



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NASIONALE
SENIOR SERTIFIKAAT**

GRAAD 12

WISKUNDE V2

NOVEMBER 2019

PUNTE: 150

TYD: 3 uur

**Hierdie vraestel bestaan uit 14 bladsye, 1 inligtingsblad
en 'n antwoordeboek van 24 bladsye.**

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat die vraestel beantwoord word.

1. Hierdie vraestel bestaan uit 10 vrae.
2. Beantwoord AL die vrae in die SPESIALE ANTWOORDEBOEK wat verskaf word.
3. Dui ALLE berekeninge, diagramme, grafieke, ens. wat jy gebruik in die beantwoording van die vrae, duidelik aan.
4. Slegs antwoorde sal NIE noodwendig volpunte verdien NIE.
5. Jy kan 'n goedgekeurde wetenskaplike sakrekenaar gebruik (nie-programmeerbaar en nie-grafies), tensy anders vermeld.
6. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.
7. Diagramme is NIE noodwendig volgens skaal geteken NIE.
8. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
9. Skryf netjies en leesbaar.

VRAAG 1

Die tabel hieronder toon die maandelikse inkomste (in rand) van 6 verskillende persone en die bedrag (in rand) wat elke persoon maandeliks aan 'n motorpaaieiment spandeer.

MAANDELIKSE INKOMSTE (IN RAND)	9 000	13 500	15 000	16 500	17 000	20 000
MAANDELIKSE PAAIEMENT (IN RAND)	2 000	3 000	3 500	5 200	5 500	6 000

- 1.1 Bepaal die vergelyking van die kleinstekwadrade-regressielyn vir die data. (3)
- 1.2 Indien 'n persoon R14 000 per maand verdien, voorspel die maandelikse paaieiment wat hierdie persoon aan 'n motorvoertuig kan spandeer. (2)
- 1.3 Bepaal die korrelasiekoëffisiënt tussen die maandelikse inkomste en die maandelikse paaieiment van 'n motorvoertuig. (1)
- 1.4 'n Persoon wat R18 000 per maand verdien, moet besluit of 'n maandelikse paaieiment van R9 000 aan 'n motorvoertuig spandeer kan word, al dan nie. Indien die inligting hierbo 'n ware weerspieëling van die bevolkingsdata is, op watter van die volgende sal die persoon heel moontlik besluit:
- A. Spandeer R9 000 per maand aangesien daar 'n baie sterk positiewe korrelasie tussen die bedrag verdien en die maandelikse paaieiment is.
- B. Spandeer NIE R9 000 per maand NIE aangesien daar 'n baie swak positiewe korrelasie tussen die bedrag verdien en die maandelikse paaieiment is.
- C. Spandeer R9 000 per maand, want die punt (18 000 ; 9 000) lê baie naby aan die kleinstekwadrade-regressielyn.
- D. Spandeer NIE R9 000 per maand NIE, want die punt (18 000 ; 9 000) lê baie ver van die kleinstekwadrade-regressielyn af. (2)

[8]

VRAAG 2

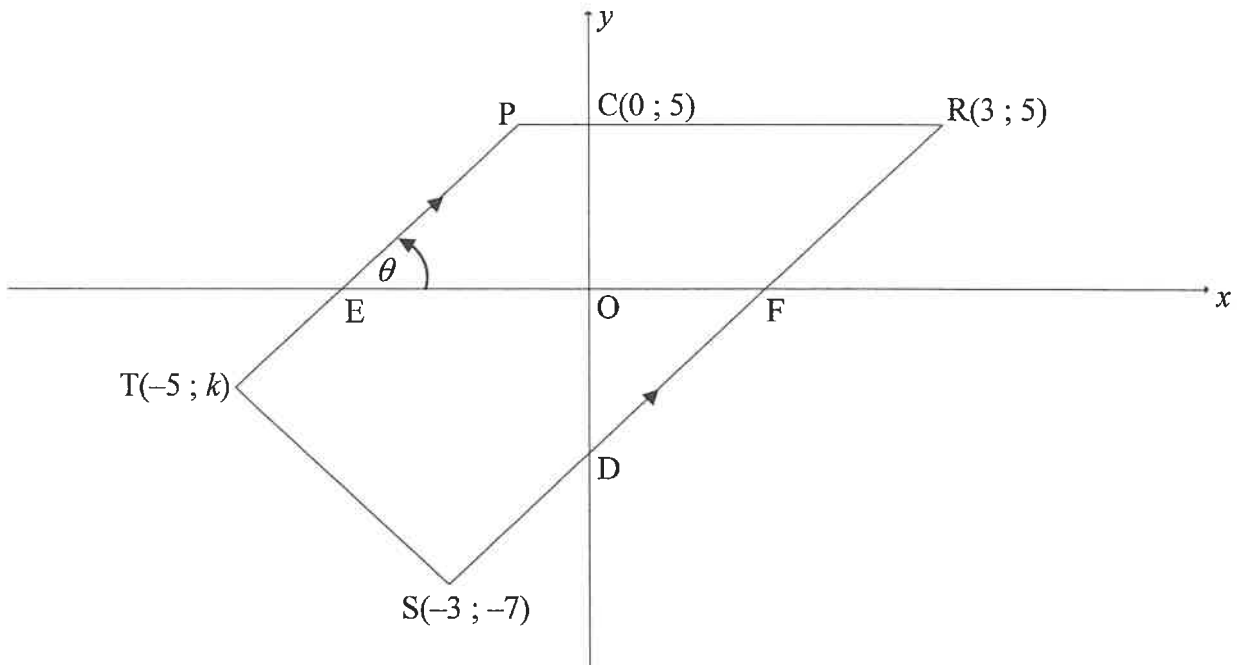
'n Opname is onder 100 mense gedoen oor die bedrag wat hulle maandeliks vir hulle selfoonkontrakte betaal het. Die persoon wat die opname gedoen het, het bereken dat die geskatte gemiddelde betaling R309 per maand is. Ongelukkig het hy na die opname van die data verloor. Die frekwensietabel hieronder toon die gedeeltelike resultate van die ondersoek:

BEDRAG BETAAL (IN RAND)	FREKWENSIE
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	a
$300 < x \leq 400$	35
$400 < x \leq 500$	b
$500 < x \leq 600$	6

- 2.1 Hoeveel mense het R200 of minder vir hulle maandelikse selfoonkontrakte betaal? (1)
- 2.2 Gebruik die inligting hierbo om te toon dat $a = 24$ en $b = 16$. (5)
- 2.3 Skryf die modale klas van die data neer. (1)
- 2.4 Gebruik die rooster wat in die ANTWOORDEBOEK verskaf word en teken 'n ogief (kumulatiewefrekwensie-grafiek) om die data voor te stel. (4)
- 2.5 Bepaal hoeveel mense meer as R420 per maand vir hulle selfoonkontrakte betaal het. (2)
- [13]

VRAAG 3

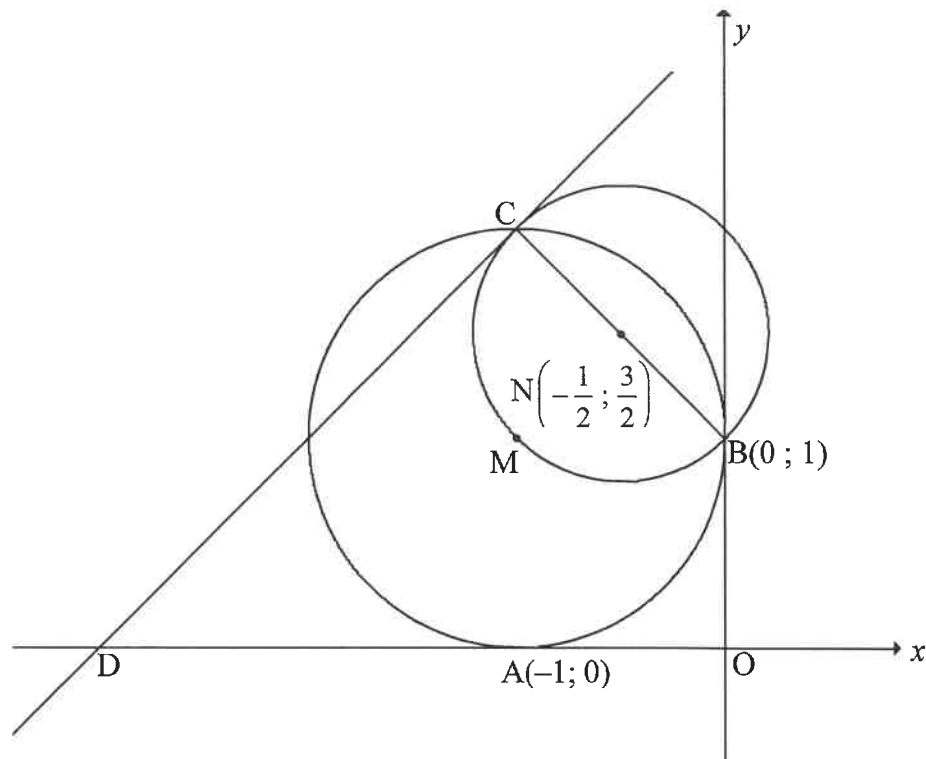
In die diagram is P , $R(3; 5)$, $S(-3; -7)$ en $T(-5; k)$ hoekpunte van trapesium $PRST$ en $PT \parallel RS$. RS en PR sny die y -as by D en $C(0; 5)$ onderskeidelik. PT en RS sny die x -as by E en F onderskeidelik. $\hat{P}EF = \theta$.



- 3.1 Skryf die vergelyking van PR neer. (1)
- 3.2 Bereken die:
- 3.2.1 Gradiënt van RS (2)
- 3.2.2 Grootte van θ (3)
- 3.2.3 Koördinate van D (3)
- 3.3 Indien gegee word dat $TS = 2\sqrt{5}$, bereken die waarde van k . (4)
- 3.4 Parallelogram $TDNS$, met N in die 4^{de} kwadrant, word getrek. Bereken die koördinate van N . (3)
- 3.5 $\triangle PRD$ word om die y -as gereflekteer om $\triangle P'R'D'$ te vorm. Bereken die grootte van $\hat{R}D'R'$. (3)
- [19]

VRAAG 4

In die diagram raak 'n sirkel met middelpunt M die x -as by $A(-1; 0)$ en die y -as by $B(0; 1)$. 'n Kleiner sirkel met middelpunt $N\left(-\frac{1}{2}; \frac{3}{2}\right)$ gaan deur M en sny die groter sirkel by B en C . BNC is 'n middellyn van die kleiner sirkel. 'n Raaklyn aan die kleiner sirkel by C , sny die x -as by D .



- 4.1 Bepaal die vergelyking van die sirkel met middelpunt M in die vorm $(x-a)^2 + (y-b)^2 = r^2$ (3)
- 4.2 Bereken die koördinate van C . (2)
- 4.3 Toon dat $y - x = 3$ die vergelyking van die raaklyn CD is. (4)
- 4.4 Bepaal die waardes van t waarvoor die lyn $y = x + t$ NIE die kleiner sirkel sal raak of sny NIE. (3)
- 4.5 Die kleiner sirkel met middelpunt N ondergaan 'n transformasie sodanig dat punt C langs die raaklyn na D getransleer word. Bereken die koördinate van E , die nuwe middelpunt van die kleiner sirkel. (3)
- 4.6 As gegee word dat die oppervlakte van vierhoek $OBCD$ is $2a^2$ vierkante eenhede en $a > 0$, toon dat $a = \frac{\sqrt{7}}{2}$ eenhede. (5)

[20]

VRAAG 5

5.1 Vereenvoudig die volgende uitdrukking na EEN trigonometriese term:

$$\frac{\sin x}{\cos x \cdot \tan x} + \sin(180^\circ + x) \cos(90^\circ - x) \quad (5)$$

5.2 **Sonder om 'n sakrekenaar te gebruik**, bepaal die waarde van: $\frac{\sin^2 35^\circ - \cos^2 35^\circ}{4 \sin 10^\circ \cos 10^\circ}$ (4)

5.3 Gegee: $\cos 26^\circ = m$

Sonder om 'n sakrekenaar te gebruik, bepaal $2 \sin^2 77^\circ$ in terme van m . (4)

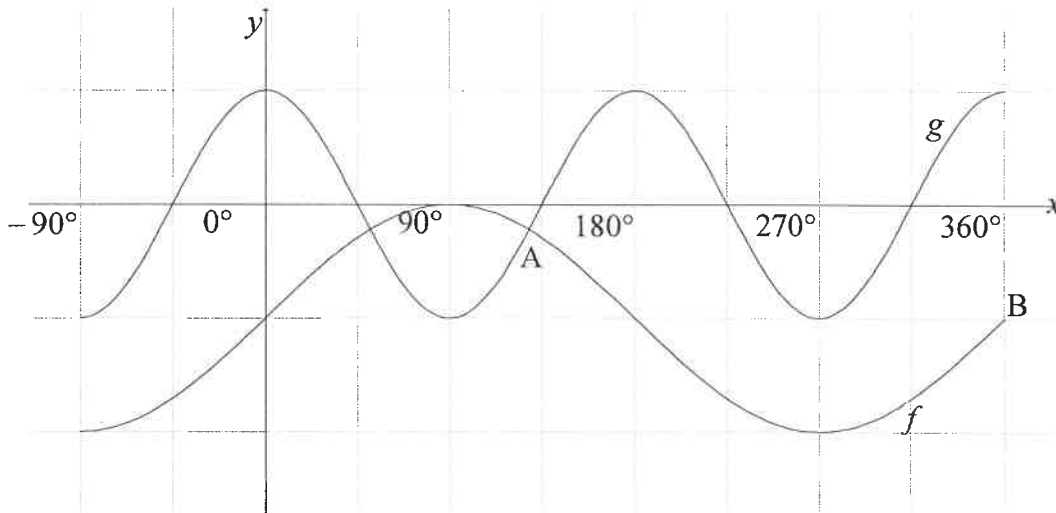
5.4 Beskou: $f(x) = \sin(x + 25^\circ) \cos 15^\circ - \cos(x + 25^\circ) \sin 15^\circ$

5.4.1 Bepaal die algemene oplossing van $f(x) = \tan 165^\circ$ (6)

5.4.2 Bepaal die waarde(s) van x waarvoor $f(x)$, in die interval $x \in [0^\circ; 360^\circ]$, 'n minimum waarde sal hê. (3)
[22]

VRAAG 6

In die diagram is die grafieke van $f(x) = \sin x - 1$ en $g(x) = \cos 2x$ vir die interval $x \in [-90^\circ; 360^\circ]$ geskets. Grafieke f en g sny by A. B($360^\circ; -1$) is 'n punt op f .

