



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

NOVEMBER 2021

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 13 bladsye en 1 inligtingsblad.

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat die vraestel beantwoord word.

1. Hierdie vraestel bestaan uit 11 vrae.
2. Beantwoord AL die vrae in die SPESIALE ANTWOORDEBOEK wat verskaf word.
3. Dui ALLE berekeninge, diagramme, grafieke, ens. wat jy in die beantwoording van die vrae gebruik, duidelik aan.
4. Slegs antwoorde sal NIE noodwendig volpunte verdien NIE.
5. Jy kan 'n goedgekeurde wetenskaplike sakrekenaar gebruik (nieprogrammeerbaar en niegrafies), tensy anders vermeld.
6. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.
7. Diagramme is NIE noodwendig volgens skaal geteken NIE.
8. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
9. Skryf netjies en leesbaar.

VRAAG 1

'n Bakkerij teken die aantal brode aan wat 'n snoepwinkel daaglik oor die laaste 18 dae bestel het. Die inligting word in die tabel hieronder aangetoon.

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

1.1 Bereken die:

1.1.1 Gemiddelde aantal brode wat per dag bestel is (2)

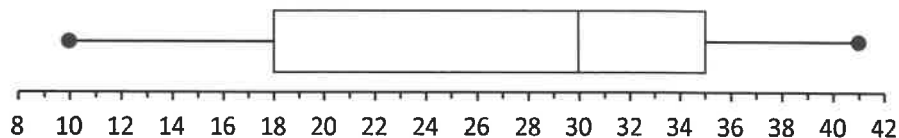
1.1.2 Standaardafwyking van die data (1)

1.1.3 Aantal dae waarop die aantal brode wat bestel is een standaardafwyking bokant die gemiddelde was (2)

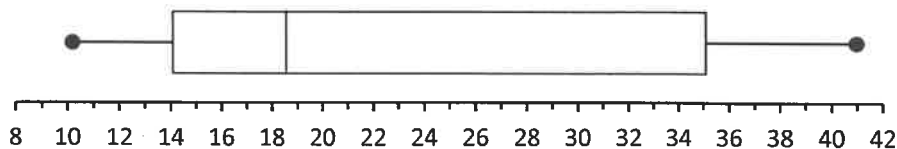
1.2 Die eienaar van die snoepwinkel kon nie al die brode verkoop wat daaglik afgelewer is nie. Hy het bereken dat die gemiddelde aantal brode wat oor die 18 dae verkoop is, 20 is. Bereken die aantal brode wat NIE oor die 18 dae verkoop is NIE. (2)

1.3 Een van die twee mond-en-snordiagramme hieronder geteken, verteenwoordig die data gegee in die tabel hierbo.

Grafiek A:



Grafiek B:



1.3.1 Watter EEN van die twee mond-en-snordiagramme hierbo geteken, is die korrekte verteenwoordiging van die data? Skryf 'n rede vir jou antwoord neer. (2)

1.3.2 Beskryf die skeefheid van die data. (1)

[10]

VRAAG 2

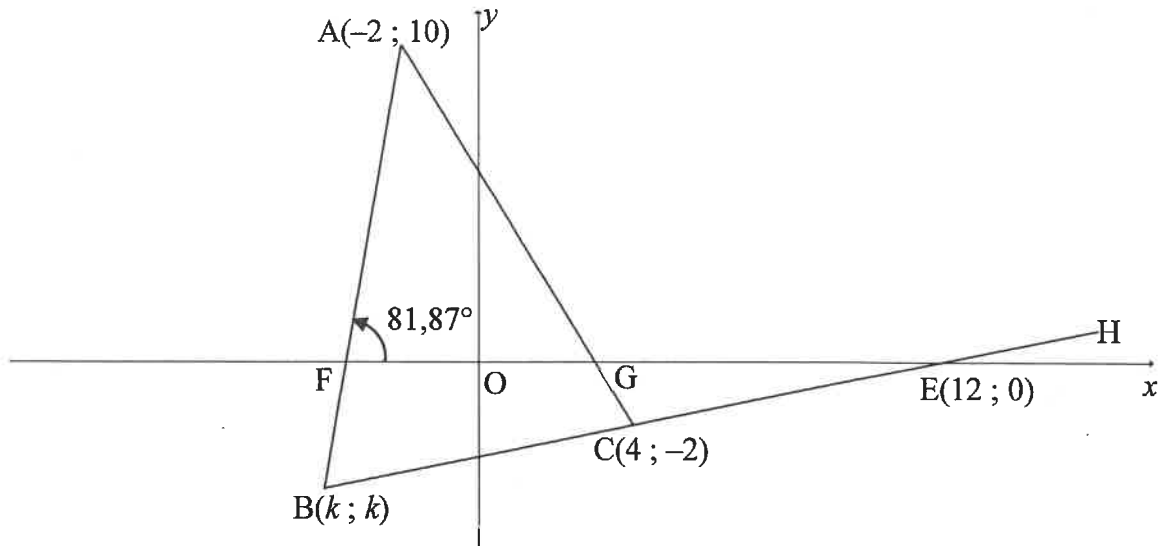
'n Padstal verkoop melk in 5 liter-houers aan die plaaslike gemeenskap. Die prys wissel volgens die beskikbaarheid van melk by die padstal. Die prys van melk, in rand per 5 liter-houer, en die aantal 5 liter-houers melk verkoop, word in die tabel hieronder aangeteken.

Prys van melk in rand per 5 liter-houer (x)	26	32	36	28	40	33	29	34	27	30
Aantal 5 liter-houers melk verkoop (y)	48	30	26	44	23	32	39	29	42	33

- 2.1 Op die rooster wat in die ANTWOORDEBOEK verskaf word, teken die spreidiagram om die data te verteenwoordig. (3)
- 2.2 Bepaal die vergelyking van die kleinstekwadrade-regressielyn vir die data. (3)
- 2.3 Indien die boer 'n 5 liter-houer melk vir R38 verkoop, voorspel die aantal 5 liter-houers melk wat hy sal verkoop. (2)
- 2.4 Verwys na die korrelasie tussen die prys van 5 liter-houers melk en die aantal 5 liter-houers melk verkoop, en lewer kommentaar op die akkuraatheid van jou antwoord op VRAAG 2.3. (2)
- [10]**

VRAAG 3

In die diagram is $A(-2; 10)$, $B(k; k)$ en $C(4; -2)$ die hoekpunte van $\triangle ABC$. Lyn BC word verleng na H en sny die x -as by $E(12; 0)$. AB en AC sny die x -as by F en G onderskeidelik. Die inklinasiehoek van lyn AB is $81,87^\circ$.

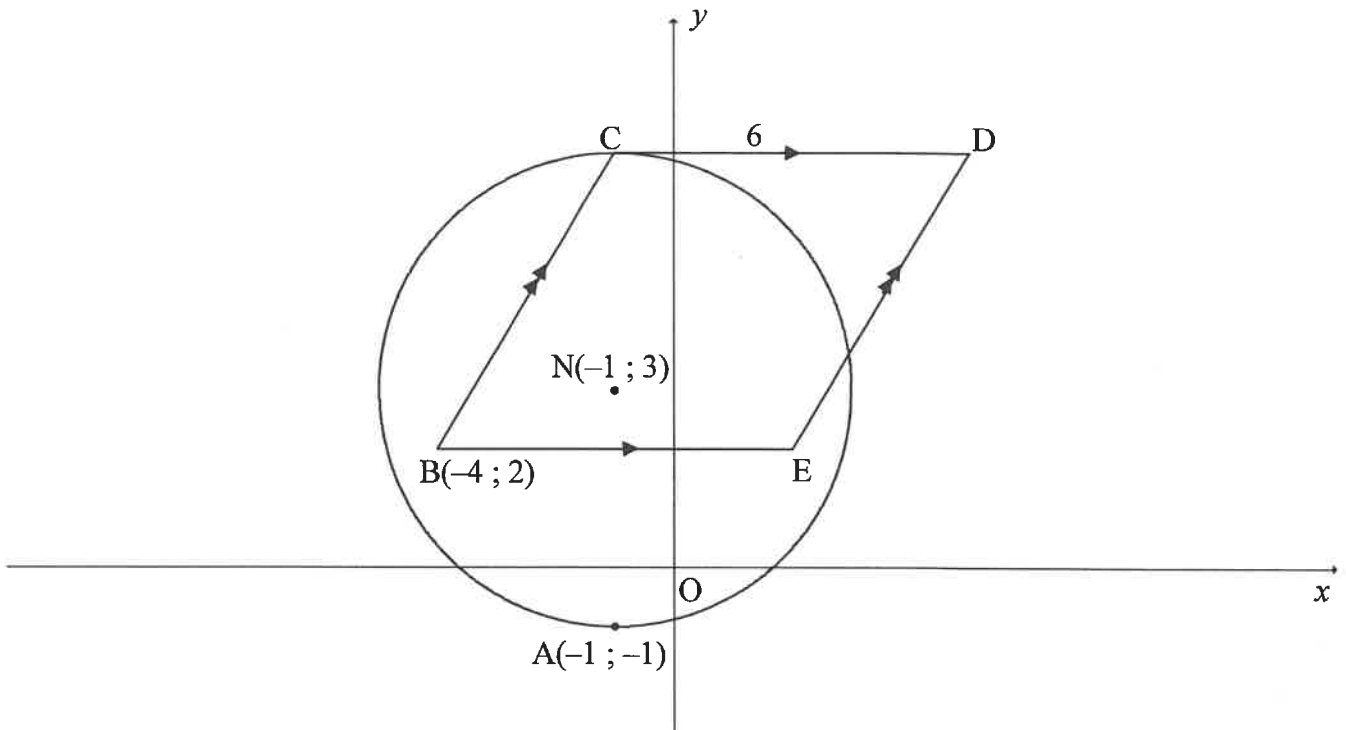


- 3.1 Bereken die gradiënt van:
- 3.1.1 BE (2)
- 3.1.2 AB (2)
- 3.2 Bepaal die vergelyking van BE in die vorm $y = mx + c$ (2)
- 3.3 Bereken die:
- 3.3.1 Koördinate van B, waar $k < 0$ (2)
- 3.3.2 Grootte van \hat{A} (4)
- 3.3.3 Koördinate van die snypunt van die hoeklyne van parallellogram ACES, waar S 'n punt in die eerste kwadrant is (2)
- 3.4 Nog 'n punt $T(p; p)$, waar $p > 0$, word gestip sodat $ET = BE = 4\sqrt{17}$ eenhede.
- 3.4.1 Bereken die koördinate van T. (5)
- 3.4.2 Bepaal die vergelyking van die:
- (a) Sirkel met middelpunt E wat deur punte B en T gaan in die vorm $(x - a)^2 + (y - b)^2 = r^2$ (2)
- (b) Raaklyn aan die sirkel by punt $B(k; k)$ (3)

[24]

VRAAG 4

In die diagram gaan die sirkel, met middelpunt $N(-1 ; 3)$, deur $A(-1 ; -1)$ en C . $B(-4 ; 2)$, C , D en E word verbind om 'n parallelogram te vorm sodat BE ewewydig aan die x -as is. CD is 'n raaklyn aan die sirkel by C en $CD = 6$ eenhede.



