$ or/*of* $k \in (-8 ; 8)$	✓ $-2x + k = \frac{8}{x}$ ✓ standard form ✓substitution into Δ ✓ $\Delta < 0$ or $\Delta = 0$ ✓inequality (5) **OR/*OF*** ✓ $-\frac{8}{x^2}$ ✓ $= -2$ ✓ x-values ✓ y-values ✓inequality (5)

5.7	$h(x) = -2x + 8$ $-2x + 8 = \frac{8}{x}$ $-2x^2 + 8x = 8$ $-2x^2 + 8x - 8 = 0$ $x^2 - 4x + 4 = 0$ $(x-2)^2 = 0$ $\therefore x = 2$	✓ $x = 2$
	$f(2) = \frac{15}{2}$ $h(2) = 4$ $4 = \frac{15}{2} + t$ $\therefore t = -\frac{7}{2}$	✓ $f(2)$ ✓ $h(2)$ ✓answer (4)
	OR/*OF* $f(2) = \frac{15}{2}$ Tangent point of contact (2 ; 4) $\therefore 4 = -\frac{1}{2}(2-1)^2 + 8 + t$ $4 = \frac{15}{2} + t$ $\therefore t = -\frac{7}{2}$	**OR/*OF*** ✓ $x = 2$ ✓ $f(2)$ ✓ $h(2)$ ✓answer (4)
	OR/*OF* $g(x) = 8x^{-1}$ $g'(x) = -8x^{-2}$ $-2 = -8x^{-2}$ $\frac{1}{4} = \frac{1}{x^2}$ $x = 2$ $y = \frac{8}{2} = 4$ $R(2 ; 4)$ $y = -\frac{1}{2}(x-1)^2 + 8 + t$ $4 = -\frac{1}{2}(2-1)^2 + 8 + t$ $t = -\frac{7}{2}$	**OR/*OF*** ✓ $x = 2$ ✓ $h(2)$ ✓ $f(2)$ ✓answer (4)
		[18]

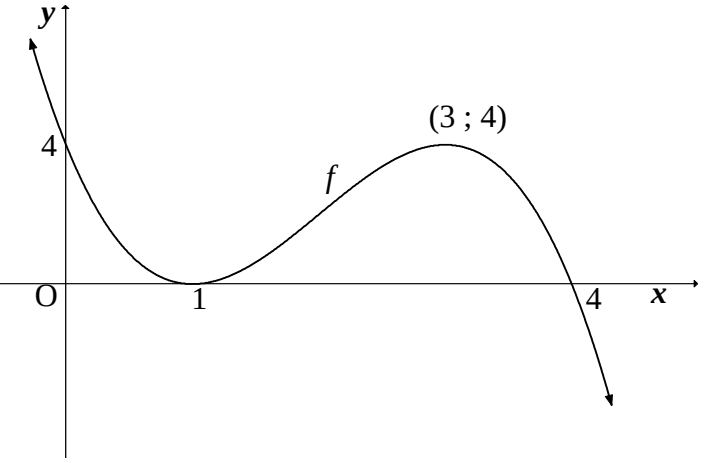
QUESTION 6/*VRAAG* 6

6.1.1	$A = P(1+i)^n$ $19\,319{,}48 = 18\,500\left(1+\frac{r}{1200}\right)^6$ $\left(1+\frac{r}{1200}\right) = \sqrt[6]{1{,}04429...}$ $\frac{r}{1200} = 0{,}00725...$ $r = 8{,}7\%$	✓ $n = 6$ ✓substitution into correct formula ✓answer (3)
6.1.2	$1+\frac{i}{100} = \left(1+\frac{8{,}7}{1200}\right)^{12}$ $r = 9{,}06\%$	✓substitution into correct formula ✓answer (2)
6.2.1	$A = P(1-in)$ $0 = 10\ 000(1-0{,}2n)$ $n = 5$	✓substitution into correct formula ✓answer (2)
6.2.2	$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $20\,000 = \frac{x\left[\left(1+\frac{8{,}7}{1200}\right)^{60} - 1\right]}{\frac{8{,}7}{1200}}$ $x = \text{R}267{,}26$	✓ i ✓ n ✓substitution into correct formula ✓answer (4)
6.3	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $1\,600\,000 = \frac{20\,000\left[1-\left(1+\frac{0{,}112}{12}\right)^{-n}\right]}{\frac{0{,}112}{12}}$ $\frac{56}{75} = 1-\left(1+\frac{0{,}112}{12}\right)^{-n}$ $\left(1+\frac{0{,}112}{12}\right)^{-n} = \frac{19}{75}$ $-n = \log_{\left(1+\frac{0{,}112}{12}\right)}\left(\frac{19}{75}\right)$ $-n = -147{,}80$ Tino will make 147 withdrawals of R20 000	✓i ✓substitution into correct formula ✓correct use of logs ✓ $-n = -147{,}80$ ✓ $n = 147$ (5)
		[16]