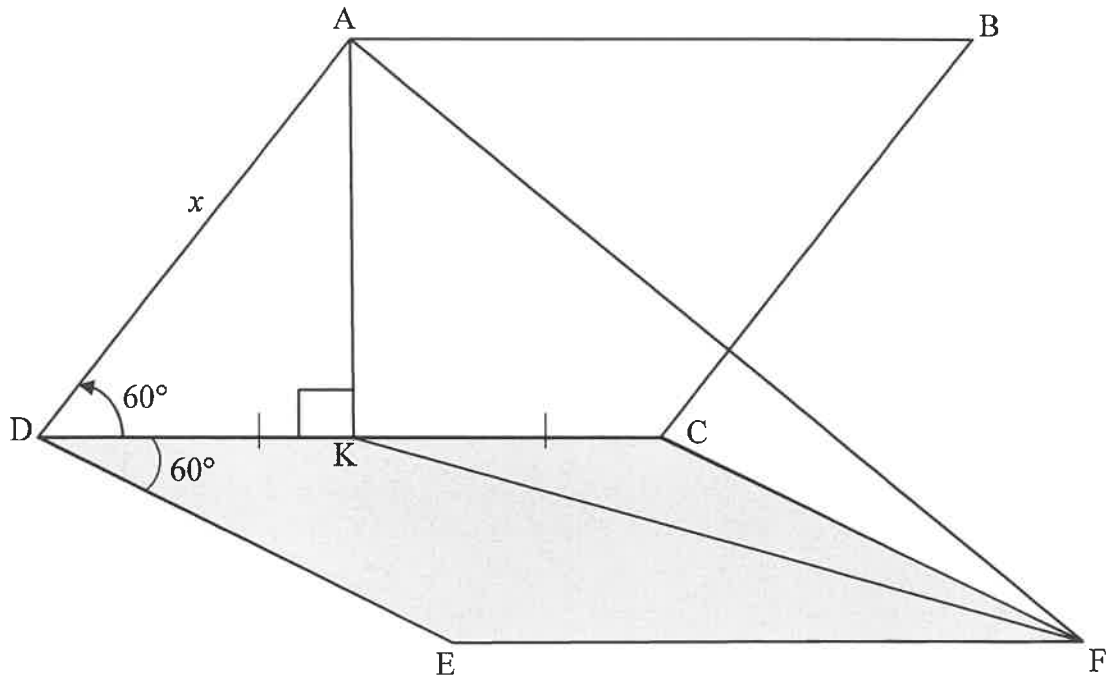


- 6.1 Skryf die waardeversameling van f neer. (2)
- 6.2 Skryf die waardes van x neer in die interval $x \in [-90^\circ; 360^\circ]$ waarvoor grafiek f afnemend is. (2)
- 6.3 P en Q is punte op grafieke g en f onderskeidelik sodanig dat PQ ewewydig aan die y -as is. Indien PQ tussen A en B lê, bepaal die waarde(s) van x waarvoor PQ 'n maksimum sal wees. (6)

[10]

VRAAG 7

Die diagram hieronder toon 'n sonpaneel, $ABCD$, wat aan 'n plat stuk sementblad, $EFCD$, vasgeheg is. $ABCD$ en $EFCD$ is twee identiese ruite. K is 'n punt op DC sodanig dat $DK = KC$ en $AK \perp DC$. AF en KF is getrek. $\hat{ADC} = \hat{CDE} = 60^\circ$ en $AD = x$ eenhede.

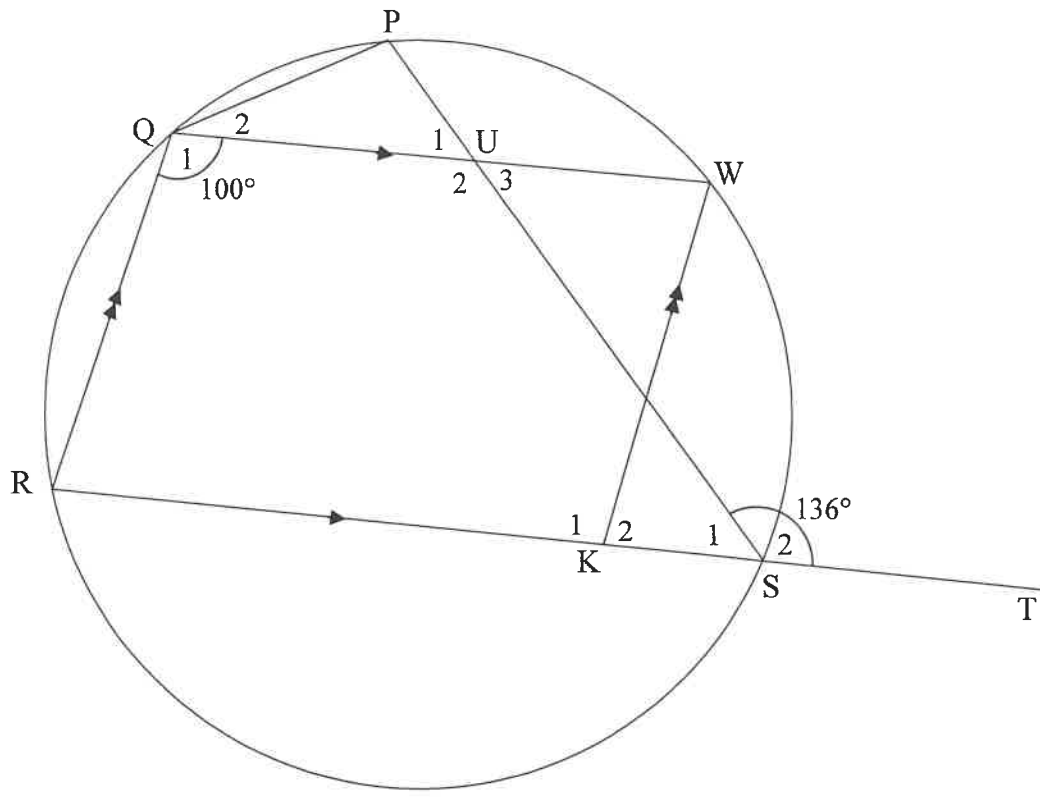


- 7.1 Bepaal AK in terme van x . (2)
- 7.2 Skryf die grootte van \hat{KCF} neer. (1)
- 7.3 Verder word gegee dat \hat{AKF} , die hoek tussen die sonpaneel en die sementblad, gelyk is aan y . Bepaal die oppervlakte van $\triangle AKF$ in terme van x en y . (7)

[10]

VRAAG 8

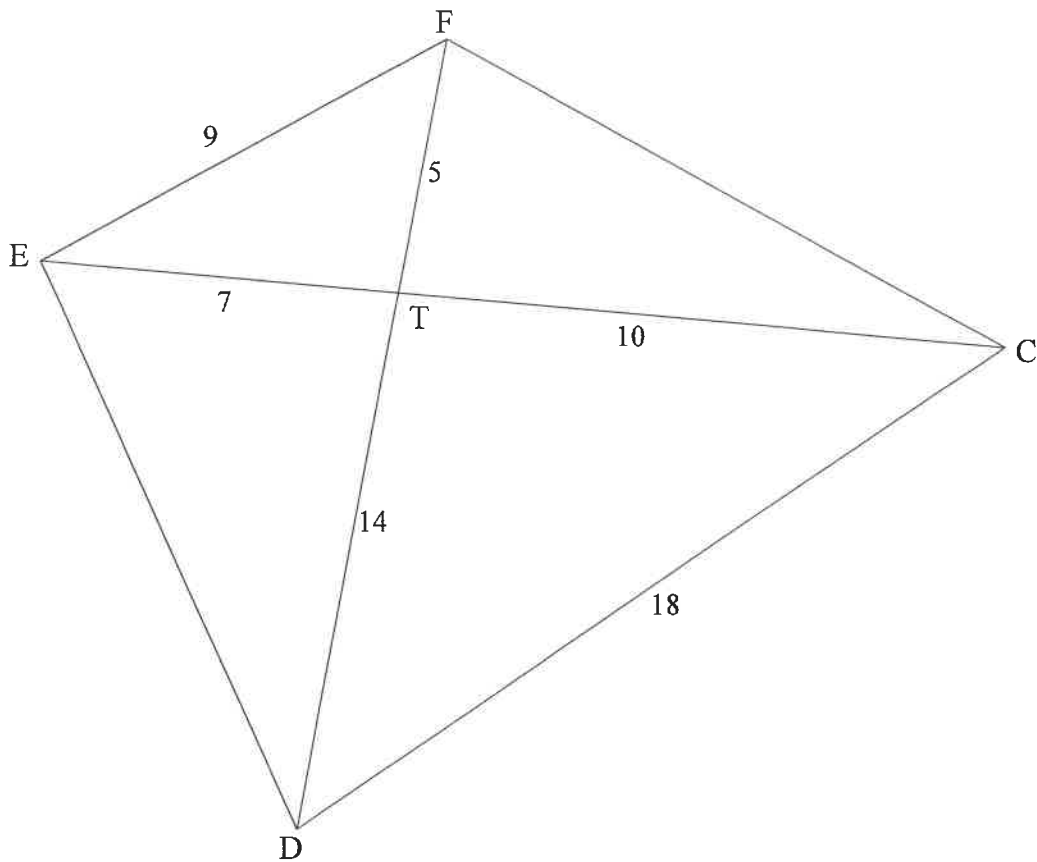
8.1 In die diagram is PQRS 'n koordevierhoek. Koord RS is verleng na T. K is 'n punt op RS en W is 'n punt op die sirkel sodanig dat QRKW 'n parallelogram is. PS en QW sny mekaar by U. $\hat{PST} = 136^\circ$ en $\hat{Q}_1 = 100^\circ$.



Bepaal, met redes, die grootte van:

- 8.1.1 \hat{R} (2)
- 8.1.2 \hat{P} (2)
- 8.1.3 \hat{PQW} (3)
- 8.1.4 \hat{U}_2 (2)

- 8.2 In die diagram sny die hoeklyne van vierhoek CDEF mekaar by T.
 EF = 9 eenhede, DC = 18 eenhede, ET = 7 eenhede, TC = 10 eenhede,
 FT = 5 eenhede en TD = 14 eenhede.



Bewys, met redes, dat:

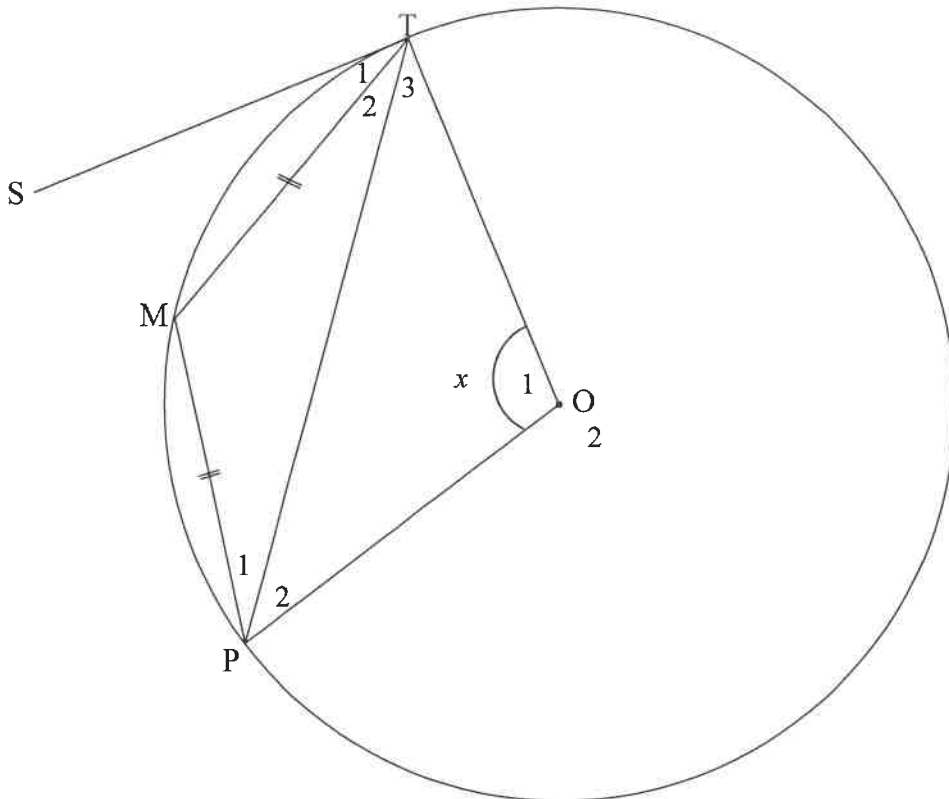
8.2.1 $\hat{E}FD = \hat{E}CD$ (4)

8.2.2 $\hat{D}FC = \hat{D}EC$ (3)

[16]

VRAAG 9

In die diagram is O die middelpunt van die sirkel. ST is 'n raaklyn aan die sirkel by T . M en P is punte op die sirkel sodanig dat $TM = MP$. OT , OP en TP is getrek. Stel $\hat{O}_1 = x$.

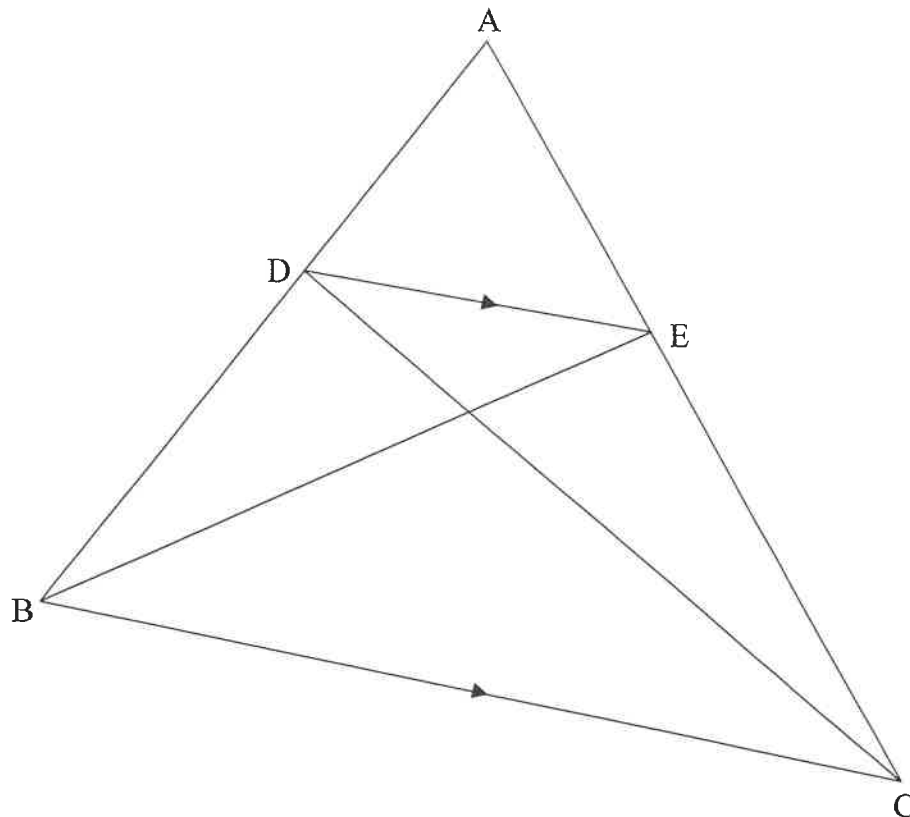


Bewys, met redes, dat $\hat{S}TM = \frac{1}{4}x$.