- 4.1 Skryf die lengte van die radius van die sirkel neer. (1)
- 4.2 Bereken die:
- 4.2.1 Koördinate van C (2)
- 4.2.2 Koördinate van D (2)
- 4.2.3 Oppervlakte van $\triangle BCD$ (3)
- 4.3 Die sirkel, met middelpunt N , word om die lyn $y = x$ gereflekteer. M is die middelpunt van die nuwe sirkel wat gevorm word. Die twee sirkels sny mekaar by A en F .
- Bereken die:
- 4.3.1 Lengte van NM (3)
- 4.3.2 Middelpunt van AF (4)

[15]

VRAAG 5

- 5.1 Vereenvoudig, **sonder om 'n sakrekenaar te gebruik**, die volgende uitdrukking tot 'n ENKELE trigonometriese verhouding:

$$\frac{\sin 140^\circ \cdot \sin(360^\circ - x)}{\cos 50^\circ \cdot \tan(-x)} \quad (6)$$

- 5.2 Bewys die identiteit: $\frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540^\circ - x)} = 2\cos x - 1$ (4)

- 5.3 Gegee: $\sin 36^\circ = \sqrt{1 - p^2}$

Bepaal, **sonder om 'n sakrekenaar te gebruik**, ELK van die volgende in terme van p :

5.3.1 $\tan 36^\circ$ (3)

5.3.2 $\cos 108^\circ$ (4)

[17]**VRAAG 6**

- 6.1 Gegee: $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

6.1.1 Gebruik die identiteit gegee om 'n formule vir $\cos(\alpha + \beta)$ af te lei. (3)

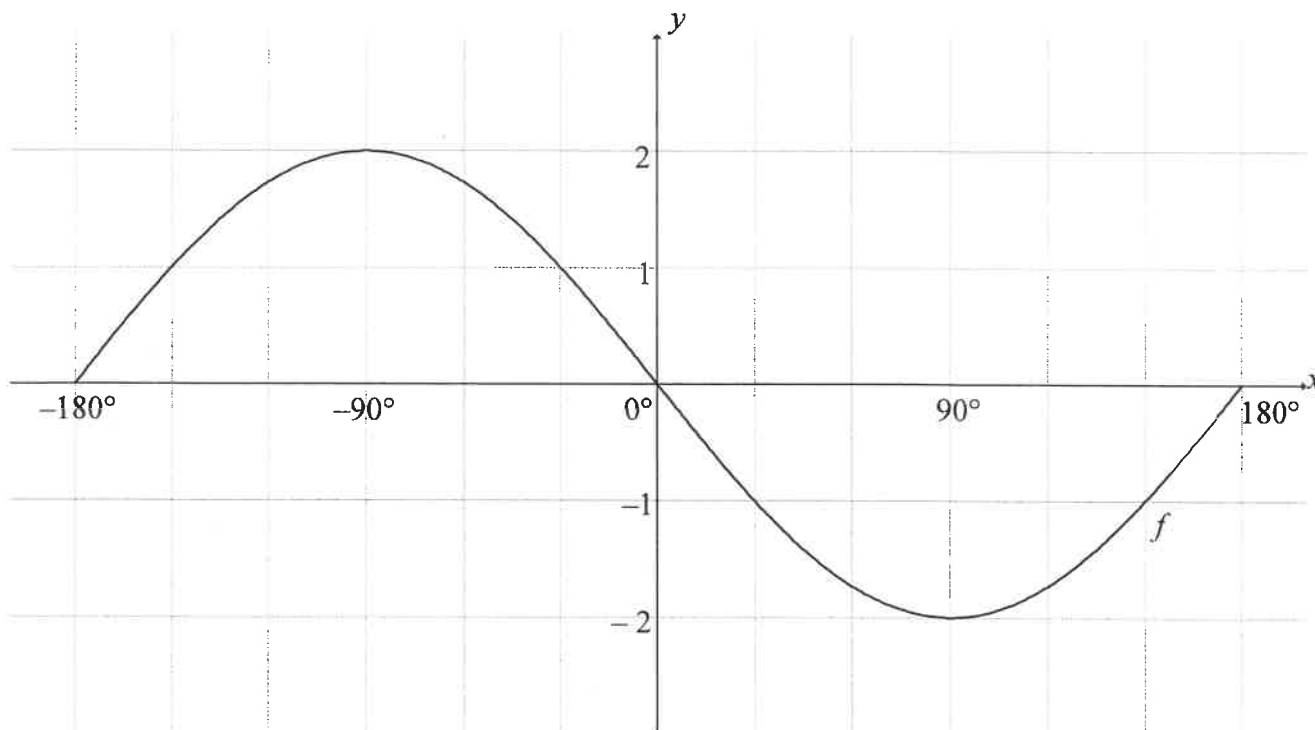
6.1.2 Vereenvoudig volledig: $2\cos 6x \cos 4x - \cos 10x + 2\sin^2 x$ (5)

- 6.2 Bepaal die algemene oplossing van $\tan x = 2\sin 2x$, waar $\cos x < 0$. (7)

[15]

VRAAG 7

In die diagram hieronder is die grafiek van $f(x) = -2\sin x$ vir die interval $x \in [-180^\circ; 180^\circ]$ geskets.

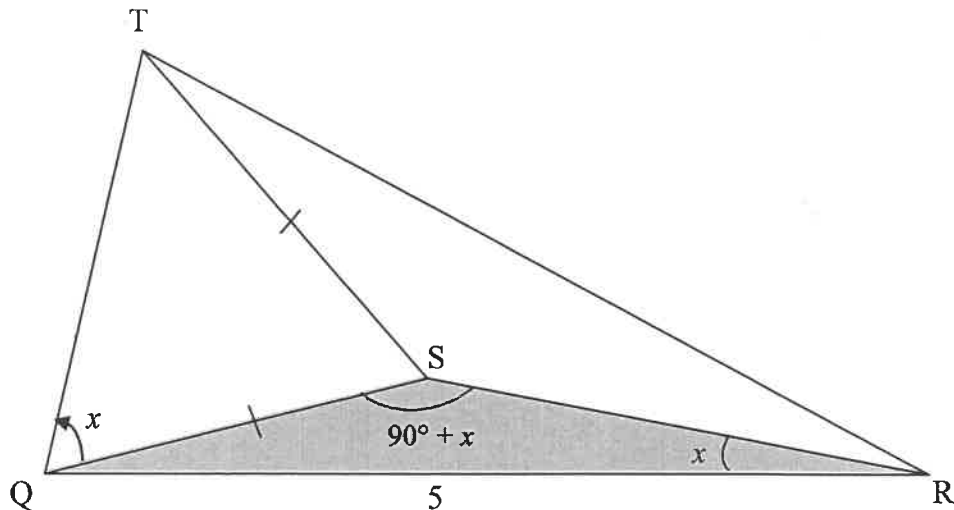


- 7.1 Op die rooster wat in die ANTWOORDEBOEK verskaf word, skets die grafiek van $g(x) = \cos(x - 60^\circ)$ vir $x \in [-180^\circ; 180^\circ]$. Toon duidelik ALLE afsnitte met die asse en die draaipunte van die grafiek aan. (3)
- 7.2 Skryf die periode van $f(3x)$ neer. (2)
- 7.3 **Gebruik die grafieke** om die waarde van x in die interval $x \in [-180^\circ; 180^\circ]$ te bepaal waarvoor $f(x) - g(x) = 1$. (1)
- 7.4 Skryf die waardeversameling van k neer, indien $k(x) = \frac{1}{2}g(x) + 1$. (2)
- [8]

VRAAG 8

In die diagram hieronder is T 'n haak in die plafon van 'n kunsgalery. Q , S en R is punte op dieselfde horisontale vlak waarvandaan drie persone na die haak T kyk. Die hoogtehoek vanaf Q na T is x .

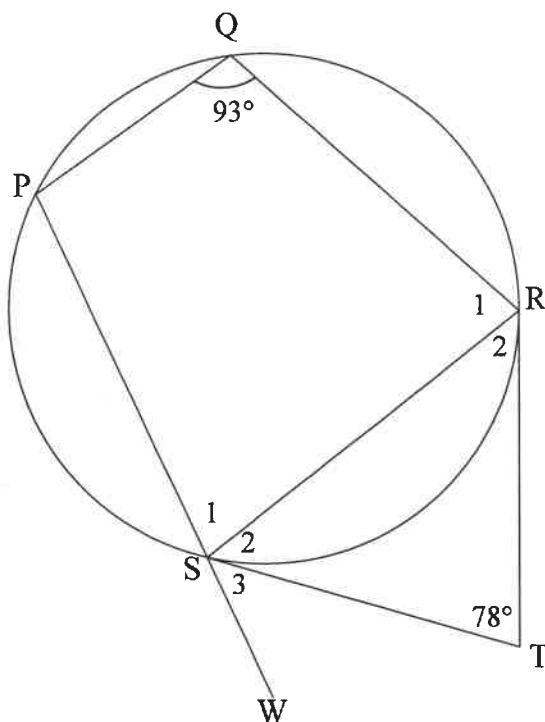
$\widehat{QSR} = 90^\circ + x$, $\widehat{QRS} = x$, $QR = 5$ eenhede en $TS = SQ$.



- 8.1 Bewys dat $QS = 5 \tan x$ (3)
- 8.2 Bewys dat die lengte van $QT = 10 \sin x$ (5)
- 8.3 Bereken die oppervlakte van ΔTQR as $\widehat{TQR} = 70^\circ$ en $x = 25^\circ$. (2)
- [10]**

VRAAG 9

In die diagram is PQRS 'n koordevierhoek. PS word verleng na W. TR en TS is raaklyne aan die sirkel by R en S onderskeidelik. $\hat{T} = 78^\circ$ en $\hat{Q} = 93^\circ$.



9.1 Gee 'n rede waarom $ST = TR$. (1)

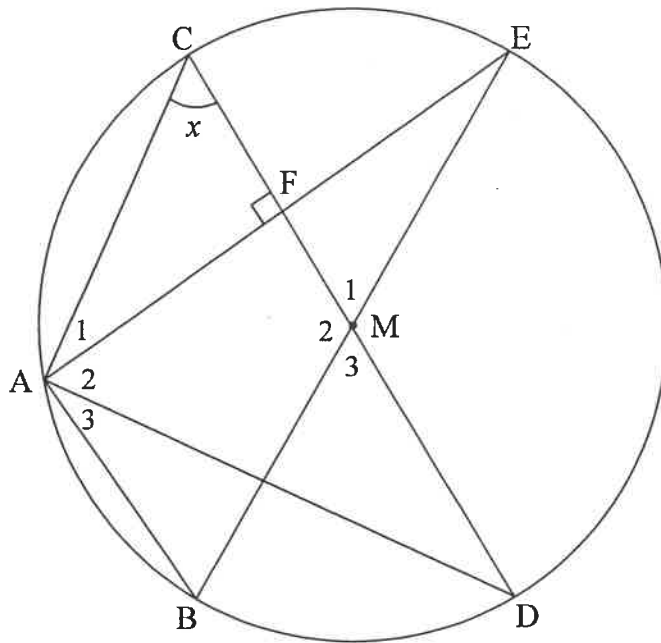
9.2 Bereken, met redes, die grootte van:

9.2.1 \hat{S}_2 (2)

9.2.2 \hat{S}_3 (2)
[5]

VRAAG 10

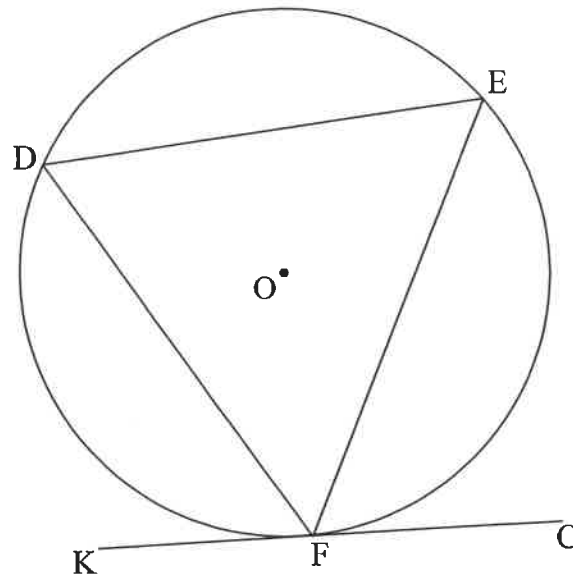
In die diagram is BE en CD middellyne van 'n sirkel met middelpunt M. Koord AE word getrek om CD by F te sny. $AE \perp CD$. Laat $\hat{C} = x$.