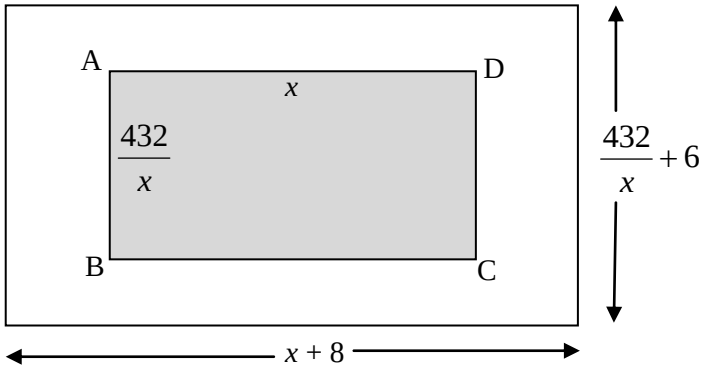
QUESTION 7/*VRAAG* 7

7.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-4(x+h)^2-\left(-4x^2\right)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-4x^2-8xh-4h^2+4x^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(-8x-4h)$ $f'(x)=-8x$ **OR/*OF*** $f(x+h)=-4(x+h)^2=-4x^2-8xh-4h^2$ $f(x+h)-f(x)=-4x^2-8xh-4h^2-(-4x^2)$ $=-8xh-4h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(-8x-4h)$ $f'(x)=-8x$	✓substitution into correct formula ✓ $f(x+h) = -4x^2-8xh-4h^2$ ✓simplification ✓common factor ✓answer (5) **OR/*OF*** ✓ $f(x+h) = -4x^2-8xh-4h^2$ ✓simplification ✓substitution into correct formula ✓common factor ✓answer (5)
7.2.1	$f(x)=2x^3-3x$ $f'(x)=6x^2-3$	✓$6x^2$ ✓-3 (2)
7.2.2	$D_x\left[7.\sqrt[3]{x^2}+2x^{-5}\right]$ $D_x\left[7.x^{\frac{2}{3}}+2x^{-5}\right]$ $=\dfrac{14}{3}x^{-\frac{1}{3}}-10x^{-6}$	✓ $x^{\frac{2}{3}}$ ✓derivative with rational exp ✓$-10x^{-6}$ (3)
7.3	$-6x^2+8>0$ $x^2<\dfrac{8}{6}$ CV's: $x=-\dfrac{2}{\sqrt{3}}$ or $x=\dfrac{2}{\sqrt{3}}$ Positive for : $-\dfrac{2}{\sqrt{3}}<x<\dfrac{2}{\sqrt{3}}$	✓CV's: $x=\pm\dfrac{2}{\sqrt{3}}$ ✓✓answer (3)
		[13]