[7]

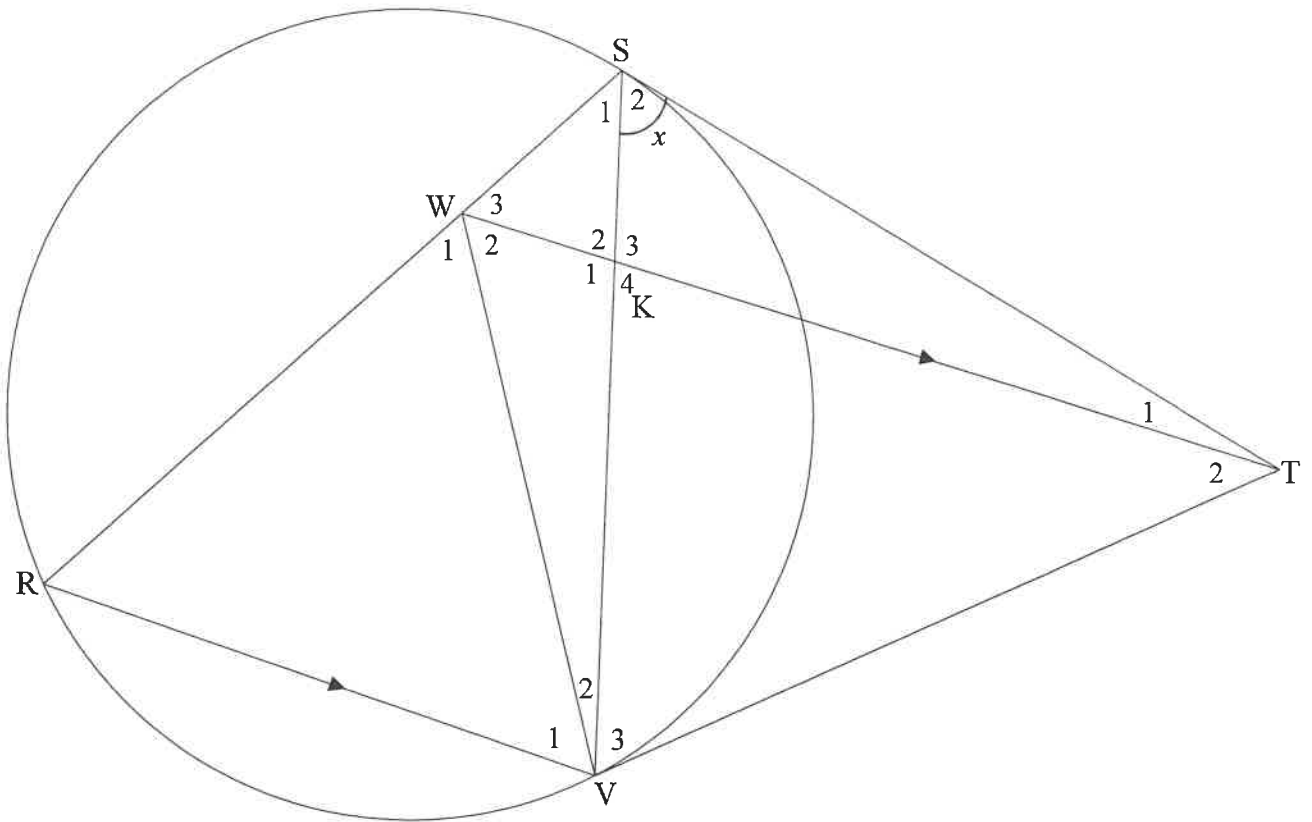
VRAAG 10

- 10.1 In die diagram is $\triangle ABC$ geskets. D is 'n punt op AB en E is 'n punt op AC sodanig dat $DE \parallel BC$. BE en DC is getrek.



Gebruik die diagram om die stelling te bewys wat beweer dat 'n lyn wat ewewydig aan een sy van 'n driehoek getrek word die ander twee sye in dieselfde verhouding verdeel, met ander woorde bewys dat $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

- 10.2 In die diagram is ST en VT raaklyne aan die sirkel by S en V onderskeidelik. R is 'n punt op die sirkel en W is 'n punt op koord RS sodanig dat WT ewewydig is aan RV . SV en WV is getrek. WT sny SV by K . Stel $\hat{S}_2 = x$.



- 10.2.1 Skryf, met redes, DRIE ander hoeke neer wat ELK aan x gelyk is. (6)
- 10.2.2 Bewys, met redes, dat:
- (a) $WSTV$ 'n koordevierhoek is (2)
 - (b) $\triangle WRV$ gelykbenig is (4)
 - (c) $\triangle WRV \parallel \triangle TSV$ (3)
 - (d) $\frac{RV}{SR} = \frac{KV}{TS}$ (4)

[25]

TOTAAL: 150

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{opp } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG HIERDIE INSTRUKSIES NOUKEURIG
1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.	1. <i>Skryf jou eksamennommer en sentrumnummer duidelik in die ruimtes verskaf en plak jou stafieskodeplakker in die ruimte verskaf.</i>
2. Remember that your own name (or the name of your school) may not appear anywhere on or in this answer book.	2. <i>Onthou dat jou eie naam (of die naam van jou skool) NIE op of in hierdie antwoordeboek mag voorkom NIE.</i>
3. Answer ALL questions in the spaces provided.	3. <i>Beantwoord ALLE vrae in die ruimtes wat verskaf is.</i>
4. No pages may be torn from this answer book.	4. <i>GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.</i>
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	5. <i>Lees die instruksies, wat op jou eksamenrooster gedruk is, sorgvuldig deur, asook enige ander instruksies wat in elke vraestel gegee word.</i>
6. Candidates may not retain an answer book or remove it from the examination room.	6. <i>GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE.</i>
7. Answers must be written in black/blue ink as distinctly as possible. Do not write in the margins.	7. <i>Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.</i>
8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.	8. <i>Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.</i>
9. If you require additional space for your answers: 9.1 Use the additional space provided at the end of the answer book. 9.2 When answering a question in the additional space, indicate clearly the question number in the column on the LHS. 9.3 Rule off after each answer.	9. <i>In geval jy bykomende ruimte benodig vir jou antwoorde:</i> 9.1 <i>Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek verskaf word.</i> 9.2 <i>As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan.</i> 9.3 <i>Trek 'n lyn na elke antwoord.</i>
10. Draw a neat line through any work/rough work that must not be marked.	10. <i>Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.</i>

QUESTION/VRAAG 1

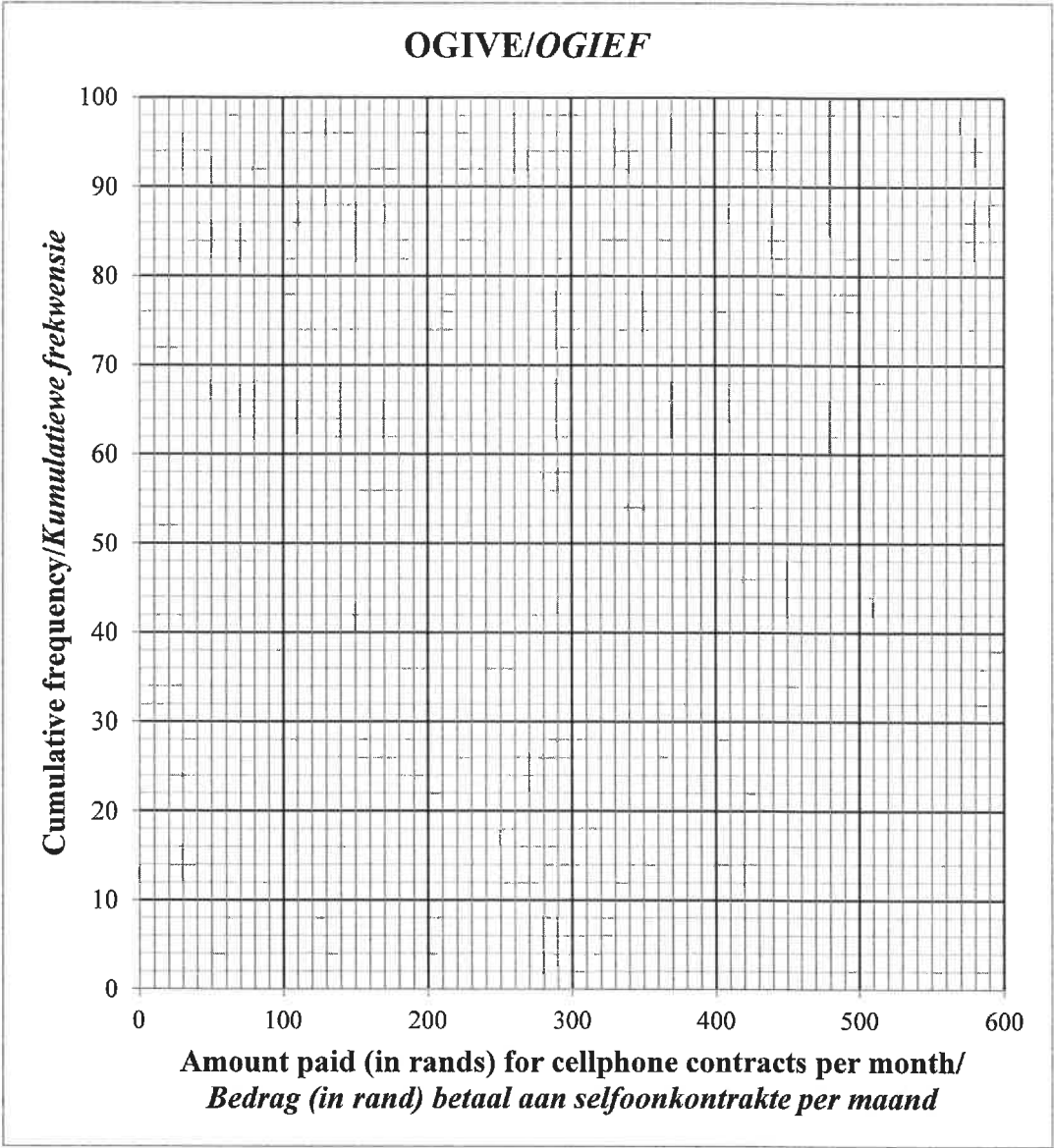
MONTHLY INCOME (IN RANDS) MAANDELIKSE INKOMSTE (IN RAND)	9 000	13 500	15 000	16 500	17 000	20 000
MONTHLY REPAYMENT (IN RANDS) MAANDELIKSE PAAIEMENT (IN RAND)	2 000	3 000	3 500	5 200	5 500	6 000

	Solution/Oplissing	Marks Punte
1.1		(3)
1.2		(2)
1.3		(1)
1.4		(2)
		8

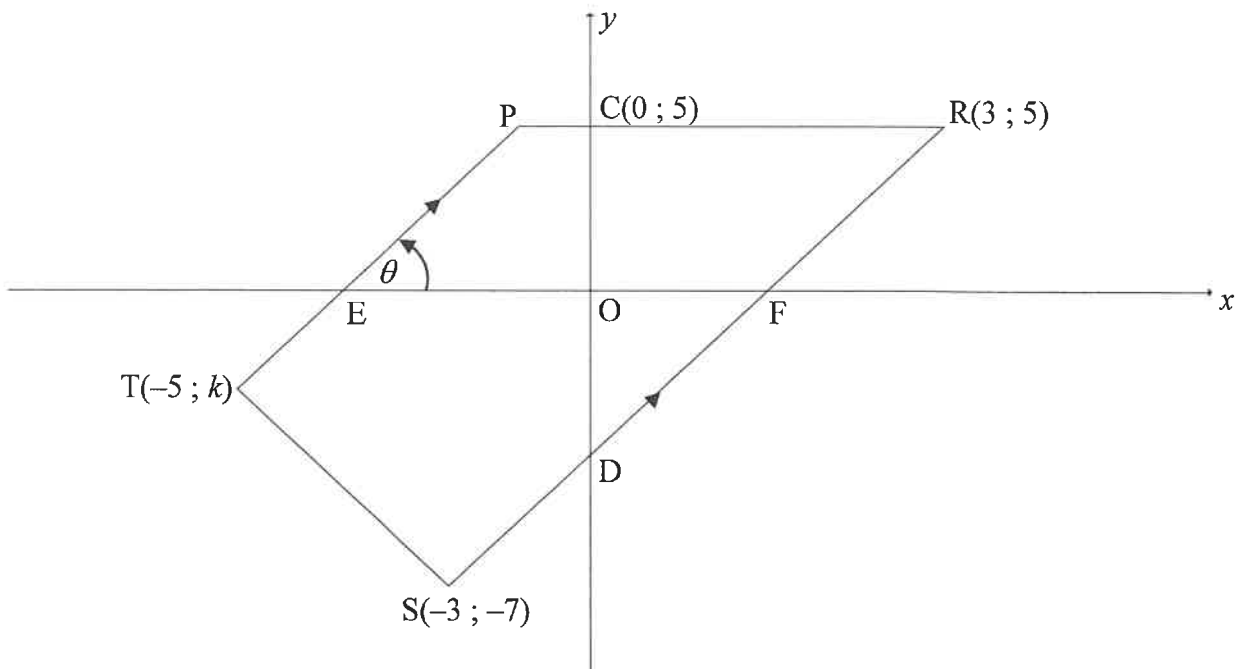
QUESTION/VRAAG 2

AMOUNT PAID (IN RANDS) BEDRAG BETAAL (IN RAND)	FREQUENCY FREKWENSIE
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	<i>a</i>
$300 < x \leq 400$	35
$400 < x \leq 500$	<i>b</i>
$500 < x \leq 600$	6

	Solution/Oplissing	Marks Punte
2.1		(1)
2.2		(5)
2.3		(1)

	Solution/Oplissing	Marks Punte
2.4	<p style="text-align: center;">OGIVE/OGIEF</p> 	(4)
2.5	<hr/> <hr/> <hr/> <hr/>	(2)
		[13]

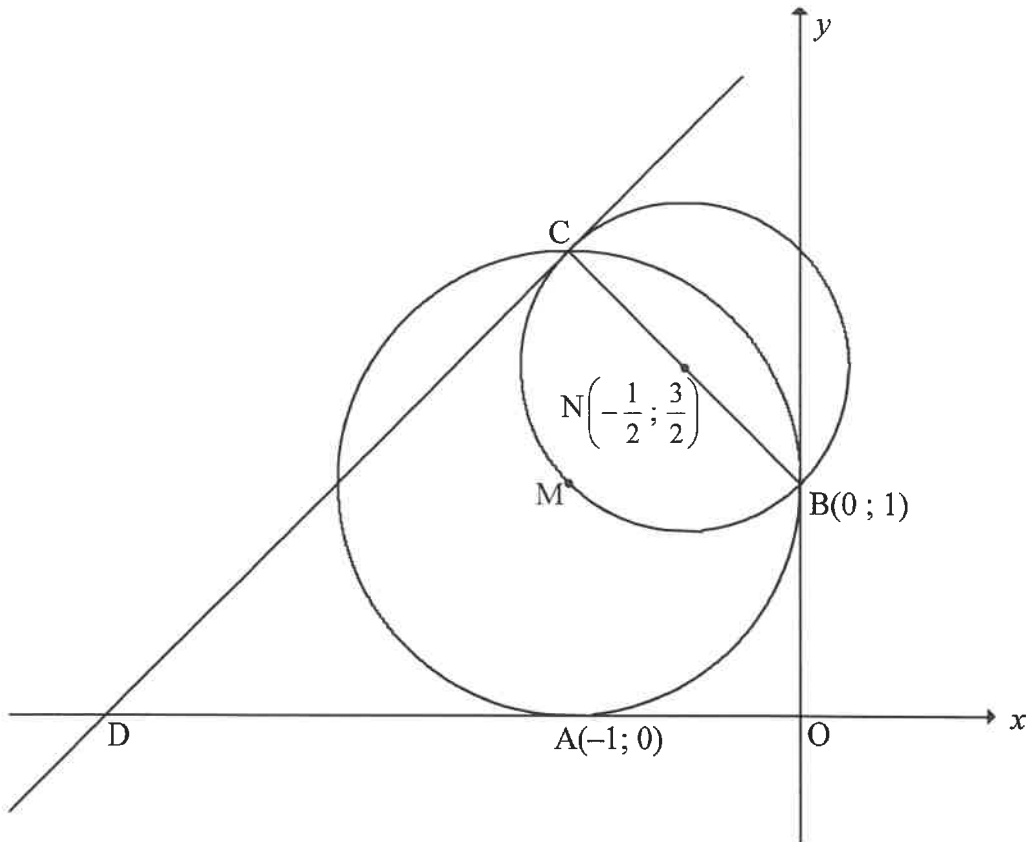
QUESTION/VRAAG 3



	Solution/Oplissing	Marks Punte
3.1		(1)
3.2.1		(2)
3.2.2		(3)