- 10.1 Gee 'n rede waarom $AF = FE$. (1)
 - 10.2 Bepaal, met redes, die grootte van \hat{M}_1 in terme van x . (3)
 - 10.3 Bewys, met redes, dat AD 'n raaklyn aan die sirkel is wat deur A, C en F gaan. (4)
 - 10.4 Indien dit gegee word dat $CF = 6$ eenhede en $AB = 24$ eenhede, bereken, met redes, die lengte van AE. (5)
- [13]**

VRAAG 11

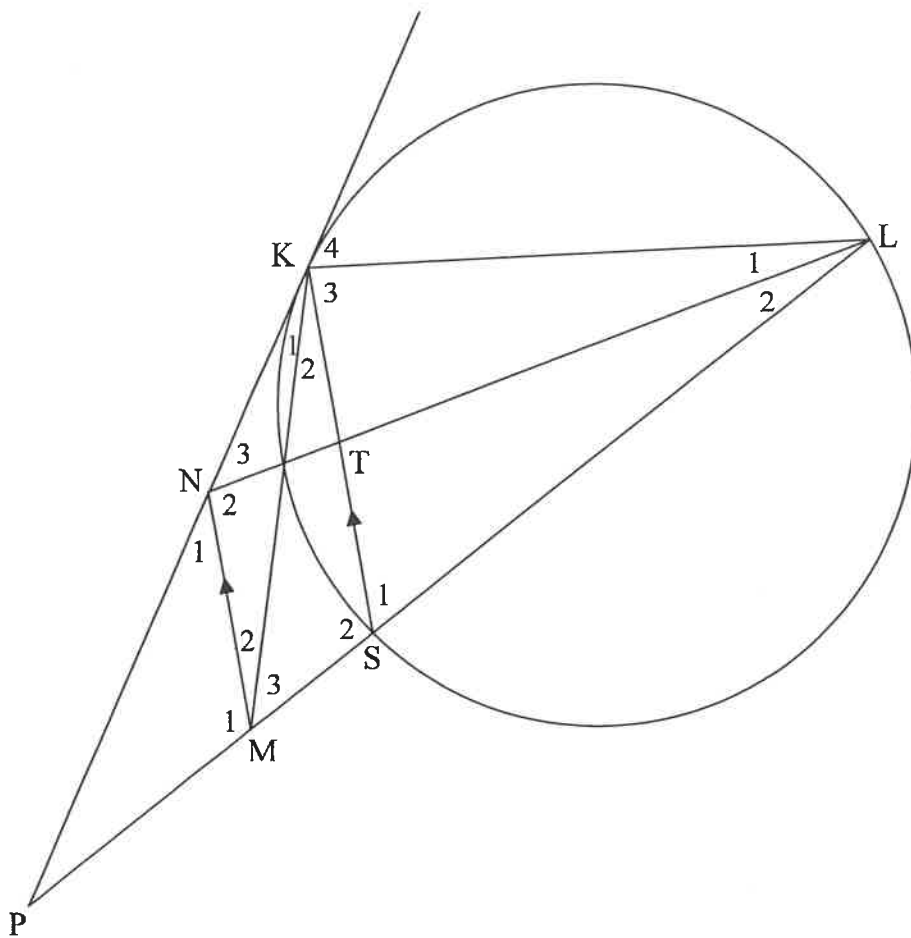
- 11.1 In die diagram is DE, EF en DF koorde in die sirkel met middelpunt O. KFC is 'n raaklyn aan die sirkel by F.



Bewys die stelling wat beweer dat $\hat{DFK} = \hat{E}$.

(5)

- 11.2 In die diagram is PK 'n raaklyn aan die sirkel by K. Koord LS is verleng na P. N en M is punte op KP en SP onderskeidelik sodanig dat $MN \parallel SK$. Koord KS en LN sny mekaar by T.



- 11.2.1 Bewys, met redes, dat:
- (a) $\hat{K}_4 = \hat{NML}$ (4)
 - b) KLMN 'n koordevierhoek is (1)
- 11.2.2 Bewys, met redes, dat $\Delta LKN \parallel \Delta KSM$. (5)
- 11.2.3 As $LK = 12$ eenhede en $3KN = 4SM$, bepaal die lengte van KS. (4)
- 11.2.4 As verder gegee word dat $NL = 16$ eenhede, $LS = 13$ eenhede en $KN = 8$ eenhede, bepaal, met redes, die lengte van LT. (4)

[23]

TOTAAL: 150

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) \quad P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG
1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.	1. Skryf jou eksamennummer en sentrumnummer duidelik in die ruimtes soos verskaf en plak jou stafieskodeplakker in die ruimte soos verskaf.
2. Remember that your own name (or the name of your school) may NOT appear anywhere on or in this answer book.	2. Onthou dat jou eie naam (of die naam van jou skool) NIE op of in hierdie antwoordeboek mag voorkom NIE.
3. Answer ALL questions in the spaces provided.	3. Beantwoord ALLE vrae in die ruimtes wat voorsien is.
4. NO pages may be torn from this answer book.	4. GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	5. Lees die instruksies wat op jou eksamenrooster gedruk is sorgvuldig deur, asook enige ander instruksies wat op elke eksamenvraestel gegee word.
6. Candidates may NOT retain an answer book or remove it from the examination room.	6. GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE.
7. Answers must be written in black/blue ink as distinctly as possible. Do NOT write in the margins.	7. Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.
8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.	8. Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.
9. If you require additional space for your answers: 9.1 Use the additional space provided at the end of the answer book. 9.2 When answering a question in the additional space, indicate clearly the question number in the column on the LHS. 9.3 Rule off after each answer.	9. In geval jy bykomende ruimte benodig vir jou antwoorde: 9.1 Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek verskaf word. 9.2 As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan. 9.3 Trek 'n lyn na elke antwoord.
10. Draw a neat line through any work/rough work that must NOT be marked.	10. Trek 'n netjiese lyn deur enige werk/rofwerk wat NIE nagesien moet word NIE.

QUESTION/VRAAG 1

1.1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

	Solution/Oplissing	Marks Punte
1.1.1		(2)
1.1.2		(1)
1.1.3		(2)
1.2		(2)
1.3.1		(2)
1.3.2		(1)
		[10]

QUESTION/VRAAG 2

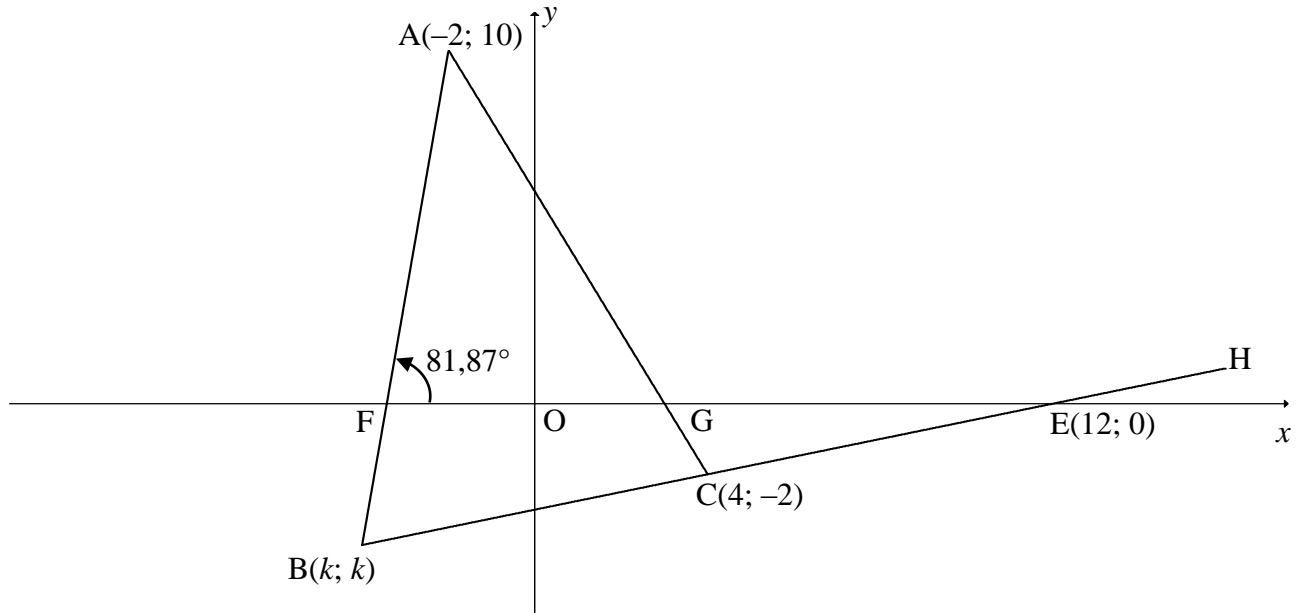
Price of milk in rands per 5-litre container (x)/ Prys van melk, in rand, per 5 liter-houer (x)	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y)/ Aantal 5 liter-houers melk verkoop (y)	48	30	26	44	23	32	39	29	42	33

	Solution/Oplissing	Marks Punte
2.1	<p style="text-align: center;">SCATTER PLOT/SPREIDIAGRAM</p>	

(3)

	Solution/Oplissing	Marks Punte
2.2		(3)
2.3		(2)
2.4		(2)
		[10]

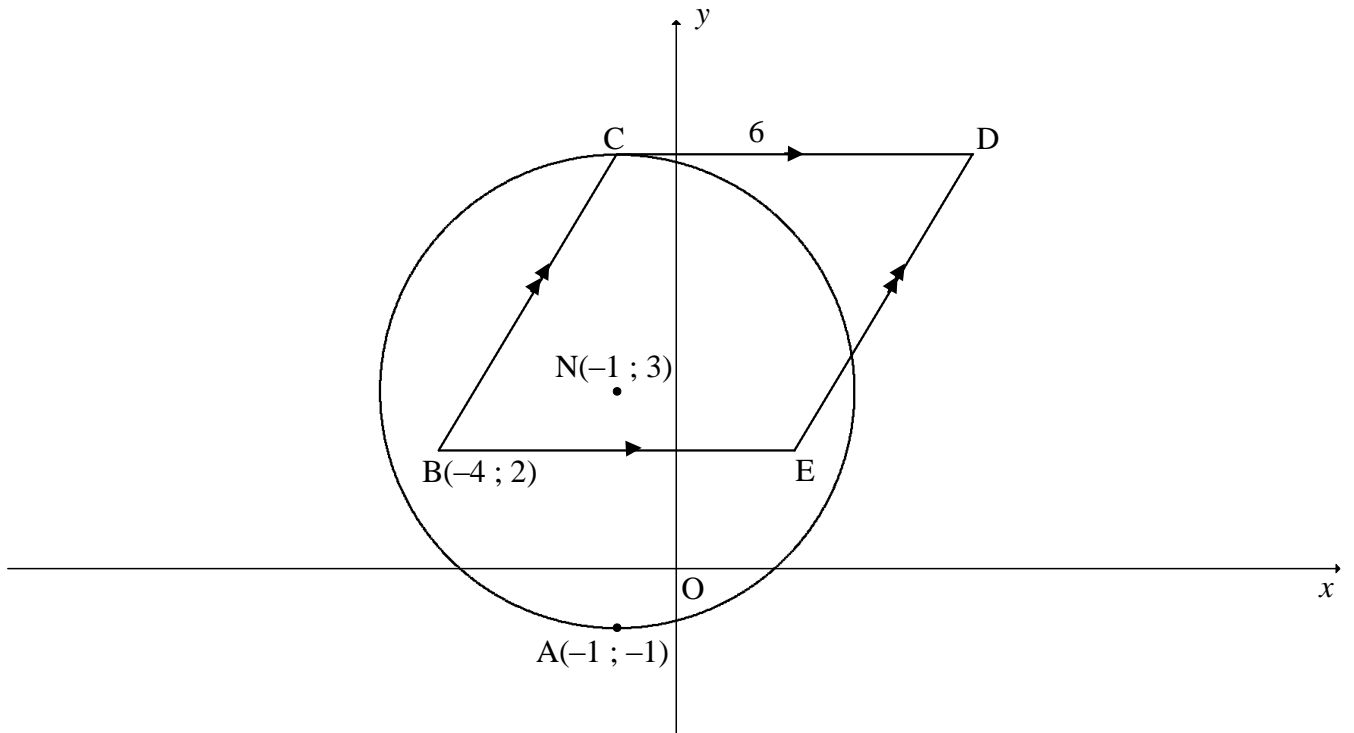
QUESTION/VRAAG 3



	Solution/Oplissing	Marks Punte
3.1.1		(2)
3.1.2		(2)
3.2		(2)
3.3.1		(2)

	Solution/Oplissing	Marks Punte
3.3.2		(4)
3.3.3		(2)
3.4.1		(5)
3.4.2(a)		(2)
3.4.2(b)		(3)
		[24]

QUESTION/VRAAG 4



	Solution/Oplissing	Marks Punte
4.1		(1)
4.2.1		(2)
4.2.2		(2)
4.2.3		(3)

	Solution/Oplissing	Marks Punte
4.3.1		
4.3.2		
		(4) [15]

QUESTION/VRAAG 5

	Solution/Oplissing	Marks Punte
5.1		(6)
5.2		(4)
5.3.1		(3)

5.3.2		
		[17]

QUESTION/VRAAG 6

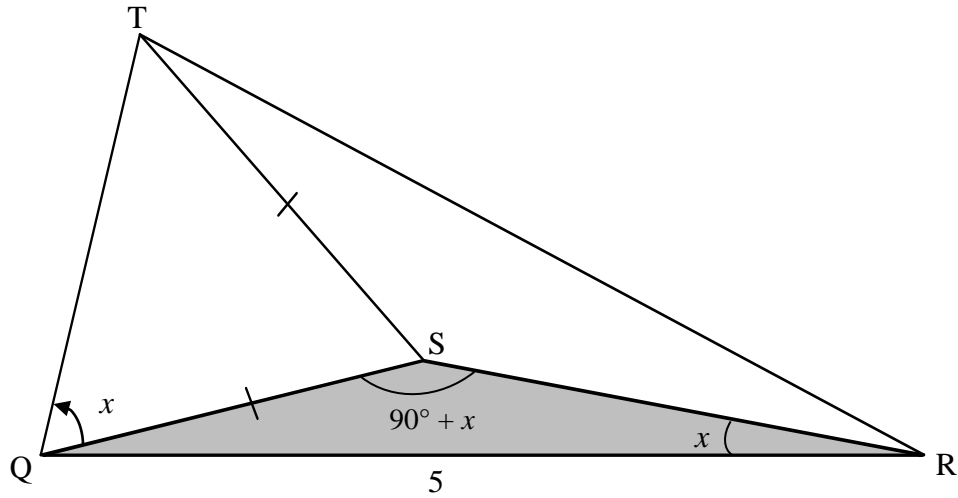
	Solution/Oplissing	Marks Punte
6.1.1	<div style="border: 1px solid black; height: 150px; width: 100%;"></div>	(3)
6.1.2	<div style="border: 1px solid black; height: 250px; width: 100%;"></div>	(5)

QUESTION/VRAAG 7

	Solution/Oplissing	Marks Punte
7.1		(3)
7.2	<hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	(2)
7.3	<hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	(1)
7.4	<hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	(2)
		[8]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.
 Gee redes vir jou bewerings in VRAAG 8, 9, 10 en 11.