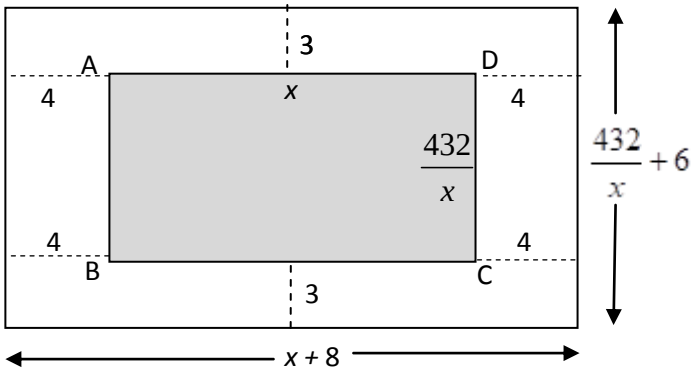
QUESTION 8/*VRAAG* 8

8.1	$f'(x) = -3x^2 + 12x - 9$ $-3x^2 + 12x - 9 = 0$ $x^2 - 4x + 3 = 0$ $(x-3)(x-1) = 0$ $\therefore x = 3$ or $x = 1$ $f(3) = -(3)^3 + 6(3)^2 - 9(3) + 4 = 4$ $f(1) = -(1)^3 + 6(1)^2 - 9(1) + 4 = 0$ $\therefore$ turning points are: (3 ; 4) and (1 ; 0)	✓ $f'(x) = -3x^2 + 12x - 9$ ✓ $f'(x) = 0$ ✓ both x-values ✓ both y-values (4)
8.2		✓ y-intercept ✓ both x-intercepts ✓ both turning points ✓ shape (4)
8.3	$0 < k < 4$ or/*of* $k \in (0 ; 4)$	✓✓k between y-values of turning points (2)
8.4	$f''(x) = -6x + 12 = 0$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$ **OR/*OF*** Point of inflection: $x = \frac{3+1}{2}$ $x = 2$ Max at (2 ; 2) $f'(2) = 3$ $\therefore y - 2 = 3(x-2)$ or $2 = 3(2) + c$ $g(x) = 3x - 4$ $g(x) = 3x - 4$	✓ $f''(x) = -6x + 12$ ✓ $f''(x) = 0$ ✓ x-value ✓ y-value ✓gradient at x-value ✓equation of tangent (6) **OR/*OF*** ✓✓$\frac{3+1}{2}$ ✓ x-value ✓ y-value ✓gradient at x-value ✓equation of tangent (6)
8.5	$\tan\theta = 3$ $\therefore \theta = 71{,}57°$	✓ gradient of g ✓ answer (2)
		[18]

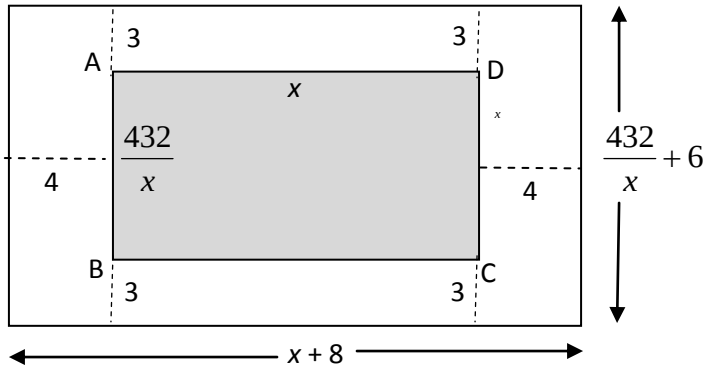
QUESTION 9/*VRAAG* 9



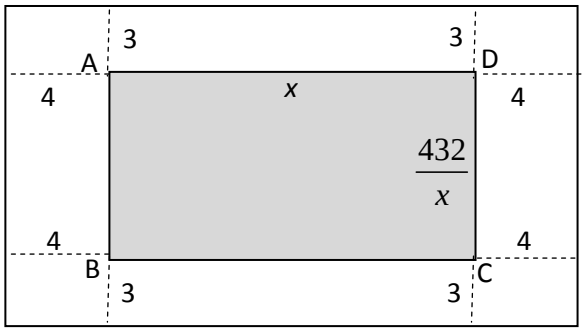
9.1	$432 = xb$ $\therefore b = \frac{432}{x}$ $A(x) = (x+8)\left(\frac{432}{x}+6\right)$ $A(x) = 432 + 6x + \frac{3456}{x} + 480$ $A(x) = \frac{3456}{x} + 6x + 480$	✓ $b = \frac{432}{x}$ ✓ $(x+8)$ ✓ $\left(\frac{432}{x}+6\right)$ (3)
9.2	$A(x) = 3456x^{-1} + 6x + 480$ $A'(x) = -\frac{3456}{x^2} + 6$ $-\frac{3456}{x^2} + 6 = 0$ $3456 = 6x^2$ $\therefore x = \sqrt{576} = 24\text{ cm}$	✓ $3456x^{-1} + 6x + 480$ ✓ $A'(x) = -\frac{3456}{x^2} + 6$ ✓answer (3)
		[6]



$$\text{total area} = 2(x+8)(3)+2\left(\frac{432}{x}\right)(4)+\left(\frac{432}{x}\right)(x)$$