	Solution/Oplissing	Marks Punte
3.2.3		(3)
3.3		(4)
3.4		(3)
3.5		(3)
		[19]

QUESTION/VRAAG 4



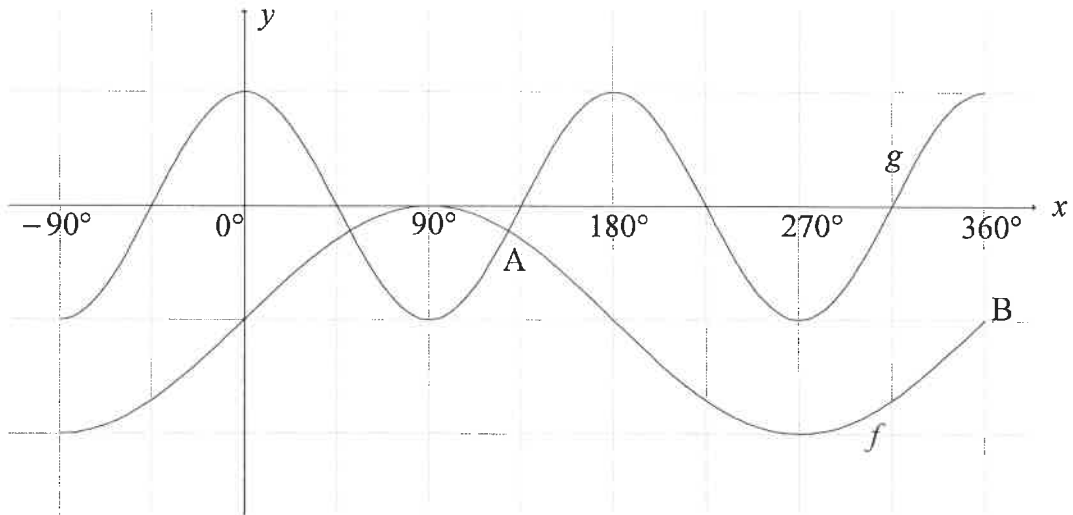
	Solution/Oplissing	Marks Punte
4.1		(3)
4.2		(2)

	Solution/Oplissing	Marks Punte
4.3		(4)
4.4		(3)
4.5		(3)
4.6		(5)

[20]

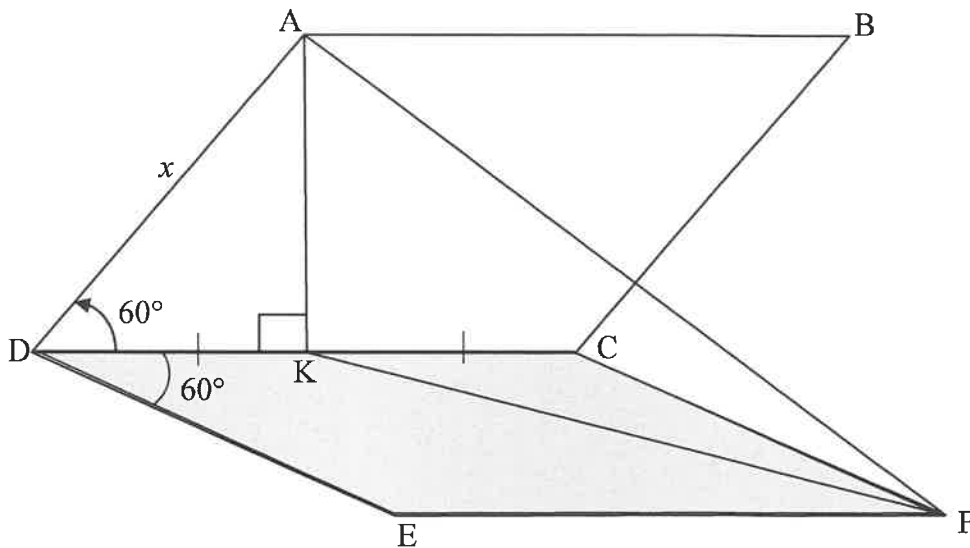
	Solution/Oplissing	Marks Punte
5.4.1		
5.4.2		(6)
		(3)
		[22]

QUESTION/VRAAG 6



	Solution/Oplissing	Marks Punte
6.1		(2)
6.2		(2)
6.3		(6)
		[10]

QUESTION/VRAAG 7

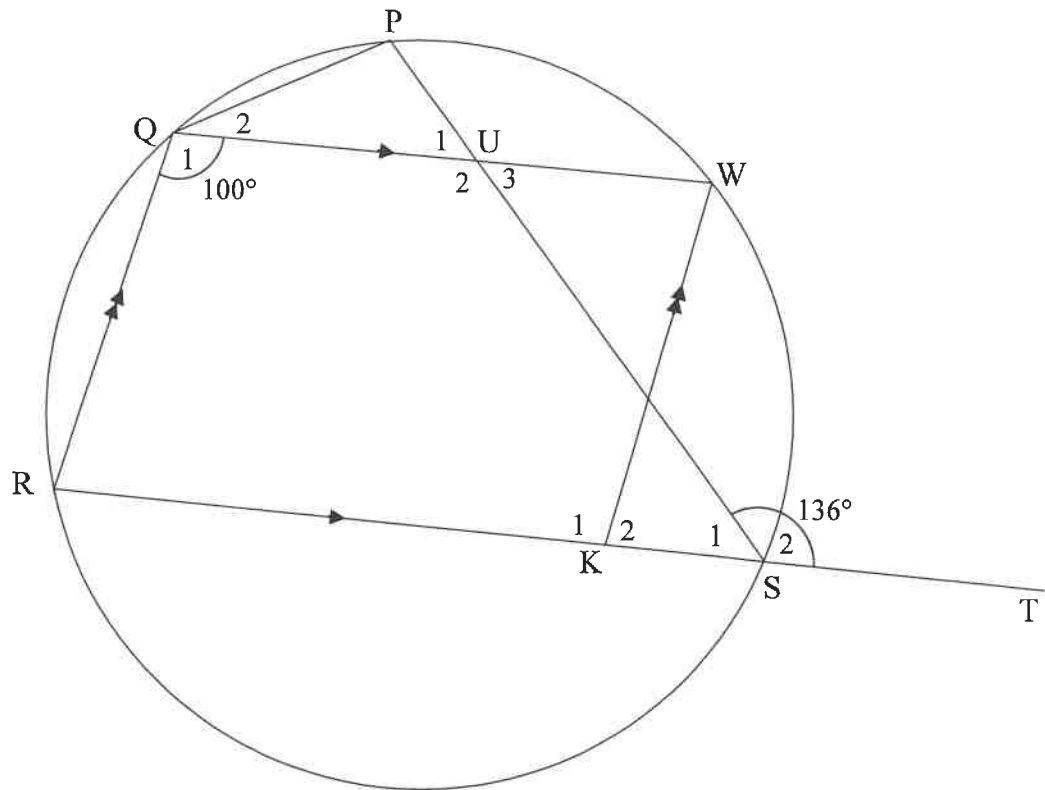


	Solution/Oplissing	Marks Punte
7.1		(2)
7.2		(1)
7.3		(7)
		[10]

Give reasons for your statements in QUESTIONS 8, 9 and 10.
 Gee redes vir jou bewerings in VRAAG 8, 9 en 10.

QUESTION/VRAAG 8

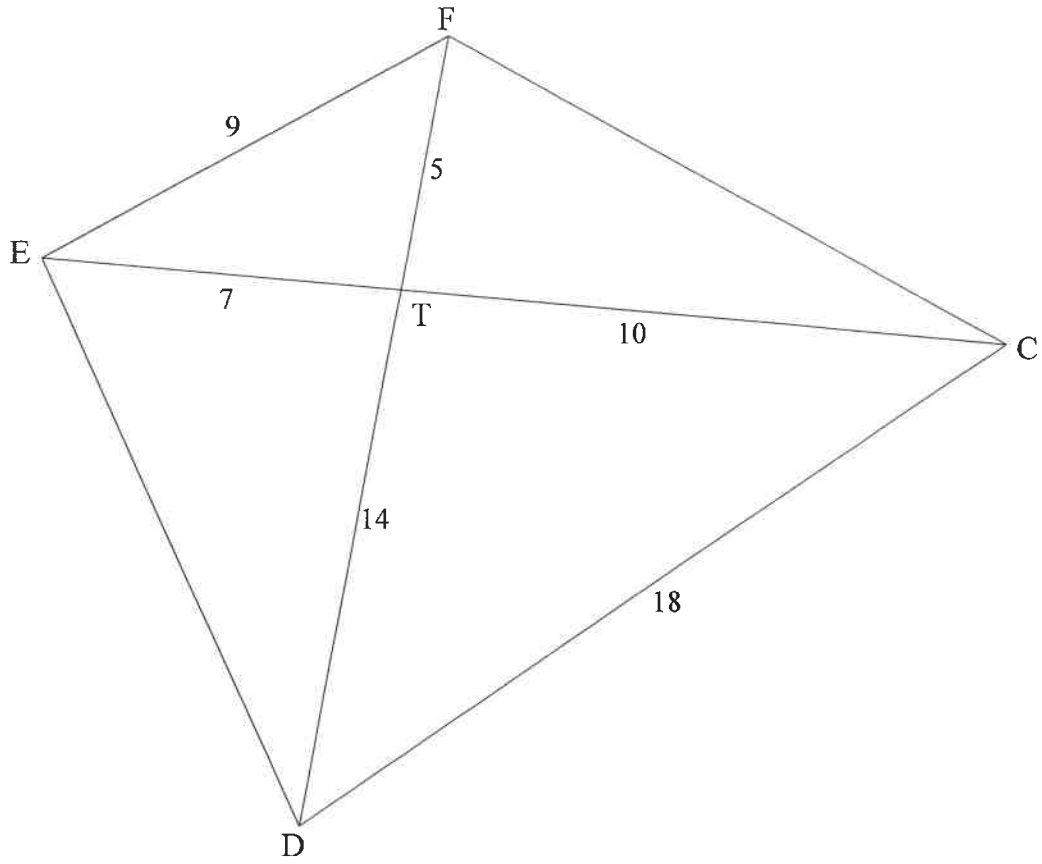
8.1



	Solution/Oplissing	Marks Punte
8.1.1		(2)
8.1.2		(2)

	Solution/Oplissing	Marks Punte
8.1.3		(3)
8.1.4		(2)

8.2

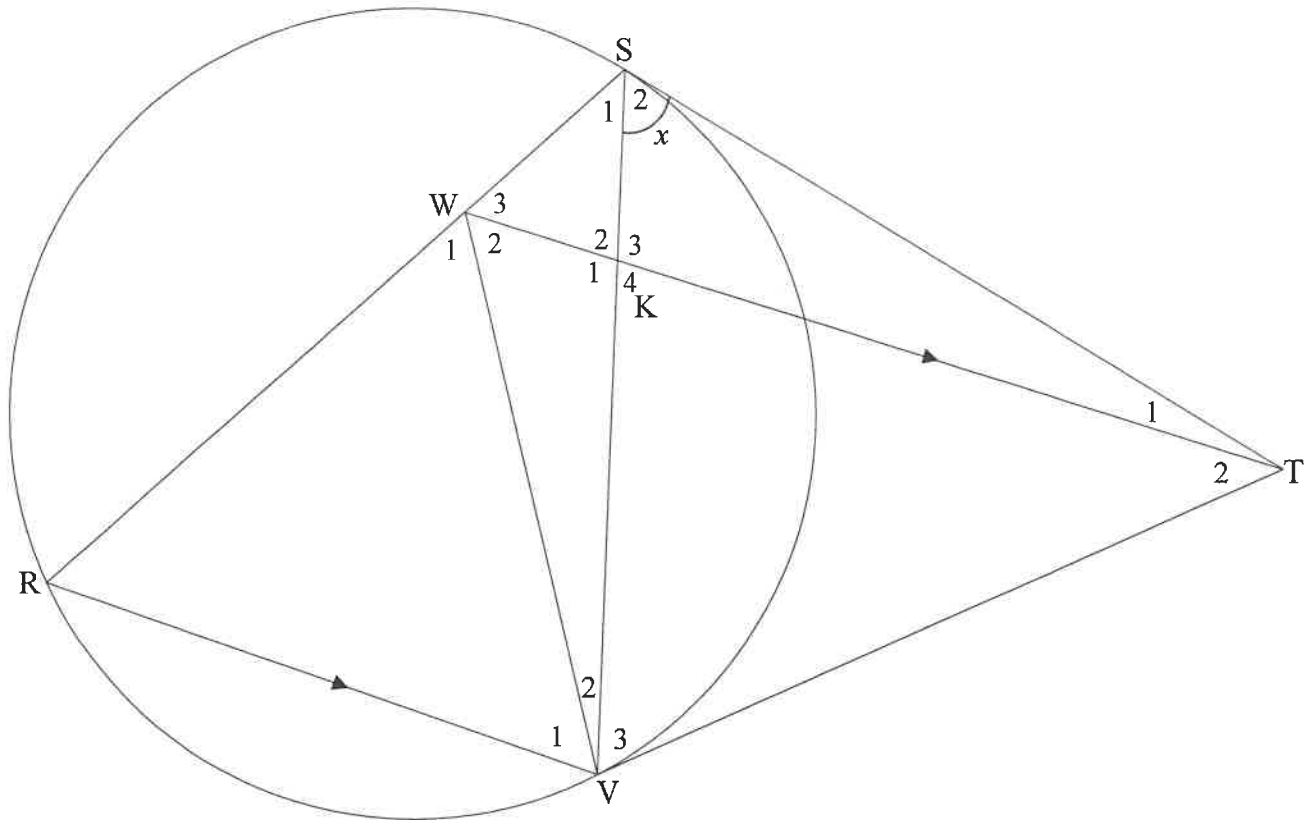


	Solution/Oplissing	Marks Punte
8.2.1		

(4)

	Solution/Oplissing	Marks Punte
8.2.2		
		(3) [16]