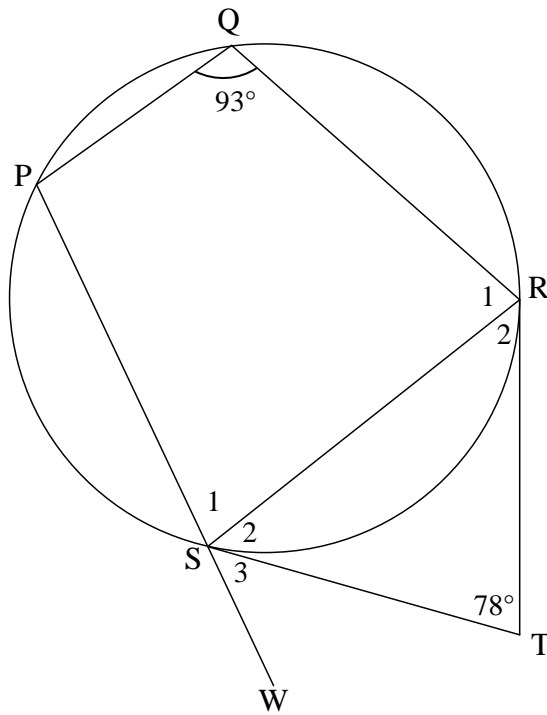
QUESTION/VRAAG 8



	Solution/Oplissing	Marks Punte
8.1		(3)
8.2		(5)

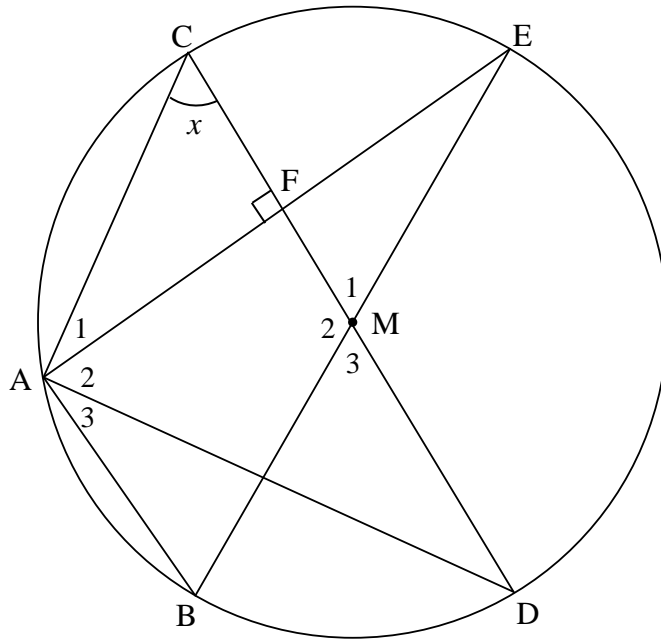
8.3		
		(2)
		[10]

QUESTION/VRAAG 9



	Solution/Oplissing	Marks Punte
9.1		(1)
9.2.1		(2)
9.2.2		(2)
		[5]

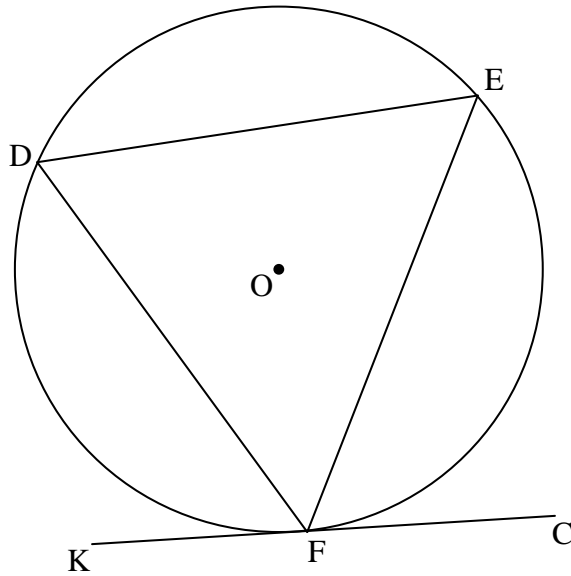
QUESTION/VRAAG 10



	Solution/Oplissing	Marks Punte
10.1		(1)
10.2		(3)
10.3		(4)
10.4		(5)
		[13]

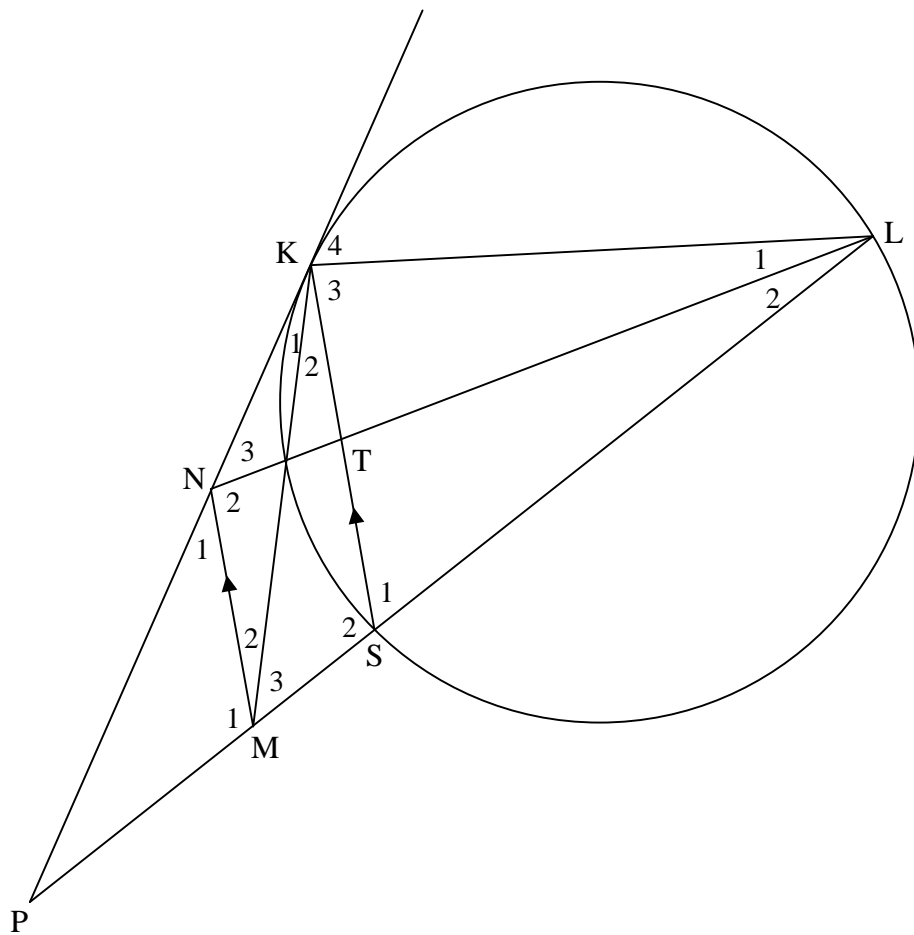
QUESTION/VRAAG 11

11.1



	Solution/Oplossing	Marks Punte
		(5)

11.2



	Solution/Oplissing	Marks Punte
11.2.1(a)		(4)
11.2.1(b)		(1)

	Solution/Oplissing	Marks Punte
11.2.2		(5)
11.2.3		(4)
11.2.4		(4)
		[23]



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL SENIOR CERTIFICATE/
*NASIONALE SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

**These marking guidelines consist of 24 pages.
*Hierdie nasienriglyne bestaan uit 24 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)</i>
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)</i>
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

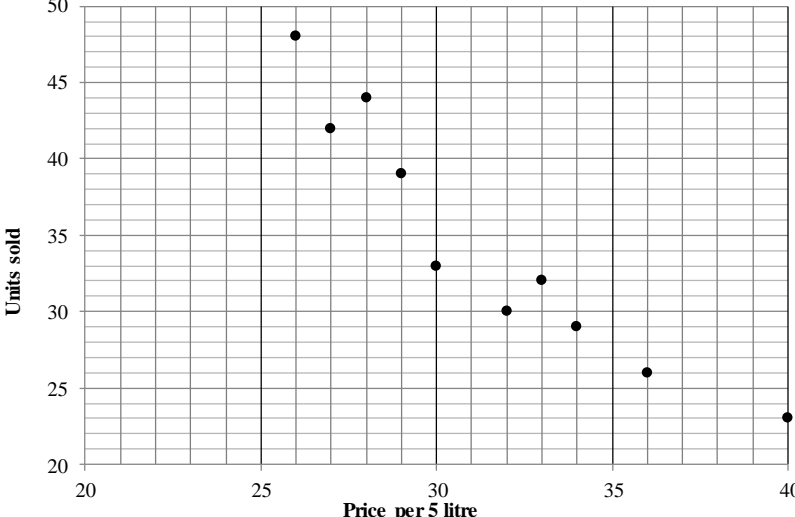
QUESTION/VRAAG 1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

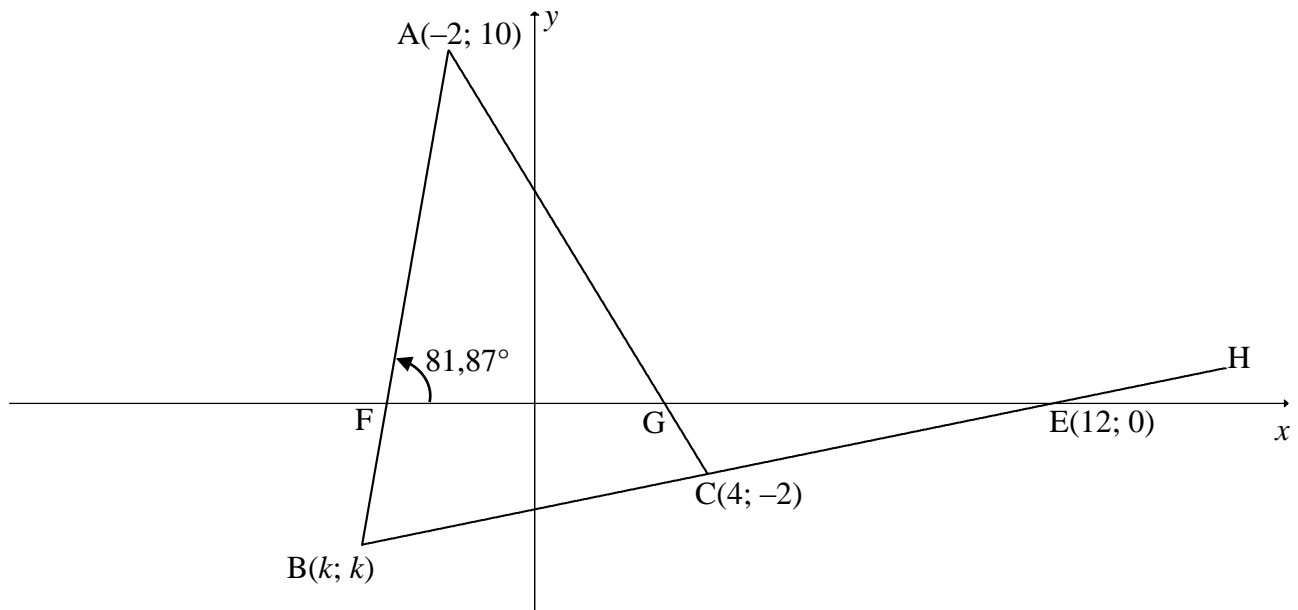
1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Answer only: Full marks <i>Slegs antw: Volpunte</i> </div>	✓ 396 ✓ answer (2)
1.1.2	$\sigma = 10,1707 \approx 10,17$		✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32,17$ ∴ 5 days		✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/ <i>bestel</i> $20 \times 18 = 360$ sold/ <i>verkoop</i> Total not sold/ <i>Totaal nie verkoop nie</i> : 36 OR/OF $22 - 20 = 2$ $2 \times 18 = 36$		✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2) ✓ $\bar{x}_1 - \bar{x}_2$ ✓ answer (2)
1.3.1	Option B/ <i>Opsie B</i> <u>Any one of the following reasons/<i>Enige een van die vlg redes</i>:</u> <ul style="list-style-type: none"> • Median/<i>Mediaan</i> = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right 		✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right <i>Data is positief skeef/skeef na regs</i>		✓ answer (1)
			[10]