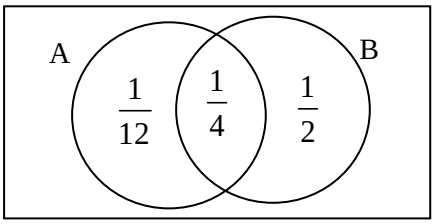
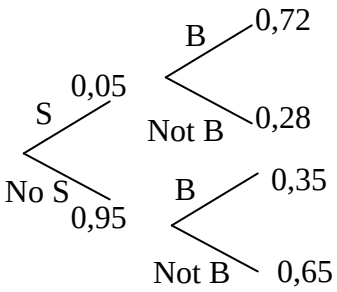


$$\text{total area} = 2(4)\left(\frac{432}{x}+6\right)+(x)\left(\frac{432}{x}+6\right)$$



$$\text{total area} = 4(4)(3)+2(x)(3)+\left(\frac{432}{x}\right)(x)+2\left(\frac{432}{x}\right)(4)$$

QUESTION 10/*VRAAG* 10

10.1.1	P(A and B) = P(A) × P(B) $= \frac{1}{3} \times \frac{3}{4}$ $= \frac{1}{4}$	✓ $\frac{1}{3} \times \frac{3}{4}$ ✓ $\frac{1}{4}$ (2)
10.1.2	P(A or B) = P(A) + P(B) – P(A and B) $= \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$ $= \frac{5}{6}$ **OR/*OF***  P(A or B) = $\frac{1}{12} + \frac{1}{4} + \frac{1}{2} = \frac{5}{6}$	✓ substitution ✓ answer (2) **OR/*OF*** ✓ substitution ✓ answer (2)
10.2.1	 	✓ branch 1 with probabilities ✓ branch 2 with probabilities ✓ branch 3 with probabilities (3)
10.2.2	P(NOT below 0^o) = P(S; NOT below 0^o) + P(NS ; NOT below 0^o) = 0,05 × 0,28 + 0,95 × 0,65 = 0,6315	✓ value of P(S; NOT below 0^o) ✓ value of P(NS ; NOT below 0^o) ✓ answer (3)
10.3.1	n(S) = 10!	✓ 10! (1)

10.3.2	4 Options; **2** ×8 ×7 × 6 ×5 ×4 ×**1** ×3 ×2 ×1 = 80 640 8 ×**2** ×7 × 6 ×5 ×4 ×3 ×**1** ×2 ×1 = 80 640 8 ×7 ×**2** × 6 ×5 ×4 ×3 ×1 ×**1** ×1 = 80 640 8 ×7 × 6 ×**2** ×5 ×4 ×3 ×2 ×1 ×**1** = 80 640 Total number of possibilities = 322 560 P(5 learners in between) = $\frac{322\,560}{10!} = \frac{4}{45}$ **OR/*OF*** **2** ×8 ×7 × 6 ×5 ×4 ×**1** ×3 ×2 ×1 4 possible starting positions $\therefore 4(2 \times 8! \times 1) = 322\,560$ $8(8!) = 322\,560$ P(5 learners in between) = $\frac{322\,560}{10!} = \frac{4}{45}$	✓ $(2 \times 8!)$ ✓✓$4(2 \times 8!)$ or 322 560 ✓ $\frac{322\,560}{n(S)}$ (4) **OR/*OF*** ✓ $(2 \times 8!)$ ✓✓$4(2 \times 8!)$ or 322 560 ✓ $\frac{322\,560}{n(S)}$ (4)
		[15]

TOTAL/*TOTAAL*: 150