10.2



	Solution/Oplissing	Marks Punte
10.2.1		
		(6)
10.2.2(a)		
		(2)

10.2.2(b)		(4)
10.2.2(c)		(3)
10.2.2(d)		(4)
		[25]



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

**MATHEMATICS P2/WISKUNDE V2
NOVEMBER 2019
MARKING GUIDELINES/NASIENRIGLYNE**

MARKS/PUNTE: 150

**These marking guidelines consist of 26pages.
Hierdie nasienriglyne bestaan uit 26 bladsye.**

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering</i> (<i>'n Punt vir 'n bewering is onafhanklik van die rede</i>)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede</i> (<i>'n Punt word slegs vir die rede toegeken as die bewering korrek is</i>)
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

QUESTION/VRAAG 1

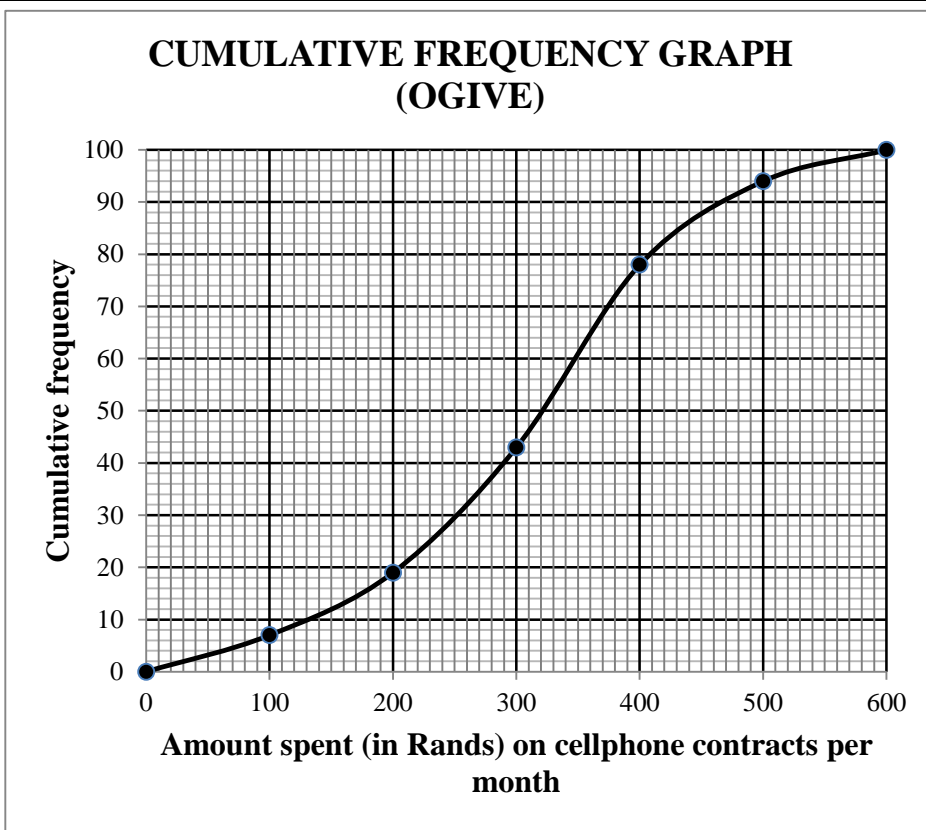
Monthly income (in rands) Maandelikse inkomste (in rand)	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayment (in rands) Maandelikse paaieiment (in rand)	2 000	3 000	3 500	5 200	5 500	6 000

1.1	$a = -1946,875... = -1946,88$ $b = 0,41$ $\hat{y} = -1946,88 + 0,41x$	Answer only: Full marks	✓ $a = -1946,88$ ✓ $b = 0,41$ ✓ equation (3)
1.2	Monthly repayment \approx R3 727,16 (calculator) <i>Maandelikse paaieiment \approx R3 727,16</i> OR $\hat{y} = -1946,88 + 0,41(14000)$ \approx R3 793,12		✓✓ answer (2) ✓ substitution ✓ answer (2)
1.3	$r = 0,946 \dots \approx 0,95$		✓ answer (1)
1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. OR D <i>Spandeer nie R9 000 per maand nie, want die punt (18 000 ; 9 000) lê baie ver van die kleinste-kwadrade regressielyn. OF D</i>		✓✓ answer (2)
			[8]

QUESTION/VRAAG 2

2.1	Number people paid R200 or less = 19 <i>Aantal mense wat R200 of minder betaal het = 19</i>	✓ answer (1)
2.2	$7 + 12 + a + 35 + b + 6 = 100$ $a = 40 - b$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times (40 - b)) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$ $350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$ $200b = 3200$ $b = 16$ $a = 24$ OR/OF $7 + 12 + a + 35 + b + 6 = 100$ $b = 40 - a$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times (40 - a)) + (550 \times 6)}{100}$ $350 + 1800 + 250a + 12250 + 1800 - 450a = 30900$ $200a = 4\ 800$ $a = 24$ $b = 16$	✓ $\sum x = 100$ ✓ $a = 40 - b$ ✓ $\sum fX$ ✓ $\sum \frac{fX}{n} = 309$ ✓ $200b = 3200$ (5) ✓ $\sum x = 100$ ✓ $b = 40 - a$ ✓ $\sum fX$ ✓ $\sum \frac{fX}{n} = 309$ ✓ $200a = 4\ 800$ (5)
2.3	Modal class/modale klas: $300 < x \leq 400$	✓ answer (1)

2.4



- ✓ grounded at (0 ; 0)
- ✓ (600 ; 100)
- ✓ cumulative frequencies for y-coordinates
- ✓ smooth shape

(4)

2.5

Number of people/Aantal mense = 100 – 82 [accept 80 – 84 people]
 18 people paid more than R420 per month/. [accept 16 – 20 people]
 18 mense betaal meer as R420 per maand

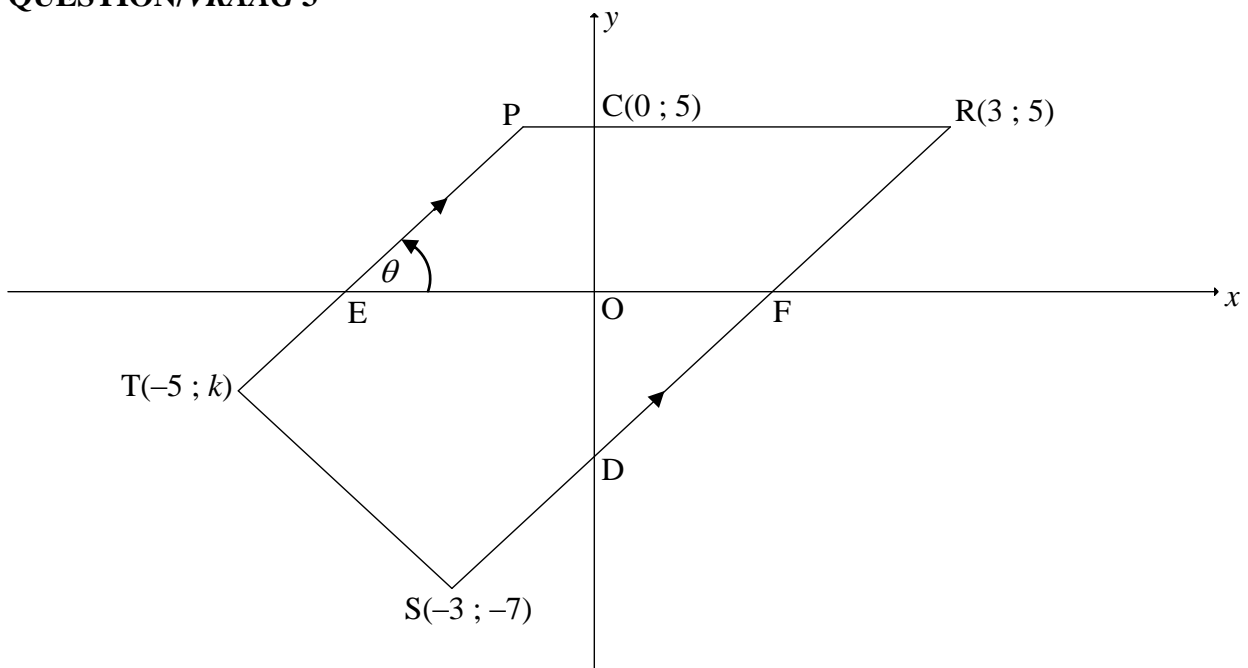
Answer only: Full marks

- ✓ 82
- ✓ answer

(2)

[13]

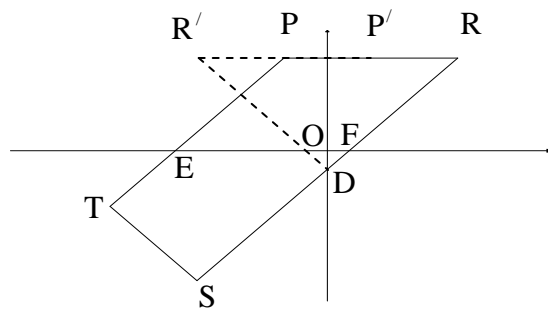
QUESTION/VRAAG 3



3.1	Equation of PR: $y = 5$	✓ answer (1)
3.2.1	$m_{RS} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{RS} = \frac{5 - (-7)}{3 - (-3)} = \frac{12}{6}$ $= 2$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px;">Answer only: Full marks</div>	✓ substitution of R & S into gradient formula ✓ answer (2)
3.2.2	$m_{RS} = m_{PT}$ [PT RS] $\tan \theta = 2$ $\theta = 63,43^\circ$	✓ $m_{RS} = m_{PT}$ ✓ $\tan \theta = 2$ ✓ $\theta = 63,43^\circ$ (3)
3.2.3	Equation of RS: $y - 5 = 2(x - 3)$ or $y - (-7) = 2(x - (-3))$ or $5 = 2(3) + c$ $y - 5 = 2x - 6$ $y + 7 = 2x + 6$ $c = -1$ $y = 2x - 1$ $y = 2x - 1$ $y = 2x - 1$ $\therefore D(0; -1)$ OR/OF $m_{RS} = m_{RD} = m_{DS}$ $2 = \frac{5 - y}{3 - 0} = \frac{y + 7}{0 - (-3)}$ $\therefore y = -1$ $\therefore D(0; -1)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px;">Answer only: Full marks</div>	✓ substitution ✓ equation of RS ✓ coordinates of D (3) ✓ equating gradients ✓ value of y ✓ coordinates of D (3)

<p>3.3</p>	$ST = 2\sqrt{5} = \sqrt{[-5 - (-3)]^2 + (k - (-7))^2}$ $20 = 4 + (k + 7)^2$ $(k + 7)^2 = 16$ $k + 7 = \pm 4$ $k = -11 \text{ or } k = -3$ $\therefore k = -3$ <p>OR</p> $ST = 2\sqrt{5} = \sqrt{[-5 - (-3)]^2 + (k - (-7))^2}$ $20 = 4 + k^2 + 14k + 49$ $k^2 + 14k + 33 = 0$ $(k + 11)(k + 3) = 0$ $k = -11 \text{ or } k = -3$ $\therefore k = -3$	<ul style="list-style-type: none"> ✓ substitute S and T into distance formula ✓ isolate square ✓ square root both sides ✓ answer <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓ substitute S and T into distance formula ✓ standard form ✓ factors ✓ answer <p style="text-align: right;">(4)</p>
<p>3.4</p>	<p>Method: translation T→S:</p> $(x; y) \rightarrow (x + 2; y - 4)$ <p>∴ by symmetry: D→N: D(0 ; -1) → N(0 + 2 ; -1 - 4) ∴ N(2 ; -5)</p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin-left: auto; margin-right: auto;">Answer only: Full marks</div> <p>OR</p> <p>Midpoint of TN = Midpoint of SD</p> $\frac{x + (-5)}{2} = \frac{-3 + 0}{2} \text{ and } \frac{y + (-3)}{2} = \frac{-7 + (-1)}{2}$ $x = 2 \text{ and } y = -5$ <p>∴ N(2 ; -5)</p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin-left: auto; margin-right: auto;">Answer only: Full marks</div>	<ul style="list-style-type: none"> ✓ method ✓ x-coordinate ✓ y-coordinate <p style="text-align: right;">(3)</p> <ul style="list-style-type: none"> ✓ method: midpoint of diagonals ✓ x-coordinate ✓ y-coordinate <p style="text-align: right;">(3)</p>

3.5



β is the inclination of RS $\therefore \beta = 63,434\dots^\circ$

$\hat{O}FD = 63,434\dots^\circ$ [vert opp \angle s]

$\hat{O}DF = 90^\circ - 63,434\dots^\circ = 26,565\dots^\circ$

$\hat{R}DR' = 2(26,565\dots^\circ) = 53,13^\circ$

OR

PEFR is a $\parallel m$ [both pairs of opp sides \parallel]

$\therefore \hat{R} = \theta = 63,434\dots^\circ$ [opp \angle s of $\parallel m$]

$\hat{R}R'D = 63,434\dots^\circ$ [\angle s opp = sides: $RD = R'D$]

$\hat{R}DR' = 180^\circ - (63,43^\circ + 63,43^\circ)$ [sum of \angle s in Δ]

$\hat{R}DR' = 53,13^\circ$

OR

$\tan \hat{O}DF = \frac{3}{6}$

$\hat{O}DF = 26,565\dots^\circ$

$\hat{R}DR' = 2(26,565\dots^\circ) = 53,13^\circ$

OR

$R'(-3; 5)$ [reflection of $R(3; 5)$ about the y-axis]

$RD = \sqrt{(3-0)^2 + (5-(-1))^2}$

$RD = \sqrt{45} = R'D$ or $3\sqrt{5}$ or 6,71

$(RR')^2 = (\sqrt{45})^2 + (\sqrt{45})^2 - 2(\sqrt{45})(\sqrt{45})(\cos \hat{R}DR')$

$6^2 = 45 + 45 - 2(45)(\cos \hat{R}DR')$

$\cos \hat{R}DR' = \frac{45 + 45 - 36}{2(45)}$

$\cos \hat{R}DR' = \frac{3}{5}$

$\therefore \hat{R}DR' = 53,13^\circ$

✓ $\beta = 63,43^\circ$

✓ $\hat{O}DF = 26,57^\circ$

✓ answer

(3)

✓ $\hat{R} = 63,43^\circ$

✓ $\hat{R}R'D = 63,43^\circ$

✓ answer

(3)

✓ trig ratio

✓ $\hat{O}DF = 26,565\dots^\circ$

✓ answer

(3)

✓ $R'(-3; 5)$ **OR**

$RD = \sqrt{45} = R'D$

✓ substitution into cosine rule

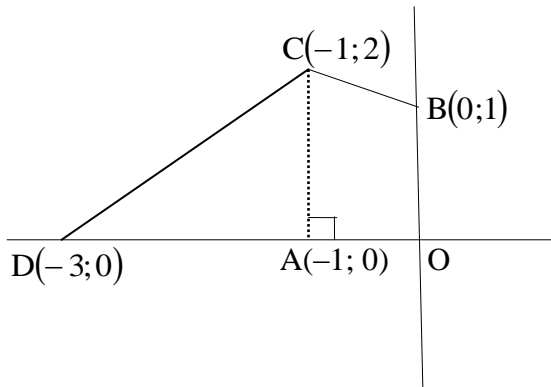
✓ answer

(3)