QUESTION/VRAAG 2

Price of milk in rands per 5-litre container (x) <i>Prys van melk in rand, per 5 liter-houer (x)</i>	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y) <i>Aantal 5 liter-houers melk verkoop (y)</i>	48	30	26	44	23	32	39	29	42	33

<p>2.1</p>	<p style="text-align: center;">SCATTER PLOT</p> 	<p>1 mark: 3 to 5 points plotted correctly</p> <p>2 marks: 6 to 9 points plotted correctly</p> <p>3 marks: all points plotted correctly</p> <p style="text-align: right;">(3)</p>
<p>2.2</p>	<p>$a = 90,478... \approx 90,48$ $b = -1,773... \approx -1,77$ $\hat{y} = 90,48 - 1,77x$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>Answer only: Full marks Slegs antw: Volpunte</p> </div>	<p>✓ a ✓ b ✓ equation</p> <p style="text-align: right;">(3)</p>
<p>2.3</p>	<p>$y = 23,069... \approx 23,07$ units/eenhede (calculator/sakrekenaar)</p> <p>OR/OF</p> <p>$y = 90,48 - 1,77(38)$ $y = 23,22$ units/eenhede</p>	<p>✓✓ answer</p> <p style="text-align: right;">(2)</p> <p>✓ substitution ✓ answer</p> <p style="text-align: right;">(2)</p>
<p>2.4</p>	<p>$r = -0,94$</p> <p>The value of r indicates a strong relationship between the cost per 5 litre and the number of units sold \therefore there is a good chance of the prediction being accurate./</p> <p><i>Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan \therefore daar is 'n goeie kans dat die voorspelling akkuraat is</i></p>	<p>✓ value of r OR/OF strong relationship/ <i>sterk verwantskap</i></p> <p>✓ accurate/akkuraat</p> <p style="text-align: right;">(2)</p>
<p>[10]</p>		

QUESTION/VRAAG 3

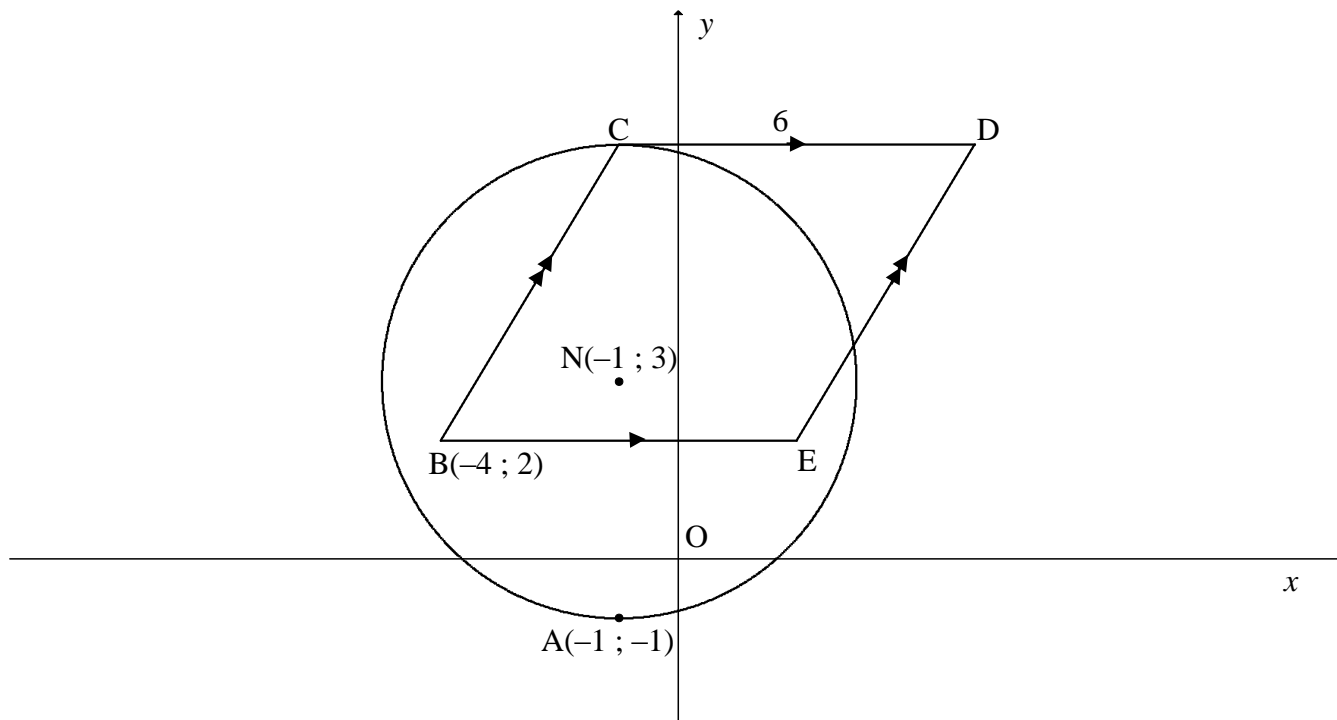


3.1.1	$m_{BE} = m_{CE} = \frac{0 - (-2)}{12 - 4} \quad \text{OR/OF} \quad m_{BE} = m_{CE} = \frac{-2 - 0}{4 - 12}$ $= \frac{1}{4} \qquad \qquad \qquad = \frac{1}{4}$	✓ substitution C & E ✓ answer (2)		
3.1.2	$m_{AB} = \tan 81,87^\circ$ $m_{AB} = 7$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-top: 10px;"> Answer only: Full marks <i>Slegs antw: Volpunte</i> </div>	✓ substitution ✓ answer (2)		
3.2	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; vertical-align: top;"> $y = mx + c$ $0 = \frac{1}{4}(12) + c$ $c = -3$ $y = \frac{1}{4}x - 3$ <p>OR/OF</p> $y = mx + c$ $-2 = \frac{1}{4}(4) + c$ $c = -3$ $y = \frac{1}{4}x - 3$ </td> <td style="width: 50%; vertical-align: top;"> $y - y_1 = m(x - x_1)$ $\text{or } y - 0 = \frac{1}{4}(x - 12)$ $y = \frac{1}{4}x - 3$ $y - y_1 = m(x - x_1)$ $\text{or } y - (-2) = \frac{1}{4}(x - 4)$ $y = \frac{1}{4}x - 3$ </td> </tr> </table>	$y = mx + c$ $0 = \frac{1}{4}(12) + c$ $c = -3$ $y = \frac{1}{4}x - 3$ <p>OR/OF</p> $y = mx + c$ $-2 = \frac{1}{4}(4) + c$ $c = -3$ $y = \frac{1}{4}x - 3$	$y - y_1 = m(x - x_1)$ $\text{or } y - 0 = \frac{1}{4}(x - 12)$ $y = \frac{1}{4}x - 3$ $y - y_1 = m(x - x_1)$ $\text{or } y - (-2) = \frac{1}{4}(x - 4)$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)
$y = mx + c$ $0 = \frac{1}{4}(12) + c$ $c = -3$ $y = \frac{1}{4}x - 3$ <p>OR/OF</p> $y = mx + c$ $-2 = \frac{1}{4}(4) + c$ $c = -3$ $y = \frac{1}{4}x - 3$	$y - y_1 = m(x - x_1)$ $\text{or } y - 0 = \frac{1}{4}(x - 12)$ $y = \frac{1}{4}x - 3$ $y - y_1 = m(x - x_1)$ $\text{or } y - (-2) = \frac{1}{4}(x - 4)$ $y = \frac{1}{4}x - 3$			

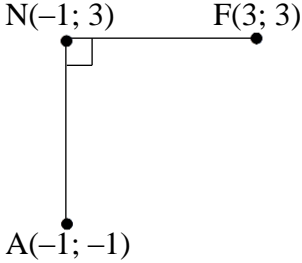
<p>3.3.1</p>	$y = \frac{1}{4}x - 3$ $k = \frac{1}{4}k - 3$ $\frac{3}{4}k = -3$ $k = -4$ $\therefore B(-4; -4)$ <p>OR/OF</p> $m_{BE} = \frac{1}{4}$ $\frac{0 - k}{12 - k} = \frac{1}{4}$ $-4k = 12 - k$ $k = -4$ $\therefore B(-4; -4)$ <p>OR/OF</p> $m_{AB} = \tan 81,87^\circ$ $m_{AB} = 7$ $m_{AB} = \frac{10 - k}{-2 - k}$ $7(-2 - k) = 10 - k$ $-14 - 7k = 10 - k$ $-6k = 24$ $k = -4$ $\therefore B(-4; -4)$ <p>OR/OF</p> <p>EB: $y = \frac{1}{4}x - 3$ and AB: $y = 7x + 24$</p> $\frac{1}{4}x - 3 = 7x + 24$ $\frac{27}{4}x = -27$ $x = k = -4$ $\therefore B(-4; -4)$	<p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ substitution</p> <p>✓ answer (2)</p> <p>✓ equating EB & AB</p> <p>✓ answer (2)</p>
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<p>3.3.2</p>	<p>In ΔAFG:</p> $m_{AC} = \frac{10 - (-2)}{-2 - 4} = -2$ <p>$\tan \theta = m_{AC} = -2$</p> <p>$\theta = 180^\circ - 63,43\dots^\circ$</p> <p>$\therefore \theta = 116,57^\circ$</p> <p>$\therefore \hat{A} = 116,57^\circ - 81,87^\circ$ [ext \angle of Δ]</p> <p>$\therefore \hat{A} = 34,70^\circ$</p> <p>OR/OF</p> <p>In ΔABC:</p> $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc \cdot \cos A$ $(2\sqrt{17})^2 = (6\sqrt{5})^2 + (10\sqrt{2})^2 - 2(6\sqrt{5})(10\sqrt{2}) \cdot \cos A$ $\cos A = \frac{(6\sqrt{5})^2 + (10\sqrt{2})^2 - (2\sqrt{17})^2}{2(6\sqrt{5})(10\sqrt{2})}$ $= 0,822\dots$ <p>$\therefore A = 34,7^\circ$</p>	<p>✓ $m_{AC} = -2$</p> <p>✓ $\tan \theta = -2$</p> <p>✓ $\theta = 116,57^\circ$</p> <p>✓ answer (4)</p> <p>✓ all 3 lengths</p> <p>✓ substitution into the correct cosine rule</p> <p>✓ cos A subject</p> <p>✓ answer (4)</p>
<p>3.3.3</p>	$M\left(\frac{12 + (-2)}{2}; \frac{10 + (0)}{2}\right)$ <p>Diagonals intersect at the point (5 ; 5)</p>	<p>✓ x-value ✓ y-value (2)</p>
<p>3.4.1</p>	<p>BE = ET</p> $4\sqrt{17} = \sqrt{(12 - p)^2 + (0 - p)^2}$ $(4\sqrt{17})^2 = (\sqrt{(12 - p)^2 + (0 - p)^2})^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p - 16)(p + 4) = 0$ <p>$\therefore p = 16$ or $p = -4$ (n.a.)</p> <p>$\therefore T(16; 16)$</p>	<p>✓ substitution of E & T</p> <p>✓ equating</p> <p>✓ standard form</p> <p>✓ factors</p> <p>✓ $p = 16$ (5)</p>
<p>3.4.2a</p>	$(x - 12)^2 + y^2 = (4\sqrt{17})^2 = 272$	<p>✓ LHS ✓ RHS (2)</p>
<p>3.4.2b</p>	$m_{\text{radius}} = \frac{1}{4}$ $m_{\text{tangent}} = -4$ <p>$y = -4x + c$ OR/OF $y - y_1 = -4(x - x_1)$</p> <p>$-4 = -4(-4) + c$ $y - (-4) = -4(x - (-4))$</p> <p>$c = -20$ $y = -4x - 20$</p> <p>$y = -4x - 20$</p>	<p>✓ m_{tangent}</p> <p>✓ substitution of B</p> <p>✓ equation (3)</p>
<p>[24]</p>		