[19]

<p>4.3</p>	$m_{\text{radius}} = \frac{2-1}{-1-0} \text{ OR } \frac{2-(-\frac{1}{2})}{-1-\frac{3}{2}} \text{ OR } \frac{0-(-\frac{1}{2})}{1-\frac{3}{2}}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y = mx + c$ $y = x + c$ $2 = 1(-1) + c$ $c = 3$ $\therefore y = x + 3$ $y - x = 3$ <p>OR</p> $m_{\text{radius}} = \frac{2-1}{-1-0}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y - y_1 = m(x - x_1)$ $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y - 2 = x + 1$ $\therefore y = x + 3$ $y - x = 3$	<p>✓ m_{radius} ✓ m_{tangent}</p> <p>✓ substitute $(-1 ; 2)$ and m ✓ simplification (4)</p> <p>✓ m_{radius} ✓ m_{tangent}</p> <p>✓ substitute $(-1 ; 2)$ and m ✓ simplification (4)</p>
<p>4.4</p>	<p>Tangents to circle: $y = x + 3$ and $y = x + 1$</p> <p>$\therefore t > 3$ or $t < 1$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Answers only: Full marks</div>	<p>✓ $y = x + 1$</p> <p>✓ $t > 3$ ✓ $t < 1$ (3)</p>
<p>4.5</p>	<p>Draw rectangle CNED:</p> <p>Midpt of DN $(-\frac{7}{4}; \frac{3}{4})$</p> <p>$\therefore E(-\frac{5}{2}; -\frac{1}{2})$</p> <div style="text-align: center;"> </div> <p>OR/OF</p> <p>D(-3 ; 0)</p> <p>C → N:</p> <p>$(x ; y) \rightarrow (x + 0,5 ; y - 0,5)$</p> <p>D → E:</p> <p>D(x ; y) → E(x + 0,5 ; y - 0,5)</p> <p>$\therefore E(-3 + 0,5 ; 0 - 0,5)$</p> <p>$\therefore E(-2,5 ; -0,5)$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Answer only: Full marks</div>	<p>✓ midpt of DN</p> <p>✓ x value ✓ y value (3)</p> <p>✓ coordinates of D</p> <p>✓ x value ✓ y value (3)</p>

4.6



$$\begin{aligned} \text{area of trapezium AOBC} &= \frac{1}{2}(1+2)(1) \\ &= 1\frac{1}{2} \text{ square units} \end{aligned}$$

$$\begin{aligned} \text{area of } \triangle ACD &= \frac{1}{2}(2)(2) \\ &= 2 \text{ square units} \end{aligned}$$

$$\text{area of quadrilateral OBCD} = 3\frac{1}{2} \text{ square units}$$

$$\begin{aligned} \therefore 2a^2 &= \frac{7}{2} \\ a^2 &= \frac{7}{4} \\ a &= \frac{\sqrt{7}}{2} \end{aligned}$$

OR

✓ substitution into area of trapezium form

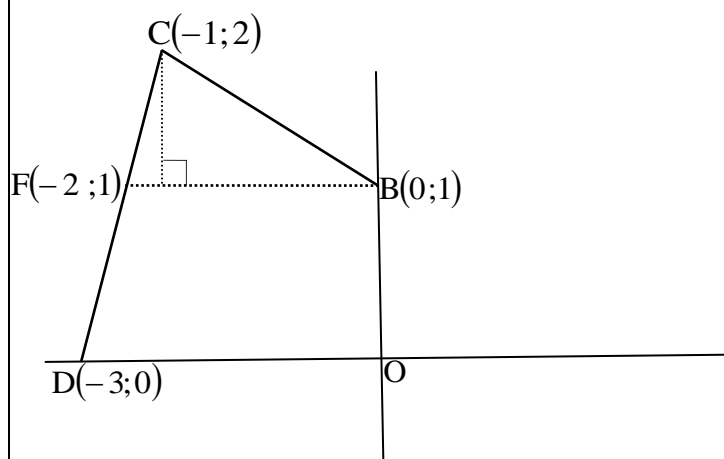
✓ area of trapezium

✓ area of triangle

✓ area of OBCD

✓ equating area OBCD to $2a^2$

(5)



BM produced cuts the tangent at F.

$$\begin{aligned} \text{area of } \triangle CFB &= \frac{1}{2}(2)(1) \\ &= 1 \text{ square unit} \end{aligned}$$

$$\begin{aligned} \text{area of trapezium BFDO} &= \frac{1}{2}(2+3)(1) \\ &= 2\frac{1}{2} \text{ square units} \end{aligned}$$

$$\text{area of quadrilateral OBCD} = 3\frac{1}{2} \text{ square units}$$

$$\therefore 2a^2 = \frac{7}{2}$$

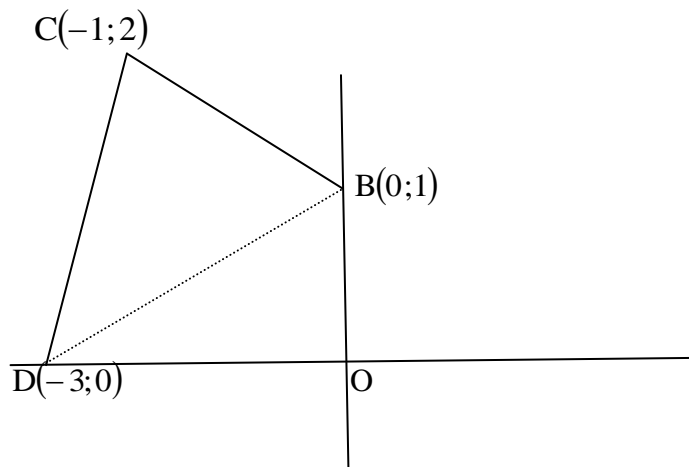
$$a^2 = \frac{7}{4}$$

$$a = \frac{\sqrt{7}}{2}$$

OR

- ✓ area of triangle
- ✓ substitution into area of trapezium
- ✓ area of trapezium
- ✓ area of OBCD
- ✓ equating area OBCD to $2a^2$

(5)



Join DB

$$\begin{aligned} \text{area of } \triangle ODB &= \frac{1}{2}(3)(1) \\ &= \frac{3}{2} \text{ square unit} \end{aligned}$$

$$\begin{aligned} \text{area of } \triangle DCB &= \frac{1}{2}(2\sqrt{2})(\sqrt{2}) \\ &= 2 \text{ square unit} \end{aligned}$$

$$\therefore \text{area of OBCD} = \frac{3}{2} + 2 = \text{square units}$$

$$2a^2 = \frac{7}{2}$$

$$a^2 = \frac{7}{4}$$

$$a = \frac{\sqrt{7}}{2}$$

OR

✓ area of Δ

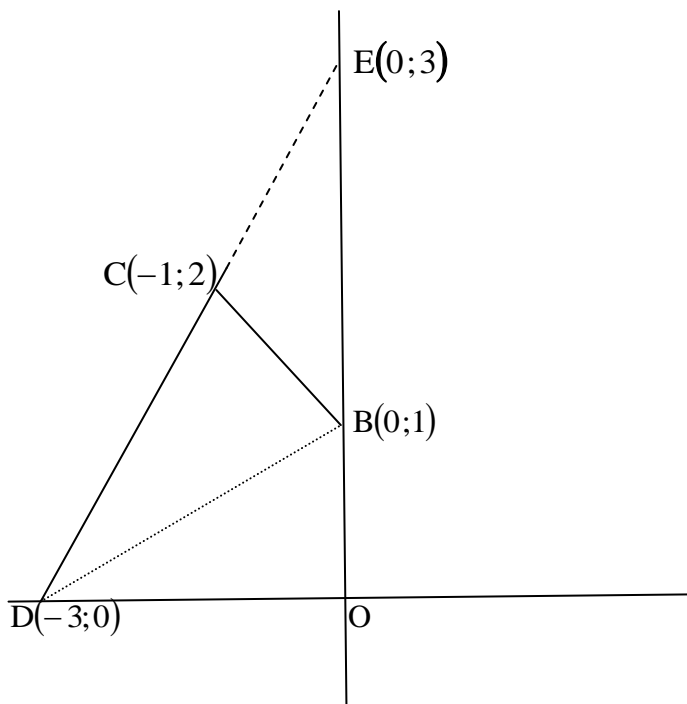
✓ subst into area of Δ

✓ area of Δ

✓ area of OBCD

✓ equating area
OBCD to $2a^2$

(5)



Let E be the point of intersection of DC with the positive y-axis.

$$\text{area of } \triangle DEO = \frac{1}{2}(3)(3)$$

$$= \frac{9}{2} \text{ square unit}$$

$$\text{area of } \triangle ECB = \frac{1}{2}(2)(1) \text{ or } \frac{1}{2}(\sqrt{2})(\sqrt{2})$$

$$= 1 \text{ square unit}$$

$$\text{area of quadrilateral OBCD} = \frac{9}{2} - 1 = 3\frac{1}{2} \text{ square units}$$

$$\therefore 2a^2 = \frac{7}{2}$$

$$a^2 = \frac{7}{4}$$

$$a = \frac{\sqrt{7}}{2}$$

✓ area of Δ

✓ subst into area of Δ

✓ area of Δ

✓ area of OBCD

✓ equating area
OBCD to $2a^2$

(5)

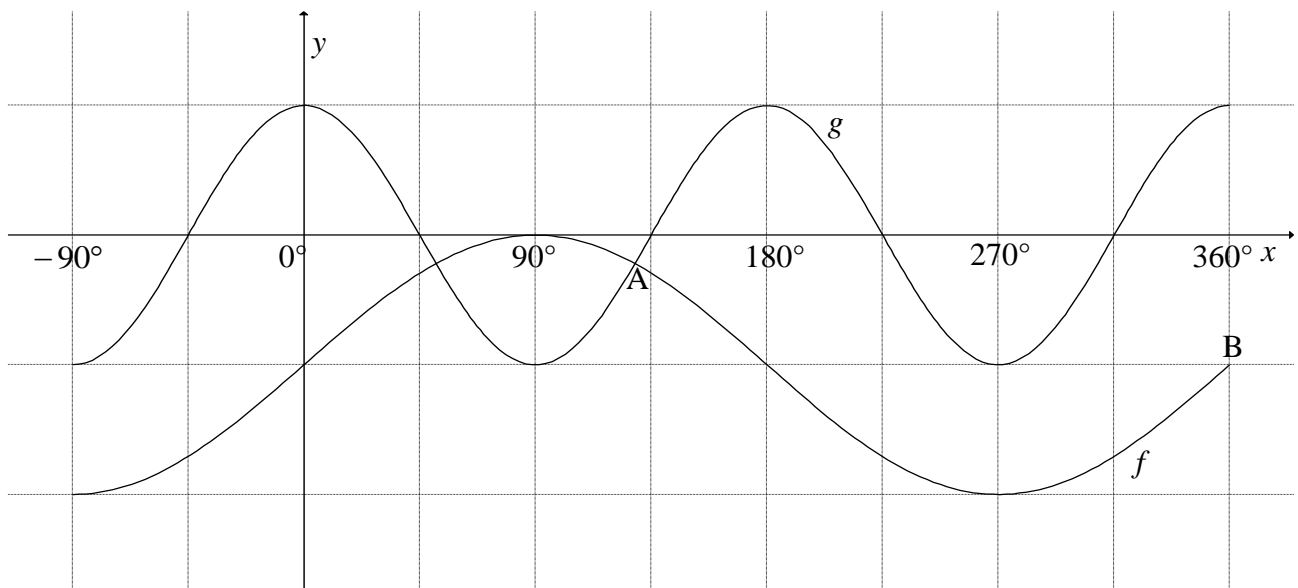
[20]

QUESTION/VRAAG 5

<p>5.1</p>	$\frac{\sin x}{\cos x \cdot \tan x} + \sin(180^\circ + x) \cos(90^\circ - x)$ $= \frac{\sin x}{\cos x \cdot \frac{\sin x}{\cos x}} + (-\sin x) \sin x$ $= 1 - \sin^2 x$ $= \cos^2 x$	<p>✓ $-\sin x$ ✓ $\sin x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $1 - \sin^2 x$ ✓ $\cos^2 x$</p> <p>(5)</p>
<p>5.2</p>	$\frac{\sin^2 35^\circ - \cos^2 35^\circ}{4 \sin 10^\circ \cos 10^\circ}$ $= \frac{-(\cos^2 35^\circ - \sin^2 35^\circ)}{2(2 \sin 10^\circ \cos 10^\circ)}$ $= \frac{-\cos 70^\circ}{2 \sin 20^\circ}$ $= \frac{-\cos 70^\circ}{2 \cos 70^\circ} \quad \text{OR} \quad = \frac{-\sin 20^\circ}{2 \sin 20^\circ} = -\frac{1}{2}$	<p>✓ $-(\cos^2 35^\circ - \sin^2 35^\circ)$ ✓ $-\cos 70^\circ$ ✓ $2 \sin 20^\circ$</p> <p>✓ answer</p> <p>(4)</p>
<p>5.3</p>	$2 \sin^2 77^\circ = 2[\sin(90^\circ - 13^\circ)]^2$ $= 2 \cos^2 13^\circ$ $= 2 \cos^2 13^\circ - 1 + 1$ $= \cos 26^\circ + 1$ $= m + 1$ <p>OR</p> $1 - 2 \sin^2 77^\circ = \cos 154^\circ$ $2 \sin^2 77^\circ = 1 - \cos 154^\circ$ $= 1 - (-\cos 26^\circ)$ $= 1 + m$	<p>✓ using co-ratio ✓ reduction ✓ $2 \cos^2 13^\circ - 1 = \cos 26^\circ$ ✓ answer</p> <p>(4)</p> <p>✓ $1 - 2 \sin^2 77^\circ = \cos 154^\circ$ ✓ $2 \sin^2 77^\circ = 1 - \cos 154^\circ$ ✓ reduction ✓ answer</p> <p>(4)</p>
<p>5.4.1</p>	$\sin(x + 25^\circ) \cos 15^\circ - \cos(x + 25^\circ) \sin 15^\circ = \tan 165^\circ$ $\sin(x + 25^\circ - 15^\circ) = -0,2679... \text{ OR } -2 + \sqrt{3}$ $\sin(x + 10^\circ) = -0,2679... \text{ OR } -2 + \sqrt{3}$ $x + 10^\circ = 195,54^\circ + k \cdot 360^\circ \quad \text{or} \quad x + 10^\circ = 344,46^\circ + k \cdot 360^\circ$ $x = 185,54^\circ + k \cdot 360^\circ; k \in \mathbb{Z} \quad \text{or} \quad x = 334,46^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ <p>OR/OF</p>	<p>✓ ✓ $\sin(x + 10^\circ)$ ✓ $-0,2679...$ ✓ $195,54^\circ$ & $344,46^\circ$ ✓ $185,54^\circ$ & $334,46^\circ$ ✓ $+ k \cdot 360^\circ; k \in \mathbb{Z}$</p> <p>(6)</p>