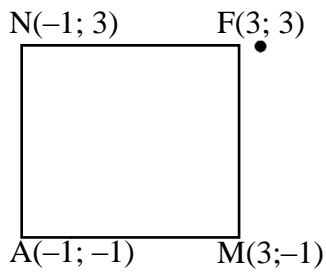
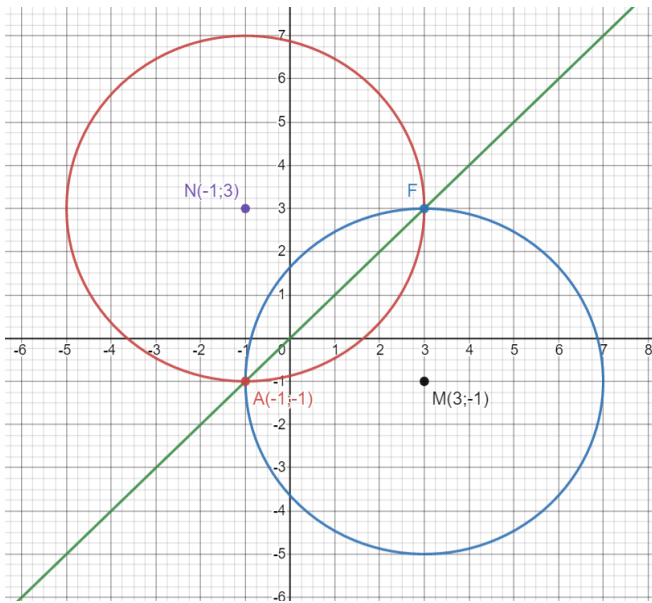
QUESTION/VRAAG 4



4.1	Radius = 4 units/eenhede	✓ answer (1)
4.2.1	CD ⊥ CN ∴ C(-1 ; 7)	✓ x value ✓ y value (2)
4.2.2	CD = 6 units ∴ D(5 ; 7)	✓ x value ✓ y value (2)
4.2.3	<p>⊥ h = 5 units DC = 6 units Area ΔBCD = $\frac{1}{2} (6)(5)$ = 15 units²</p> <p>OR/OF</p> <p>⊥ h = 5 units DC = 6 units Area ΔBCD = $\frac{1}{2}$ [Area of ^m] = $\frac{1}{2} [(5)(6)]$ = 15 units²</p>	<p>✓ ⊥ h = 5 units ✓ substitution into Area formula ✓ answer (3)</p> <p>✓ ⊥ h = 5 units ✓ substitution into Area formula ✓ answer (3)</p>

	<p>OR/OF Let angle of inclination of BC = α $\tan \alpha = \frac{5}{3}$ $\alpha = 59,036\dots^\circ$</p> <p>$\widehat{BCD} = 180^\circ - \alpha$ $\widehat{BCD} = 180^\circ - 59,036\dots^\circ$ $\widehat{BCD} = 120,96^\circ$</p> <p>Area $\triangle BCD = \frac{1}{2}(\sqrt{34})(6) \sin 120,96^\circ$ $= 15 \text{ units}^2$</p>	<p>✓ $\widehat{BCD} = 120,96^\circ$</p> <p>✓ substitution into Area rule</p> <p>✓ answer (3)</p>
<p>4.3.1</p>	<p>$M(3 ; -1)$ [reflection of $N(-1 ; 3)$ about the line $y = x$] $\therefore MN = \sqrt{(3 - (-1))^2 + (-1 - 3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5,66 \text{ units}$</p>	<p>✓ coordinates of M (A)</p> <p>✓ substitution of M&N</p> <p>✓ answer (3)</p>
<p>4.3.2</p>	<p>$M(3 ; -1)$ $m_{MN} = \frac{3 - (-1)}{-1 - 3} = -1$</p> <p>MN: $-1 = -(3) + c$ or $y - 3 = -1(x + 1)$ $c = 2$ $y - 3 = -x - 1$ $\therefore y = -x + 2$ $y = -x + 2$</p> <p>$x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1)</p> <p>OR/OF</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 20px;"> <p>$N(-1 ; 3)$ $y_F = y_N = 3$ Reflected about $y = x$ $\therefore F(3 ; 3)$</p> <p>midpoint $\left(\frac{-1 + 3}{2}; \frac{-1 + 3}{2}\right) = (1 ; 1)$</p> </div>  </div>	<p>✓ equation of MN</p> <p>✓ equating AF & MN</p> <p>✓ x value ✓ y value (4)</p> <p>✓ ✓ coordinates of F</p> <p>✓ x value ✓ y value (4)</p>

OR/OF



NAMF is a square (NA=NF=AM=MF and NA ⊥ AM)

Midpoint NM = (1 ; 1)
= Midpoint of AF

✓ NAMF = square

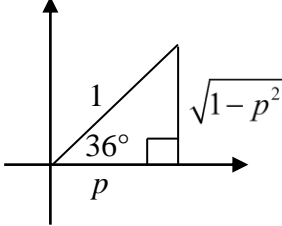
✓ x ✓ y of midpt NM

✓ midpt AF

(4)

[15]

QUESTION/VRAAG 5

<p>5.1</p>	$\frac{\sin 140^\circ \cdot \sin(360^\circ - x)}{\cos 50^\circ \cdot \tan(-x)}$ $= \frac{\sin 40^\circ (-\sin x)}{\sin 40^\circ (-\tan x)}$ $= \frac{-\sin x}{-\frac{\sin x}{\cos x}}$ $= \cos x$	<p>✓ $\sin 40^\circ$ ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p>✓ answer</p> <p style="text-align: right;">(6)</p>
<p>5.2</p>	$\text{LHS} = \frac{-2\sin^2 x + \cos x + 1}{1 - \cos(540^\circ - x)} \qquad \text{RHS} = 2\cos x - 1$ $\text{LHS} = \frac{-2(1 - \cos^2 x) + \cos x + 1}{1 - (-\cos x)}$ $\text{LHS} = \frac{-2 + 2\cos^2 x + \cos x + 1}{1 + \cos x}$ $\text{LHS} = \frac{2\cos^2 x + \cos x - 1}{1 + \cos x}$ $\text{LHS} = \frac{(2\cos x - 1)(\cos x + 1)}{1 + \cos x}$ $\text{LHS} = 2\cos x - 1$ <p>$\therefore \text{LHS} = \text{RHS}$</p>	<p>✓ identity i. t. o. $\cos x$ ✓ $\cos(540^\circ - x) = -\cos x$</p> <p>✓ standard form</p> <p>✓ factors</p> <p style="text-align: right;">(4)</p>
<p>5.3.1</p>	$\sin 36^\circ = \sqrt{1 - p^2}$ $\tan 36^\circ = \frac{\sqrt{1 - p^2}}{p}$ <p>OR/OF</p> $\cos^2 36^\circ = 1 - \sin^2 36^\circ$ $\cos 36^\circ = \sqrt{1 - (1 - p^2)}$ $= p$ $\tan 36^\circ = \frac{\sin 36^\circ}{\cos 36^\circ}$ $= \frac{\sqrt{1 - p^2}}{p}$	<div style="text-align: center;">  </div> <p>✓ method ✓ value of p ✓ answer</p> <p style="text-align: right;">(3)</p> <p>✓ method</p> <p>✓ $\cos 36^\circ = p$</p> <p>✓ answer</p> <p style="text-align: right;">(3)</p>

<p>5.3.2</p>	<p> $\cos 108^\circ$ $= -\cos 72^\circ$ $= -\cos (2 \times 36^\circ)$ $= -(2 \cos^2 36^\circ - 1)$ $= -2p^2 + 1$ </p> <p>OR/OF</p> <p> $\cos 108^\circ$ $= -\cos 72^\circ$ $= -\cos (2 \times 36^\circ)$ $= -(1 - 2 \sin^2 36^\circ)$ $= -1 + 2(\sqrt{1 - p^2})^2$ $= -1 + 2(1 - p^2)$ $= -2p^2 + 1$ </p> <p>OR/OF</p> <p> $\cos 108^\circ$ $= -\cos 72^\circ$ $= -\cos (2 \times 36^\circ)$ $= -(\cos^2 36^\circ - \sin^2 36^\circ)$ $= -\left(p^2 - (\sqrt{1 - p^2})^2\right)$ $= -(p^2 - (1 - p^2))$ $= -2p^2 + 1$ </p> <p>OR/OF</p> <p> $\cos 108^\circ$ $= \cos(2 \times 54^\circ)$ $= 2 \cos^2 54^\circ - 1$ $= 2(1 - p^2) - 1$ $= 1 - 2p^2$ </p> <p>OR/OF</p> <p> $\cos 108^\circ = \cos(72^\circ + 36^\circ)$ $= \cos 72^\circ \cos 36^\circ - \sin 72^\circ \sin 36^\circ$ $= (2 \cos^2 36^\circ - 1) \cos 36^\circ - (2 \sin 36^\circ \cos 36^\circ) \sin 36^\circ$ $= 2 \cos^3 36^\circ - \cos 36^\circ - 2 \cos 36^\circ \sin^2 36^\circ$ $= 2p^3 - p - 2p(\sqrt{1 - p^2})^2$ $= 2p^3 - p - 2p + 2p^3$ $= 4p^3 - 3p$ </p>	<p> ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) </p> <p> ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) </p> <p> ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) </p> <p> ✓ double angle ✓✓ expansion ✓ answer i. t. o. p (4) </p> <p> ✓ expansion ✓ both double angle identities ✓ value of $\sin 36^\circ$ ✓ answer i. t. o. p (4) </p>
<p>[17]</p>		

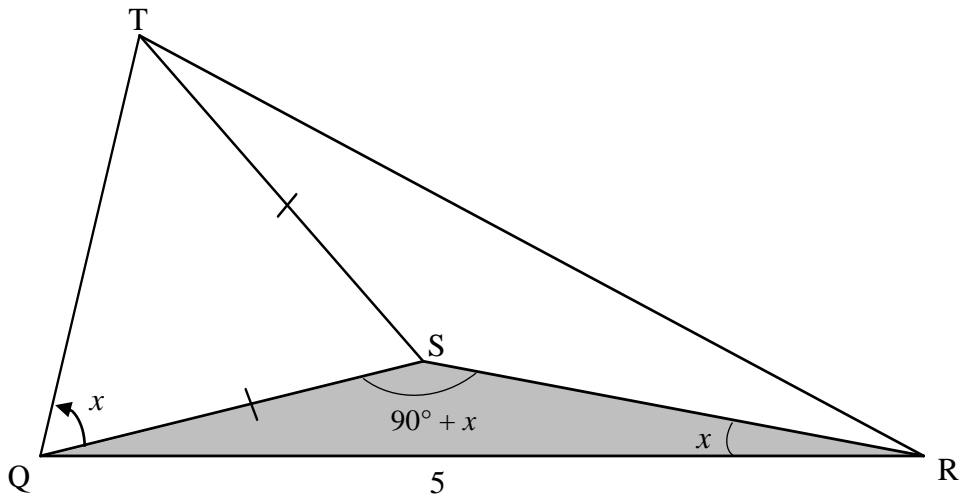
QUESTION/VRAAG 6

<p>6.1.1</p>	$\begin{aligned} &\cos(\alpha + \beta) \\ &= \cos(\alpha - (-\beta)) \\ &= \cos \alpha \cos(-\beta) + \sin \alpha \sin(-\beta) \\ &= \cos \alpha \cos \beta + \sin \alpha(-\sin \beta) \\ &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{aligned}$	<p>✓ $\cos(\alpha - (-\beta))$ ✓ expansion ✓ reduction (3)</p>
<p>6.1.2</p>	$\begin{aligned} &2 \cos 6x \cos 4x - \cos 10x + 2 \sin^2 x \\ &= 2 \cos 6x \cos 4x - \cos(6x + 4x) + 2 \sin^2 x \\ &= 2 \cos 6x \cos 4x - (\cos 6x \cos 4x - \sin 6x \sin 4x) + 2 \sin^2 x \\ &= \cos 6x \cos 4x + \sin 6x \sin 4x + 2 \sin^2 x \\ &= \cos 2x + 2 \sin^2 x \\ &= 1 - 2 \sin^2 x + 2 \sin^2 x \\ &= 1 \end{aligned}$	<p>✓ $\cos 10x = \cos(6x + 4x)$ ✓ expansion of $\cos(6x + 4x)$ ✓ $\cos 2x$ ✓ $1 - 2 \sin^2 x$ ✓ answer (5)</p>
<p>6.2</p>	$\begin{aligned} &\tan x = 2 \sin 2x \\ &\frac{\sin x}{\cos x} = 2(2 \sin x \cos x) \\ &\sin x = 4 \sin x \cos^2 x \\ &4 \sin x \cos^2 x - \sin x = 0 \\ &\sin x(4 \cos^2 x - 1) = 0 \\ &\sin x = 0 \qquad \qquad \qquad \text{or} \qquad \qquad \qquad \cos^2 x = \frac{1}{4} \\ &\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \cos x = -\frac{1}{2} \\ &x = 180^\circ + k.360^\circ; k \in Z \qquad \text{or} \qquad x = 120^\circ + k.360^\circ; k \in Z \\ &\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad x = 240^\circ + k.360^\circ; k \in Z \\ &\mathbf{OR/OF} \\ &\tan x = 2 \sin 2x \\ &\frac{\sin x}{\cos x} = 4 \sin x \cos x \\ &\sin x = 4 \sin x \cos^2 x \\ &4 \sin x \cos^2 x - \sin x = 0 \\ &4 \sin x(1 - \sin^2 x) - \sin x = 0 \\ &3 \sin x - 4 \sin^3 x = 0 \\ &\sin x(3 - 4 \sin^2 x) = 0 \\ &\sin x = 0 \qquad \text{or} \qquad \sin^2 x = \frac{3}{4} \\ &\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \sin x = \frac{\sqrt{3}}{2} \qquad \text{or} \qquad \sin x = -\frac{\sqrt{3}}{2} \\ &x = 180^\circ + k.360^\circ, k \in Z \qquad \text{or} \qquad x = 120^\circ + k.360^\circ, k \in Z \\ &\qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \text{or} \qquad x = 240^\circ + k.360^\circ, k \in Z \end{aligned}$	<p>✓ quotient identity ✓ double angle identity ✓ factors ✓ both equations ✓ $x = 180^\circ$ ✓ $x = 120^\circ \& 240^\circ$ OR/OF $x = \pm 120^\circ$ ✓ $k.360^\circ; k \in Z$ (7) ✓ quotient identity ✓ identity ✓ factors ✓ both equations ✓ $x = 180^\circ$ ✓ $x = 120^\circ \& 240^\circ$ OR/OF $x = \pm 120^\circ$ ✓ $k.360^\circ; k \in Z$ (7)</p>
		<p>[15]</p>