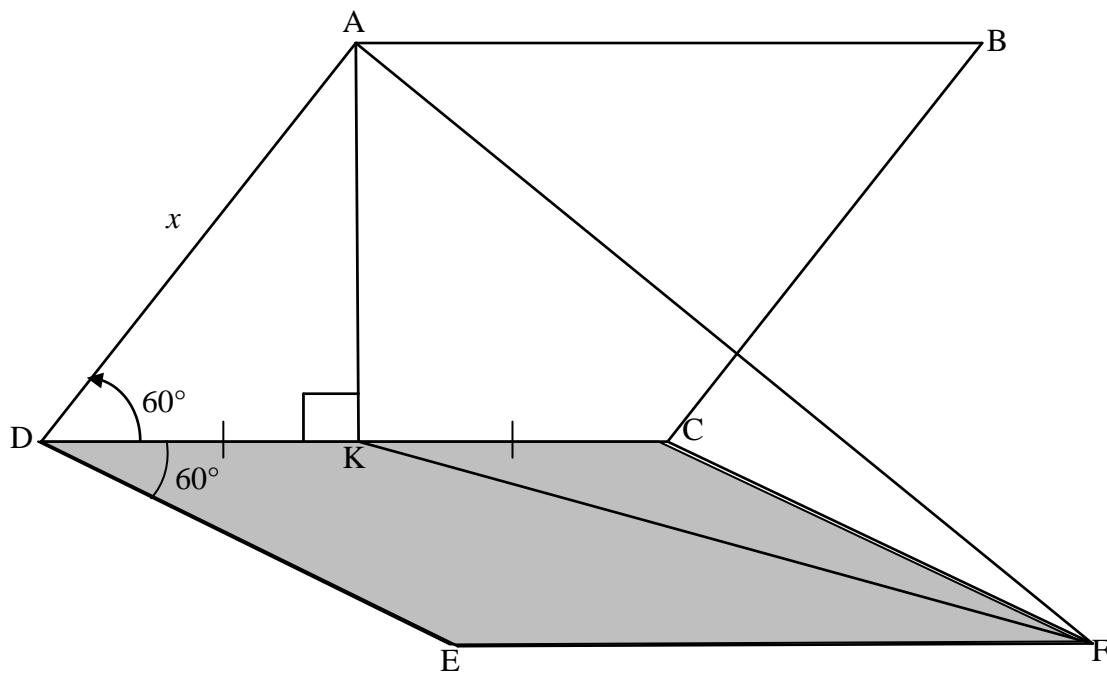
	$\sin(x + 25^\circ)\sin 75^\circ - \cos(x + 25^\circ)\cos 75^\circ = \tan 165^\circ$ $-(\cos(x + 25^\circ)\cos 75^\circ - \sin(x + 25^\circ)\sin 75^\circ) = -0,2679\dots$ $\cos(x + 100^\circ) = 0,2679\dots$ $\text{ref. } \angle = 74.4577\dots^\circ$ $x + 100^\circ = 74,46^\circ + k.360^\circ \text{ or } x + 100^\circ = 285,54^\circ + k.360^\circ$ $x = -25,54^\circ + k.360^\circ; k \in Z \text{ or } x = 185,54^\circ + k.360^\circ; k \in Z$	$\checkmark\checkmark \cos(x + 100^\circ)$ $\checkmark -0,2679\dots$ $\checkmark 74,46^\circ \text{ \& } 285,54^\circ$ $\checkmark -25,54^\circ \text{ \& } 185,54^\circ$ $\checkmark + k.360^\circ; k \in Z$ <p style="text-align: right;">(6)</p>
5.4.2	$f(x) = \sin(x + 10^\circ)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px 0;">Answers only: Full marks</div> <p>For minimum value of $\sin x$: $x = 270^\circ$ For minimum value of $\sin(x + 10^\circ)$: $x = 260^\circ$</p>	$\checkmark f(x) = \sin(x + 10^\circ)$ $\checkmark 270^\circ$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3)</p>
		[22]

QUESTION/VRAAG 6



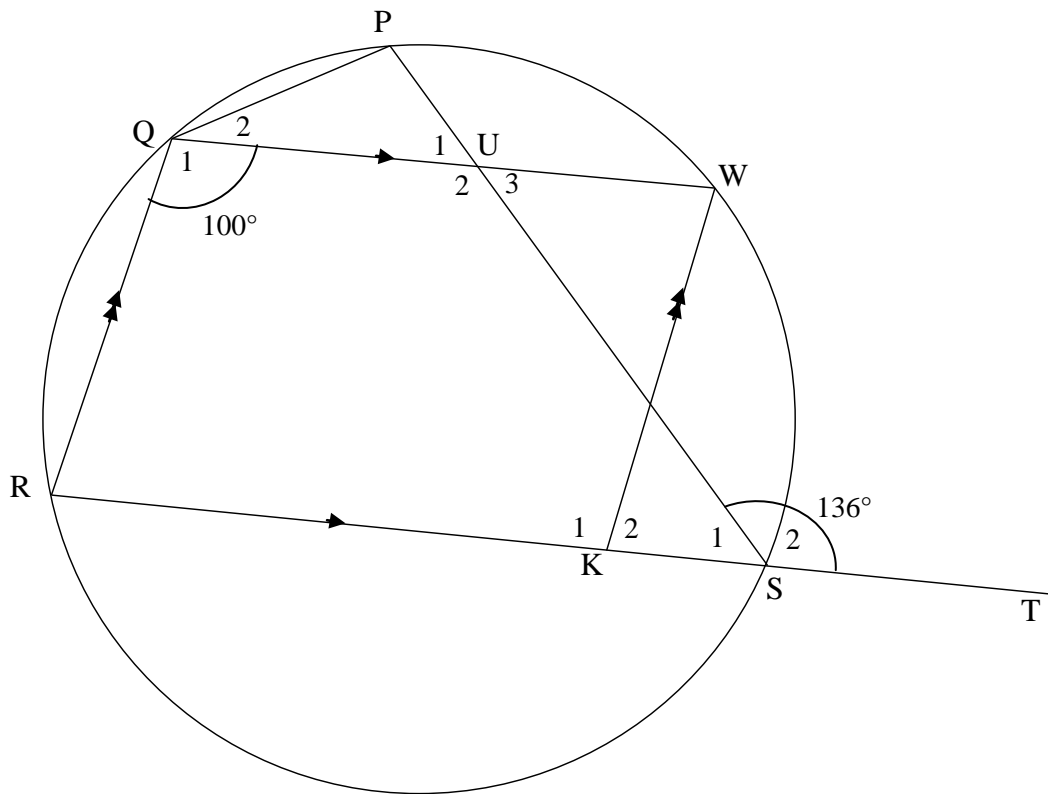
6.1	Range of f : $y \in [-2 ; 0]$ OR $-2 \leq y \leq 0$	✓ critical values ✓ notation (2)
6.2	$x \in (90^\circ ; 270^\circ)$ OR $x \in [90^\circ ; 270^\circ]$	✓ critical values ✓ notation (2)
6.3	$PQ = \cos 2x - (\sin x - 1)$ $= 1 - 2\sin^2 x - \sin x + 1$ $= -2\sin^2 x - \sin x + 2$ $\sin x = -\frac{b}{2a}$ $= \frac{-(-1)}{2(-2)}$ $\sin x = -\frac{1}{4}$ $\therefore x = 194,48^\circ \text{ or } x = 345,52^\circ$	✓ $PQ = \cos 2x - (\sin x - 1)$ ✓ $\cos 2x = 1 - 2\sin^2 x$ ✓ substitution into formula ✓ $\sin x = -\frac{1}{4}$ ✓ $194,48^\circ$ ✓ $345,52^\circ$ (6)
[10]		

QUESTION/VRAAG 7



7.1	$\sin 60^\circ = \frac{AK}{x}$ $AK = x \sin 60^\circ \text{ or } \frac{\sqrt{3}}{2}x \text{ or } 0,866x$	✓ trig ratio ✓ answer (2)
7.2	$\widehat{KCF} = 120^\circ$	✓ answer (1)
7.3	$KF^2 = CF^2 + CK^2 - 2CF \cdot CK \cos \widehat{KCF}$ $= x^2 + \left(\frac{x}{2}\right)^2 - 2x\left(\frac{x}{2}\right) \cos 120^\circ$ $= x^2 + \frac{x^2}{4} - x^2\left(-\frac{1}{2}\right)$ $= \frac{7x^2}{4}$ $KF = \frac{\sqrt{7}x}{2}$ $\widehat{AKF} = y$ $\text{Area } \triangle AKF = \frac{1}{2} \cdot AK \cdot KF \sin \widehat{AKF}$ $= \frac{1}{2} \cdot \frac{\sqrt{3}x}{2} \cdot \frac{\sqrt{7}x}{2} \sin y$ $= \frac{x^2 \sqrt{21} \sin y}{8}$	✓ correct use of cosine rule ✓ substitution ✓ $\cos 120^\circ = -\frac{1}{2}$ ✓ $KF = \frac{\sqrt{7}x}{2}$ ✓ correct use of area rule ✓ substitution ✓ answer in terms of x and y (7)
[10]		

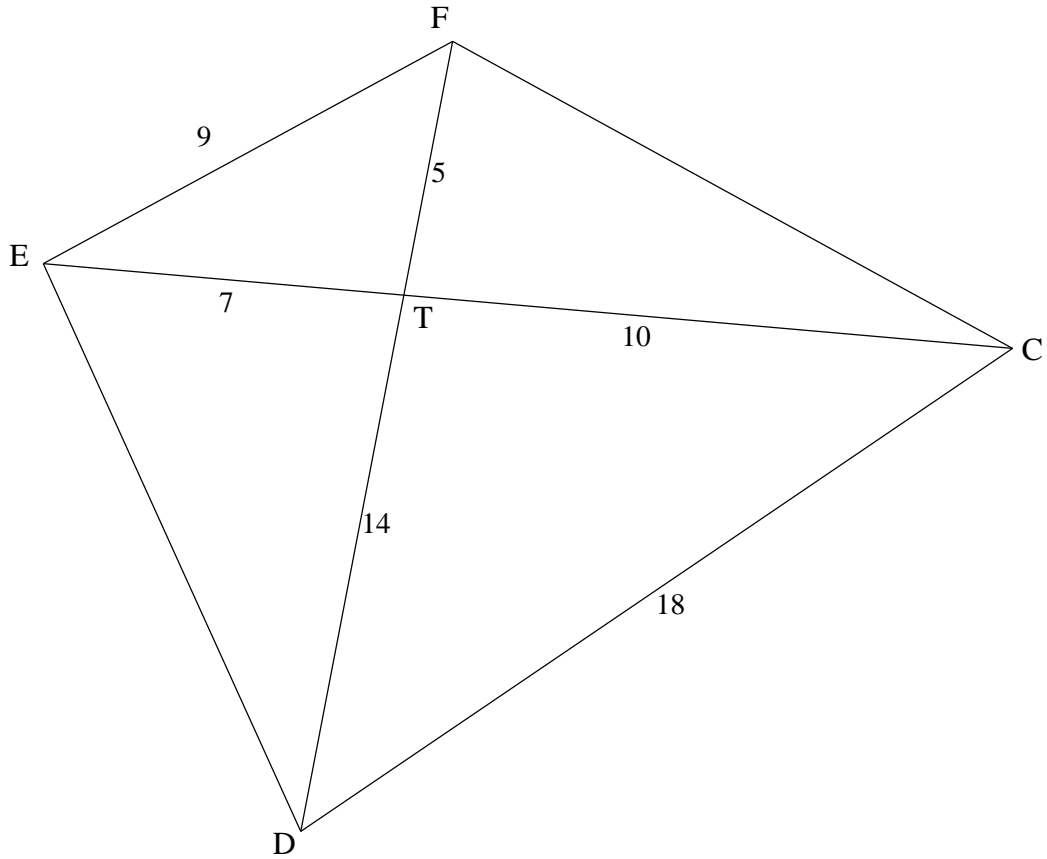
QUESTION/VRAAG 8



8.1.1	$\hat{R} = 80^\circ$ [co-int \angle s/ko-binne \angle e; QW RK]	\checkmark S \checkmark R (2)
8.1.2	$\hat{P} = 100^\circ$ [opp \angle s of cyclic quad/teenoorst \angle e v koordevh]	\checkmark S \checkmark R (2)
8.1.3	<p> $\hat{PQR} = 136^\circ$ [ext \angle of cyclic quad/buite \angle v koordevh] $\hat{Q}_2 = 36^\circ$ OR $\hat{PUW} = \hat{S}_2 = 136^\circ$ [corresp \angles/ooreenkomstige \anglee; QW RK] $\hat{PQW} + \hat{P} = \hat{PUW}$ [ext \angles of/buite \angle van Δ QPU] $\hat{PQW} + 100^\circ = 136^\circ$ $\hat{PQW} = 36^\circ$ OR $\hat{U}_3 = 180^\circ - 136^\circ = 44^\circ$ [co-int \angles/ko-binne \anglee; QW RK] $\hat{U}_1 = \hat{U}_3 = 44^\circ$ [vert opp \angles/regoorstaande \anglee] $\hat{PQW} = 180^\circ - (100 + 44^\circ)$ [sum of \angles in Δ/som \anglee van Δ] $\hat{PQW} = 36^\circ$ </p>	\checkmark S \checkmark R \checkmark S (3) \checkmark S \checkmark R \checkmark S (3)

8.1.4	$\hat{U}_2 = \hat{S}_2 = 136^\circ$ OR	[alt \angle s/verwiss \angle e ; QW RK] 	\checkmark S \checkmark R (2)
	$\hat{U}_2 = 100^\circ + 36^\circ$ $= 136^\circ$ OR	[ext \angle s of/buite \angle van Δ QPU] 	\checkmark S \checkmark R (2)
	$\hat{U}_2 = P\hat{U}W = 136^\circ$ OR	[vert opp \angle s/regoorstaande \angle e] 	\checkmark S \checkmark R (2)
	$\hat{U}_2 = 180^\circ - \hat{U}_3$ $= 180^\circ - 44^\circ$ $= 136^\circ$	[\angle s on a str line/ \angle e op reguitlyn]	\checkmark S \checkmark R (2)

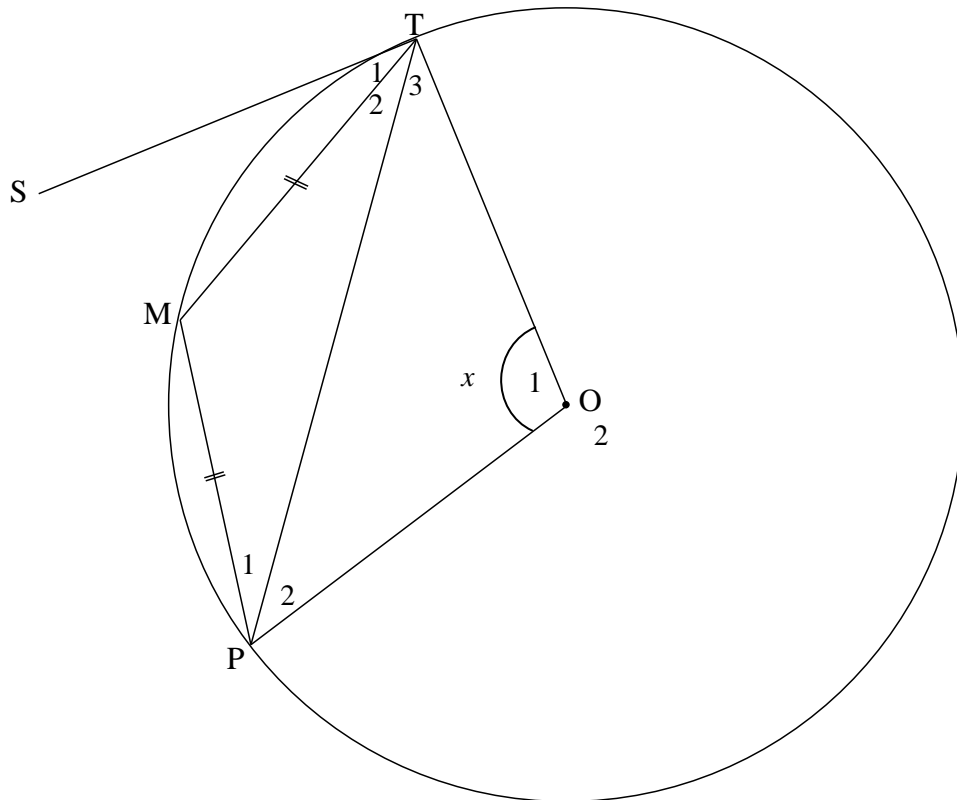
8.2



<p>8.2.1</p>	<p>In $\triangle EFT$ and $\triangle DCT$:</p> $\frac{EF}{CD} = \frac{9}{18} = \frac{1}{2}$ $\frac{FT}{TC} = \frac{5}{10} = \frac{1}{2}$ $\frac{ET}{TD} = \frac{7}{14} = \frac{1}{2}$ <p>$\therefore \triangle EFT \parallel \triangle DCT$ [Sides of Δ in prop/ sye van Δ in dieselfde verh] $\therefore \hat{EFD} = \hat{ECD}$</p> <p>OR</p> <p>In $\triangle FET$: $49 = 25 + 81 - 2(5)(9)\cos\hat{F}$ $\cos\hat{F} = \frac{19}{30}$ $\hat{F} = 50,7^\circ$</p> <p>In $\triangle TDC$: $196 = 100 + 256 - 2(10)(18)\cos\hat{C}$ $\cos\hat{C} = \frac{19}{30}$ $\hat{C} = 50,7^\circ$</p>	<p>$\checkmark\checkmark$ all 3 ratios = $\frac{1}{2}$</p> <p>$\checkmark \triangle EFT \parallel \triangle DCT \checkmark R$</p> <p>(4)</p> <p>$\checkmark\checkmark \hat{F} = 50,7^\circ$</p> <p>$\checkmark\checkmark \hat{C} = 50,7^\circ$</p> <p>(4)</p>
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8.2.2	$\hat{E}FD = \hat{E}CD$ [proved in 8.2.1] E, F, C and D are concyclic EFCD is a cyclic quad [converse \angle s in the same segment/ <i>omgekeerde \anglee in dies segment</i>] $\therefore \hat{D}FC = \hat{D}EC$ [\angle s in the same segment/ \angle e in dies segment]	\checkmark S \checkmark R \checkmark R (3)
[16]		

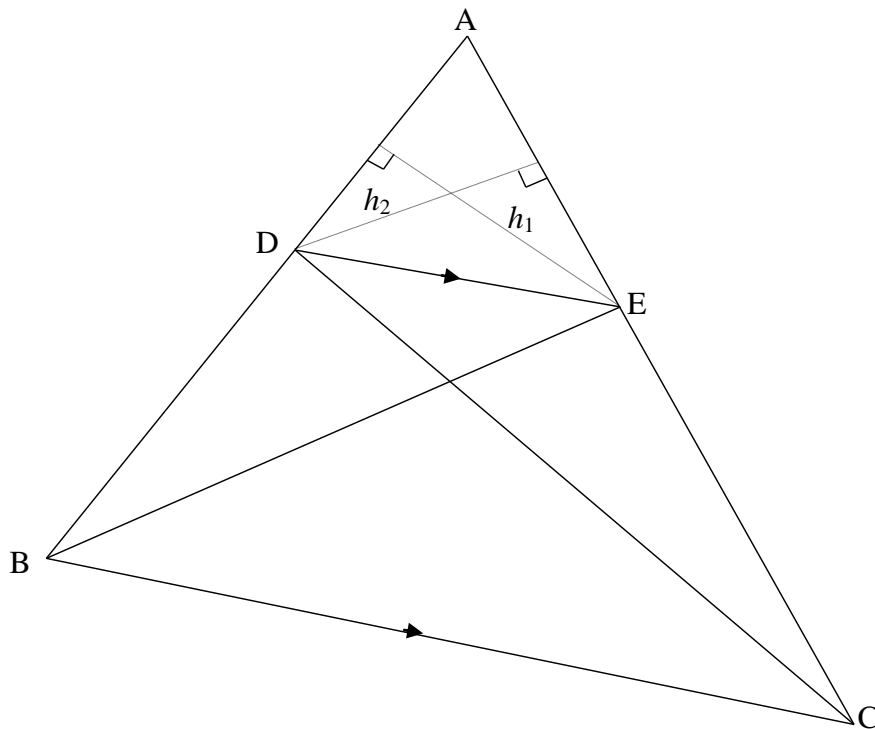
QUESTION/VRAAG 9



<p> $\hat{O}_2 = 360^\circ - x$ [\angles round a pt/\anglee om 'n punt] $\therefore \hat{M} = 180^\circ - \frac{1}{2}x$ [\angle at centre = $2 \times \angle$ at circumference/ <i>middelpunts</i> $\angle = 2 \times$ <i>omtreks</i> \angle] $\therefore \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ [sum of \angles in Δ/<i>som</i> \anglee van Δ] $\therefore \hat{T}_2 = \hat{P}_1 = \frac{1}{4}x$ [\angles opp equal sides/\anglee teenoor gelyke sye] $\therefore \hat{STM} = \hat{P}_1 = \frac{1}{4}x$ [tan chord theorem/<i>raaklyn koordstelling</i>] OR/OF $\hat{O}_2 = 360^\circ - x$ [\angles round a pt/\anglee om 'n punt] $\therefore \hat{M} = \frac{1}{2}\hat{O}_2$ [\angle at centre = $2 \times \angle$ at circumference] $\therefore \hat{T}_2 + \hat{P}_1 = 180^\circ - \hat{M}$ [sum of \angles in Δ/<i>som</i> \anglee van Δ] $\therefore \hat{T}_2 = \hat{P}_1$ [\angles opp equal sides/\anglee teenoor gelyke sye] $= \frac{180^\circ - \hat{M}}{2} = \frac{180^\circ - \frac{1}{2}\hat{O}_2}{2} = \frac{180^\circ - \frac{1}{2}(360^\circ - x)}{2} = \frac{1}{4}x$ $\therefore \hat{STM} = \frac{1}{4}x$ [tan chord theorem/<i>raaklyn koordstelling</i>] </p>	<p> $\checkmark \hat{O}_2 = 360^\circ - x$ $\checkmark \hat{M} = 180^\circ - \frac{1}{2}x$ $\checkmark R$ $\checkmark \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ $\checkmark \hat{P}_1 = \frac{1}{4}x$ $\checkmark R$ $\checkmark R$ (7) $\checkmark \hat{O}_2 = 360^\circ - x$ $\checkmark S$ $\checkmark R$ $\checkmark S$ $\checkmark R$ $\checkmark S$ $\checkmark R$ (7) </p>
[7]	

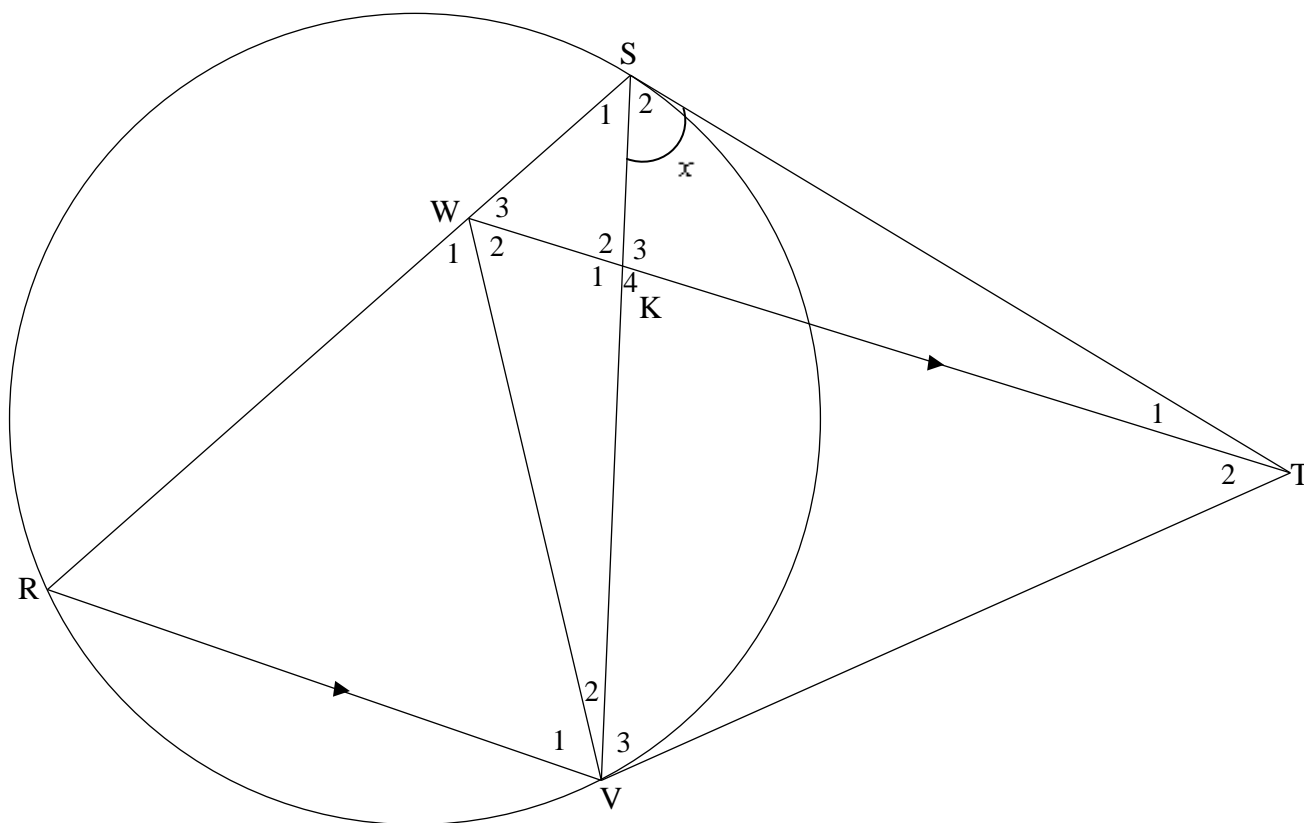
QUESTION/VRAAG 10

10.1



<p>10.1</p>	<p>Constr: Draw h_1 from $E \perp AD$ and h_2 from $D \perp AE$ <i>Konstr: Trek h_1 vanaf $E \perp AD$ en h_2 vanaf $D \perp AE$</i></p> <p>Proof/Bewys:</p> $\frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1} = \frac{AD}{DB}$ $\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{\frac{1}{2}AE \times h_2}{\frac{1}{2}EC \times h_2} = \frac{AE}{EC}$ <p>But area $\triangle BDE = \text{area } \triangle DEC$ [same base & height or $DE \parallel BC$/ <i>dies basis & hoogte; of $DE \parallel BC$</i>]</p> $\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	<p>✓ constr/konstr OR reason: common vertex or same height</p> $\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1}$ $\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{AE}{EC}$ <p>✓ S ✓ R ✓ S</p> <p style="text-align: right;">(6)</p>
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10.2



10.2.1	$\hat{V}_3 = x$ [Tans from same point/raaklyne vanaf dieselfde pt] $\hat{R} = x$ [tan chord theorem/raaklyn koordstelling] $\hat{W}_3 = x$ [corresp \angle s/ooreenkomstige \angle e; WT RV]	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R (6)
10.2.2(a)	$\hat{V}_3 = \hat{W}_3 = x$ [proved in 10.2.1] W, S, T and V are concyclic/is konsiklies WSTV is a cyclic quad [converse \angle s in the same segment/ Omgekeerde \angle e in dieselfde segment]	✓ S ✓ R (2)
10.2.2(b)	$\hat{W}_2 = \hat{S}_2 = x$ [\angle s in the same segment/ \angle e in dies segment] $\hat{V}_1 = \hat{W}_2 = x$ [alt \angle s/verwiss \angle e ; WT RV] But $\hat{R} = x$ [proved in 10.2.1] $\therefore \hat{R} = \hat{V}_1 = x$ $\therefore WR = WV$ [sides opp equal \angle s/sye teenoor gelyke \angle e] ΔWRV is isosceles/is gelykbenig OR/OF	✓ S ✓ R ✓ S/ R ✓ S (4)

	$\hat{S}_2 = \hat{W}_2 = x$ [\angle s in the same segment] $\hat{W}_2 = \hat{W}_3 = x$ $\hat{W}_2 + \hat{W}_3 = \hat{R} + \hat{V}_1$ [ext \angle of Δ] $\therefore \hat{V}_1 = x = \hat{R}$ $\therefore WR = WV$ [sides opp equal \angle s/sye teenoor gelyke \angle e] ΔWRV is isosceles/is gelykbenig	✓ S ✓R ✓ S/R ✓ S (4)
10.2.2(c)	In ΔWRV and/en ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved OR tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\therefore \Delta WRV \parallel \Delta TSV$ [\angle, \angle, \angle] OR/OF In ΔWRV and/en ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved OR tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\hat{W}_1 = \hat{S}TV = x$ [sum of \angle s in Δ/\angle e van Δ] $\therefore \Delta WRV \parallel \Delta TSV$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.2(d)	$\frac{RV}{SV} = \frac{WR}{TS}$ [$\Delta WRV \parallel \Delta TSV$] $\therefore WR \times SV = RV \times TS$ $\frac{WR}{SR} = \frac{KV}{SV}$ [prop theorem/eweredigst; $WT \parallel RV$] $\therefore WR \times SV = KV \times SR$ $\therefore RV \times TS = KV \times SR$ $\therefore \frac{RV}{SR} = \frac{KV}{TS}$ OR/OF In ΔRVS and/en ΔVKT $S\hat{V}R = \hat{K}_4$ [alt \angle s, $WT \parallel RV$] $S\hat{R}V = \hat{V}_3$ [proven] $\Delta RVS \parallel \Delta VKT$ [\angle, \angle, \angle] $\therefore \frac{RV}{SR} = \frac{KV}{VT}$ but $VT = ST$ [tans from same point] $\therefore \frac{RV}{SR} = \frac{KV}{TS}$	✓ correct ratios ✓ $\frac{WR}{SR} = \frac{KV}{SV}$ ✓ R ✓ equating $WR \times SV$ (4) ✓ identifying correct Δ s ✓ proving \parallel ✓ correct ratio ✓ S (4)
[25]		

TOTAL/TOTAAL: 150