QUESTION/VRAAG 7

<p>7.1</p>		<ul style="list-style-type: none"> ✓ both turning points ✓ both x intercepts (-30° & 150°) ✓ shape <p style="text-align: right;">(3)</p>
<p>7.2</p>	<p>Period = 120°</p>	<ul style="list-style-type: none"> ✓✓ answer <p style="text-align: right;">(2)</p>
<p>7.3</p>	<p>$x = -30°$</p>	<ul style="list-style-type: none"> ✓ answer <p style="text-align: right;">(1)</p>
<p>7.4</p>	<p>Range of/waardeversameling van g: $y \in [-1; 1]$</p> <p>Range of/Waardeversameling van $\frac{1}{2}g$: $y \in \left[-\frac{1}{2}; \frac{1}{2}\right]$</p> <p>Range of/Waardeversameling van $\frac{1}{2}g + 1$: $y \in \left[\frac{1}{2}; \frac{3}{2}\right]$</p> <p>OR/OF</p> <p>Range of/Waardeversameling van $\frac{1}{2}g + 1$: $\frac{1}{2} \leq y \leq \frac{3}{2}$</p>	<ul style="list-style-type: none"> ✓ critical values ✓ correct notation <p style="text-align: right;">(2)</p> <ul style="list-style-type: none"> ✓ critical values ✓ correct notation <p style="text-align: right;">(2)</p>
<p>[8]</p>		

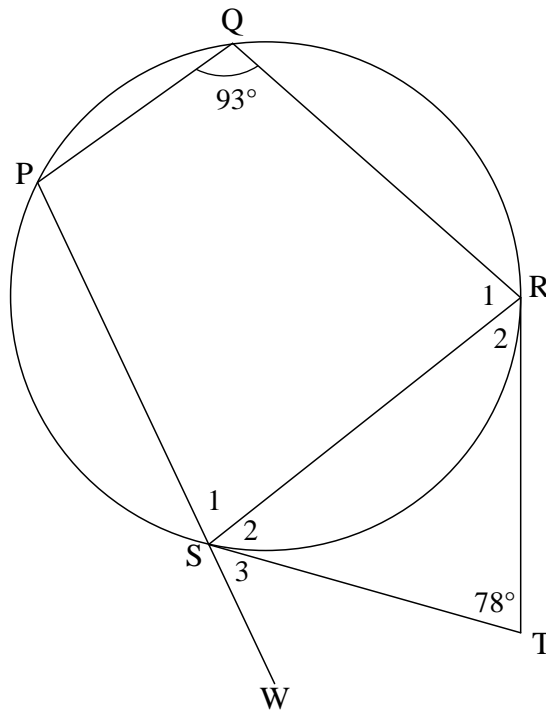
QUESTION/VRAAG 8



<p>8.1</p>	<p>In ΔSQR:</p> $\frac{QS}{\sin x} = \frac{QR}{\sin(90^\circ + x)}$ $\frac{QS}{\sin x} = \frac{5}{\cos x}$ $QS = \frac{5 \sin x}{\cos x}$ $QS = 5 \tan x$	<p>✓ correct use of sine rule</p> <p>✓ $\sin(90^\circ + x) = \cos x$</p> <p>✓ $QS = \frac{5 \sin x}{\cos x}$</p> <p style="text-align: right;">(3)</p>
<p>8.2</p>	$\frac{QT}{\sin(180^\circ - 2x)} = \frac{TS}{\sin x}$ $\frac{QT}{\sin 2x} = \frac{5 \tan x}{\sin x}$ $QT = \frac{5 \tan x \sin 2x}{\sin x}$ $QT = \frac{5 \left(\frac{\sin x}{\cos x} \right) (2 \sin x \cos x)}{\sin x}$ $QT = \frac{5 \sin x (2 \sin x)}{\sin x}$ $QT = 10 \sin x$	<p>✓ correct use of sine rule</p> <p>✓ $TS = QS = 5 \tan x$</p> <p>✓ $QT = \frac{5 \tan x \sin 2x}{\sin x}$</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p>✓ $\sin 2x = 2 \sin x \cos x$</p> <p style="text-align: right;">(5)</p>

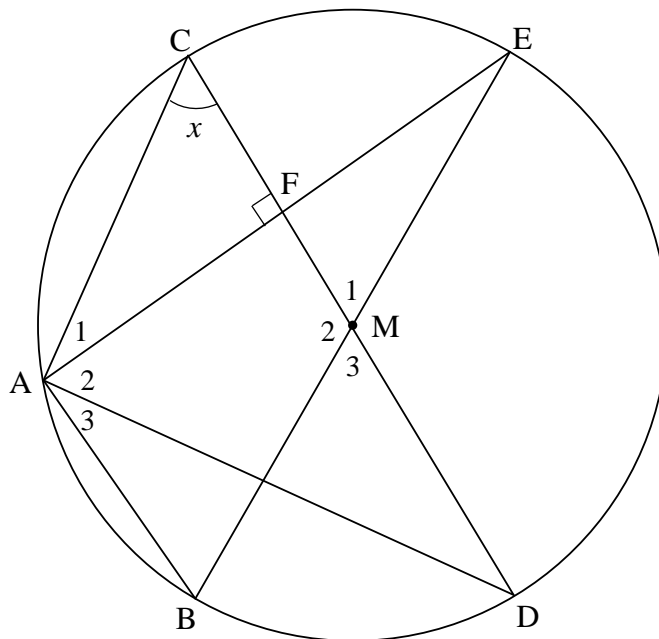
	<p>OR/OF</p> $QT^2 = QS^2 + TS^2 - 2QS.TS\cos\hat{Q}ST$ $QT^2 = (5 \tan x)^2 + (5 \tan x)^2 - 2(5 \tan x).(5 \tan x)\cos(180^\circ - 2x)$ $QT^2 = 50 \tan^2 x - 50 \tan^2 x(-\cos 2x)$ $QT^2 = 50 \tan^2 x(1 + \cos 2x)$ $QT^2 = 50 \tan^2 x(1 + 2 \cos^2 x - 1)$ $QT^2 = 50 \tan^2 x(2 \cos^2 x)$ $QT^2 = 100 \frac{\sin^2 x}{\cos^2 x} (\cos^2 x)$ $QT^2 = 100 \sin^2 x$ $QT = 10 \sin x$ <p>OR/OF</p> $TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$ $(5 \tan x)^2 = (5 \tan x)^2 + TQ^2 - 2(5 \tan x).TQ.\cos x$ $0 = TQ^2 - 2(5 \tan x).TQ.\cos x$ $0 = TQ[TQ - 10 \tan x.\cos x]$ $TQ = 10 \tan x.\cos x \quad (TQ \neq 0)$ $= 10 \frac{\sin x}{\cos x} .\cos x$ $= 10 \sin x$	<p>✓ correct use of cos rule ✓ $TS = QS = 5 \tan x$</p> <p>✓ $\cos 2x = 2 \cos^2 x - 1$ & reduction</p> <p>✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $QT^2 = 100 \sin^2 x$</p> <p style="text-align: right;">(5)</p> <p>✓ correct use of cos rule ✓ $TS = QS = 5 \tan x$ ✓ quadratic equation into TQ</p> <p>✓ $TQ = 10 \tan x . \cos x$ ✓ $\tan x = \frac{\sin x}{\cos x}$</p> <p style="text-align: right;">(5)</p>
<p>8.3</p>	<p>Area of $\Delta TQR = \frac{1}{2} . TQ . QR \sin \hat{T}QR$</p> $= \frac{1}{2} (10 \sin 25^\circ)(5)(\sin 70^\circ)$ $= 9,93 \text{ unit}^2$	<p>✓ correct substitution into the area rule ✓ answer</p> <p style="text-align: right;">(2)</p>
<p>[10]</p>		

QUESTION/VRAAG 9



9.1	tangents from same(common) point/ <i>raaklyne vanaf dieselfde punt</i>	✓ R	(1)
9.2.1	$\hat{S}_2 = \hat{SRT}$ [∠s opp equal sides/ <i>∠e teenoor gelyke sye</i>] $\therefore \hat{S}_2 = 51^\circ$ [sum of ∠s in Δ/ <i>som van ∠e in Δ</i>]	✓ R ✓ S	(2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93^\circ$ [ext ∠ of cyclic quad/ <i>buite ∠ van koordevh</i>] $\hat{S}_3 = 42^\circ$ OR/OF $\hat{S}_1 = 87^\circ$ [opp ∠s of cyclic quad/ <i>teenoorst ∠e v kdvh</i>] $\hat{S}_3 = 180^\circ - (87^\circ + 51^\circ)$ $\hat{S}_3 = 42^\circ$ [∠s on a str line/ <i>∠e op reguitlyn</i>]	✓ R ✓ answer ✓ R ✓ answer	(2) (2)
			[5]

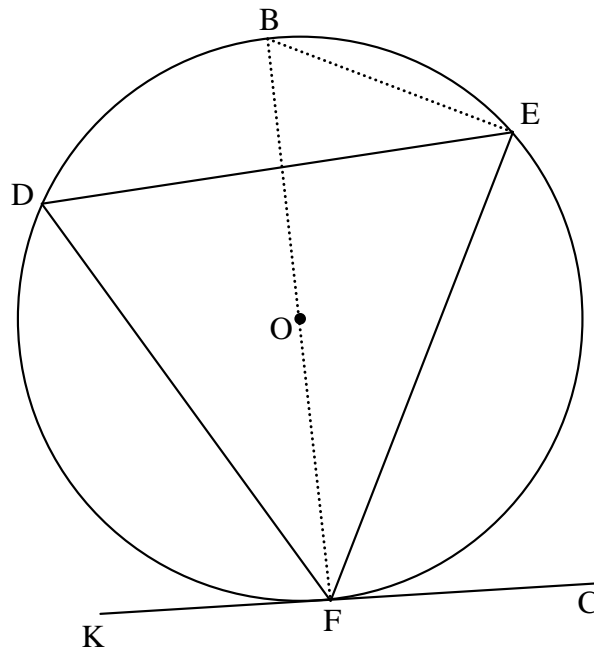
QUESTION/VRAAG 10



10.1	line from centre \perp to chord/ <i>lyn vanaf middelpunt \perp op koord</i>	\checkmark R (1)
10.2	$\therefore \hat{A}_1 = 90^\circ - x$ [sum of \angle s in Δ / <i>som van \anglee in Δ</i>] $\therefore \hat{M}_1 = 180^\circ - 2x$ [\angle at centre = $2 \times$ at circumf/ <i>midpts \angle = $2 \times$ omtreks \angle</i>]	\checkmark S \checkmark S \checkmark R (3)
10.3	$\hat{C}\hat{A}\hat{D} = 90^\circ$ [\angle in semi circle/ <i>\angle in halfsirkel</i>] $\hat{A}_2 = 90^\circ - (90^\circ - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/ <i>omgek rkl-kd st.</i>] OR/OF $\hat{E}\hat{M}\hat{D} = 2x$ [adj suppl \angle s/ <i>aanligg suppl \anglee</i>] $\therefore \hat{A}_2 = x$ [\angle at centre = $2 \times \angle$ at circumf/ <i>midpts \angle = $2 \times$ omtreks \angle</i>] $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/ <i>omgek rkl-kd st.</i>] OR/OF $\hat{M}_3 = 180^\circ - 2x$ [vert. opp/ <i>regoorstaande \anglee</i>] $\therefore \hat{A}_3 = 90^\circ - x$ [\angle at centre = $2 \times \angle$ at circumf/ <i>midpts \angle = $2 \times$ omtreks \angle</i>] $\hat{B}\hat{A}\hat{E} = 90^\circ$ [\angle in semi-circle/ <i>\angle in halfsirkel</i>] $\therefore \hat{A}_2 = \hat{C} = x$ $\therefore AD$ is a tangent [converse tan-chord theorem/ <i>omgek rkl-kd st.</i>] OR/OF	\checkmark S \checkmark R \checkmark S \checkmark R \checkmark R \checkmark S \checkmark R \checkmark S \checkmark R (4) (4) (4) (4)

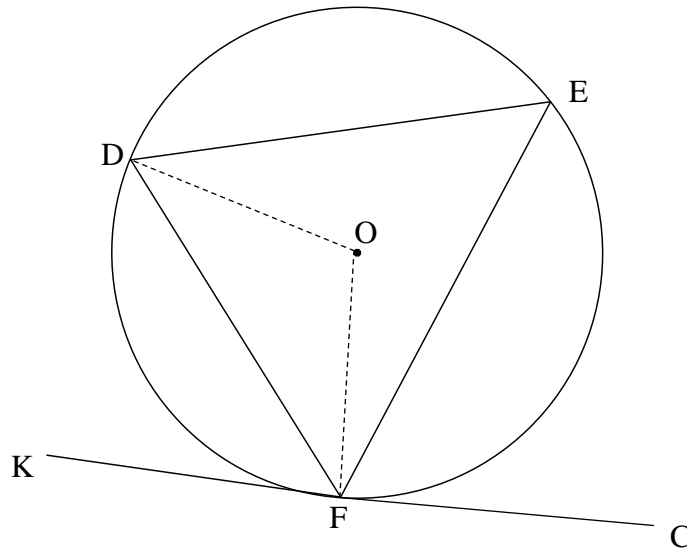
QUESTION/VRAAG 11

11.1



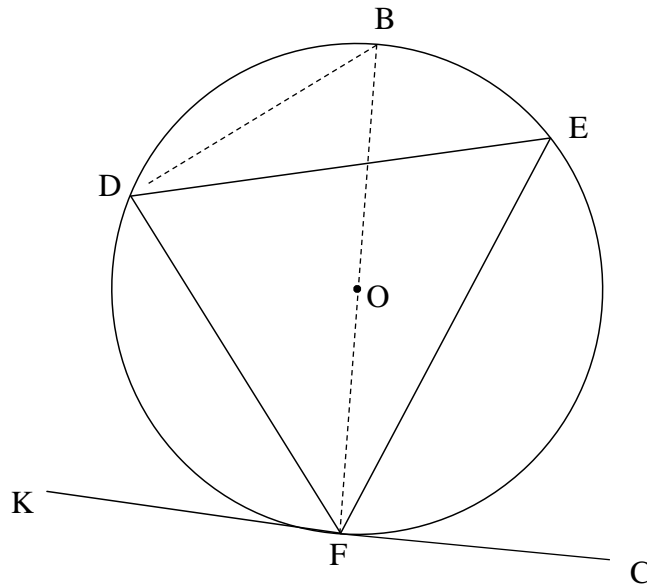
	<p>Construction: Draw diameter BF and draw BE <i>Konstruksie: Trek middellyn BF en verbind BE</i></p> <p>$\hat{B}F\hat{K} = 90^\circ$ or $\hat{D}F\hat{K} = 90^\circ - \hat{B}F\hat{D}$ [radius \perp tangent/raaklyn]</p> <p>$\hat{B}E\hat{F} = 90^\circ$ [\angle in semi-circle/semi-sirkel]</p> <p>$\therefore \hat{D}E\hat{F} = 90^\circ - \hat{B}E\hat{D}$</p> <p>$= 90^\circ - \hat{B}F\hat{D}$ [\angles same segment/\anglee dieselfde segment]</p> <p>$\therefore \hat{D}F\hat{K} = \hat{D}E\hat{F}$</p>	<p>✓ Constr</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
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OR/OF



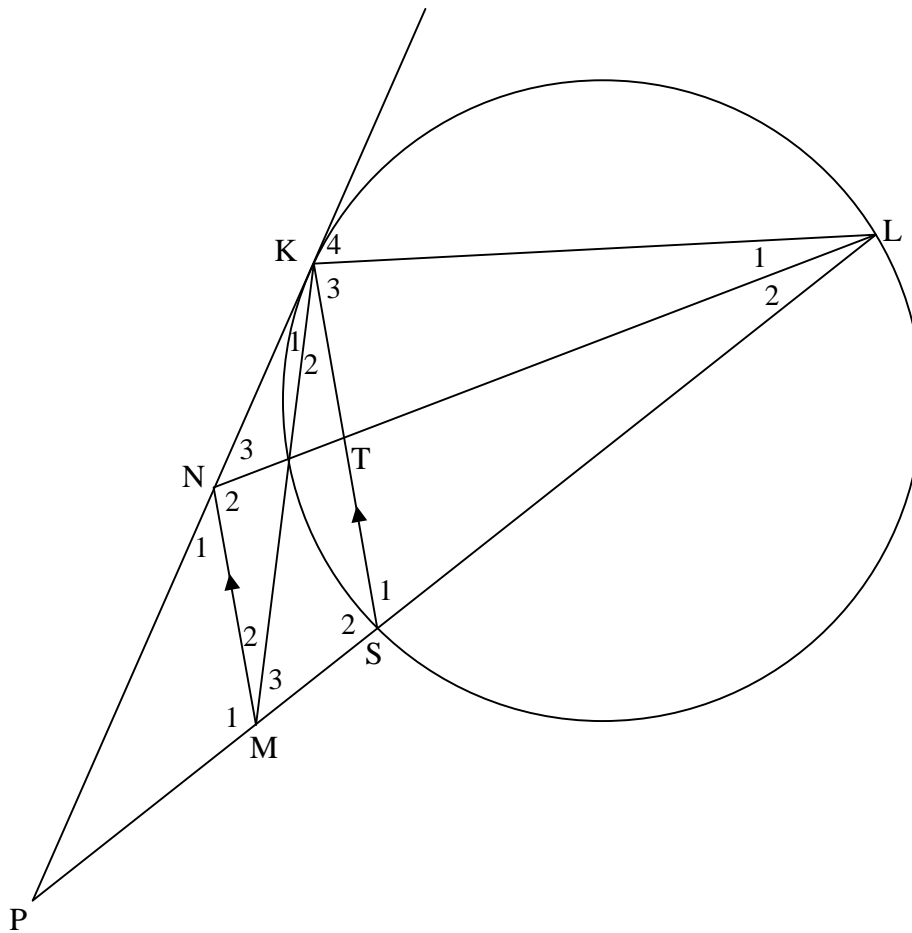
	<p>Construction: Draw radii DO and OF</p> <p><i>Konstruksie: Trek radii DO en OF</i></p> <p>$\hat{O}F\hat{K} = 90^\circ$ or $\hat{D}\hat{F}\hat{K} = 90^\circ - \hat{O}\hat{F}\hat{D}$ radius \perp tangent/raaklyn] $\hat{O}\hat{D}\hat{F} = \hat{O}\hat{F}\hat{D}$ [\angles opp = sides/\anglee teenoor = sye]</p> <p>$\therefore \hat{D}\hat{O}\hat{F} = 180^\circ - 2\hat{O}\hat{F}\hat{D}$ [\angles of Δ/\anglee van Δ]</p> <p>$\hat{D}\hat{E}\hat{F} = 90^\circ - \hat{O}\hat{F}\hat{D}$ [\angle at centre = $2 \times \angle$ circumf/ midpts $\angle = 2 \times$ omtreks \angle]</p> <p>$\therefore \hat{D}\hat{F}\hat{K} = \hat{D}\hat{E}\hat{F}$</p>	<p>✓ construction</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
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OR/OF



	<p>Construction: Draw diameter BF and join BD.</p> <p><i>Konstruksie: Trek middellyn BF en verbind BD.</i></p> <p>$\hat{B}\hat{F}K = 90^\circ$ or $\hat{D}\hat{F}K = 90^\circ - \hat{B}\hat{F}D$ [radius \perp tangent/raaklyn]</p> <p>$\hat{F}\hat{D}B = 90^\circ$ [\angle in half circle/semi-sirkel]</p> <p>$\hat{B} = 90^\circ - \hat{B}\hat{F}D$</p> <p>$\therefore \hat{D}\hat{F}K = \hat{B}$</p> <p>but $\hat{B} = \hat{E}$ [\angles same segment/\anglee dieselfde segment]</p> <p>$\therefore \hat{D}\hat{F}K = \hat{E}$</p>	<p>✓ construction</p> <p>✓ S ✓/R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
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11.2



<p>11.2.1(a)</p>	<p>$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/raaklynkoordstelling] $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp \angles; / ooreenk \angles; MN KS] $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = \hat{NML}$</p>	<p>✓ S ✓ R ✓ S ✓ R</p> <p>(4)</p>
<p>11.2.1(b)</p>	<p>$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = \hat{NML}$ \therefore KLMN is a cyclic quad [ext \angle of quad = opp int \angle / <i>buite \angle van vh = teenorst binne \angle</i>]</p> <p>OR/OF</p> <p>$N_1 = \hat{K}_1 + \hat{K}_2 = \hat{NKS}$ [corresp \angles; / ooreenk \angles; MN KS] $N\hat{K}S = K\hat{L}S$ [tan chord theorem / raaklynkoordstelling] $\hat{N}_1 = K\hat{L}S$ \therefore KLMN is a cyclic quad [ext \angle of quad = opp int \angle / <i>buite \angle van vh = teenorst binne \angle</i>]</p> <p>OR/OF</p>	<p>✓ R</p> <p>(1)</p> <p>✓ R</p> <p>(1)</p>

	$NKL = 180^\circ - K_4$ [adj. suppl.] $\therefore NKL = 180^\circ - NML$ [proved] $\therefore KLMN$ is a cyclic quad [opp. \angle s supplementary]	✓ R (1)
11.2.2	In $\triangle LKN \parallel \triangle KSM$: $\hat{N}_3 = \hat{M}_3$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $\hat{L}_1 = \hat{M}_2$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $= \hat{K}_2$ [alt \angle s; / verw \angle e; $MN \parallel KS$] $N\hat{K}L = M\hat{S}K$ [\angle s of Δ / \angle e van Δ] $\triangle LKN \parallel \triangle KSM$ OR/OF In $\triangle LKN \parallel \triangle KSM$: $\hat{N}_3 = \hat{M}_3$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $N\hat{K}L = \hat{M}_1$ [ext \angle of cyclic quad/buite \angle van koordevh] $= \hat{S}_2$ [corresp \angle s/ooreenk \angle e; $KS \parallel NM$] $\triangle LKN \parallel \triangle KSM$ [\angle , \angle , \angle] OR/OF In $\triangle LKN \parallel \triangle KSM$: $\hat{N}_3 = \hat{M}_3$ [\angle s in the same seg / \angle e in dieselfde sirkel segm] $\hat{K}_4 + N\hat{K}L = \hat{S}_1 + \hat{S}_2$ [\angle s on straight line / \angle e op reguitlyn] $\therefore N\hat{K}L = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\triangle LKN \parallel \triangle KSM$ [\angle , \angle , \angle]	✓ S ✓ R ✓ S ✓ S/R ✓ S (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
11.2.3	$\frac{LK}{KS} = \frac{KN}{SM}$ [$\triangle LKN \parallel \triangle KSM$] $\therefore \frac{12}{KS} = \frac{4}{3}$ $KS = 9$ units	✓ S ✓ R ✓ substitution ✓ answer (4)
11.2.4	$4SM = 3KN$ $SM = \frac{3(8)}{4}$ $SM = 6$ $\frac{LT}{NL} = \frac{LS}{ML}$ [line \parallel one side of Δ / lyn \parallel een sy v Δ] $\frac{LT}{16} = \frac{13}{19}$ $LT = \frac{208}{19} = 10,95$	✓ $SM = 6$ ✓ S ✓ R ✓ answer (4)
		[23]

TOTAL/TOTAAL: 150