



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NASIONALE
SENIOR SERTIFIKAAT**

GRAAD 12

WISKUNDE V2

NOVEMBER 2017

PUNTE: 150

TYD: 3 uur

**Hierdie vraestel bestaan uit 14 bladsye, 1 inligtingsblad
en 'n antwoordeboek van 28 bladsye.**

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat die vraestel beantwoord word.

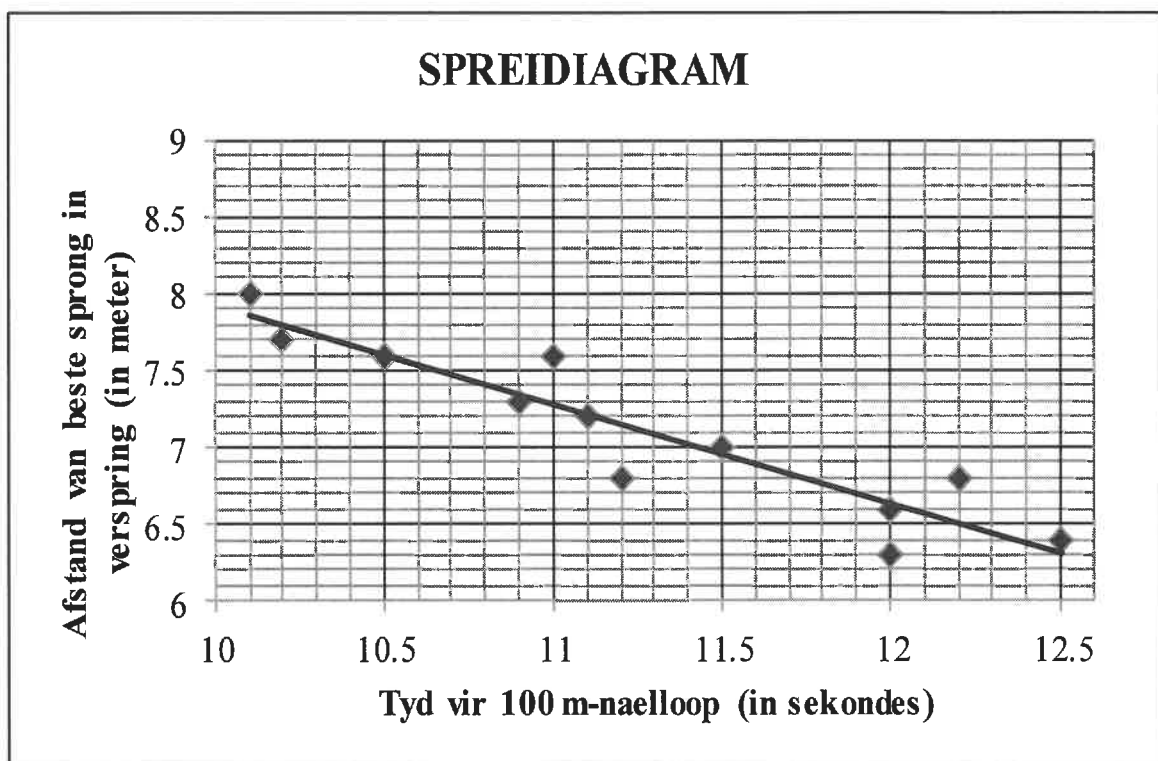
1. Hierdie vraestel bestaan uit 11 vrae.
2. Beantwoord AL die vrae in die ANTWOORDEBOEK wat verskaf word.
3. Dui ALLE berekeninge, diagramme, grafieke ensovoorts wat jy in die beantwoording van die vrae gebruik, duidelik aan.
4. Slegs antwoorde sal NIE noodwendig volpunte verdien NIE.
5. Jy mag 'n goedgekeurde wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders aangedui.
6. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders aangedui.
7. Diagramme is NIE noodwendig volgens skaal geteken NIE.
8. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
9. Skryf netjies en leesbaar.

VRAAG 1

Die tabel hieronder toon die tyd (in sekondes, tot EEN desimale syfer afgerond) wat dit 12 atlete neem om die 100 meter-naelloop te voltooi, asook die afstand (in meter, tot EEN desimale syfer afgerond) van hulle beste sprong in verspring.

Tyd vir 100 m-naelloop (in sekondes)	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Afstand van beste sprong in verspring (in meter)	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

Die data hierbo word in die spreidiagram hieronder voorgestel.



Die vergelyking van die kleinstekwadrate-regressielyn is $\hat{y} = a + bx$.

- 1.1 Bepaal die waardes van a en b . (3)
- 1.2 'n Atleet hardloop die 100 meter-naelloop in 11,7 sekondes. Gebruik $\hat{y} = a + bx$ om die afstand van hierdie atleet se beste sprong in verspring te voorspel. (2)
- 1.3 'n Ander atleet voltooi die 100 meter-naelloop in 12,3 sekondes en die afstand van sy beste sprong in verspring is 7,6 meter. Indien dit by die data ingesluit word, sal die gradiënt van die kleinstekwadrate-regressielyn toeneem of afneem? Motiveer jou antwoord sonder enige verdere berekening. (2)

[7]

VRAAG 2

'n Groep van 23 meisies word in 'n eksperiment voorsien van 'n bladsy met 30 gekleurde reghoeke. Hulle word gevra om die kleure van die reghoeke so vinnig as moontlik korrek op te noem. Die tyd, in sekondes, wat dit elkeen van die meisies geneem het, word in die tabel hieronder gegee.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1 Bereken:

2.1.1 Die gemiddelde van die data (2)

2.1.2 Die interkwartielvariasiewydte van die data (3)

2.2 Die standaardafwyking van die tye wat dit die meisies geneem het, is 5,94. Hoeveel meisies het langer as EEN standaardafwyking vanaf die gemiddelde geneem om die kleure op te noem? (2)

2.3 Trek 'n mond-en-snordiagram (boksplot) om die data voor te stel op die getallelyn wat in die ANTWOORDEBOEK verskaf word. (3)

2.4 Die vyfpunt-opsomming van die tye wat dit 'n groep van 23 seuns geneem het om die kleure van die reghoeke korrek op te noem, is (15 ; 21 ; 23,5 ; 26 ; 38).

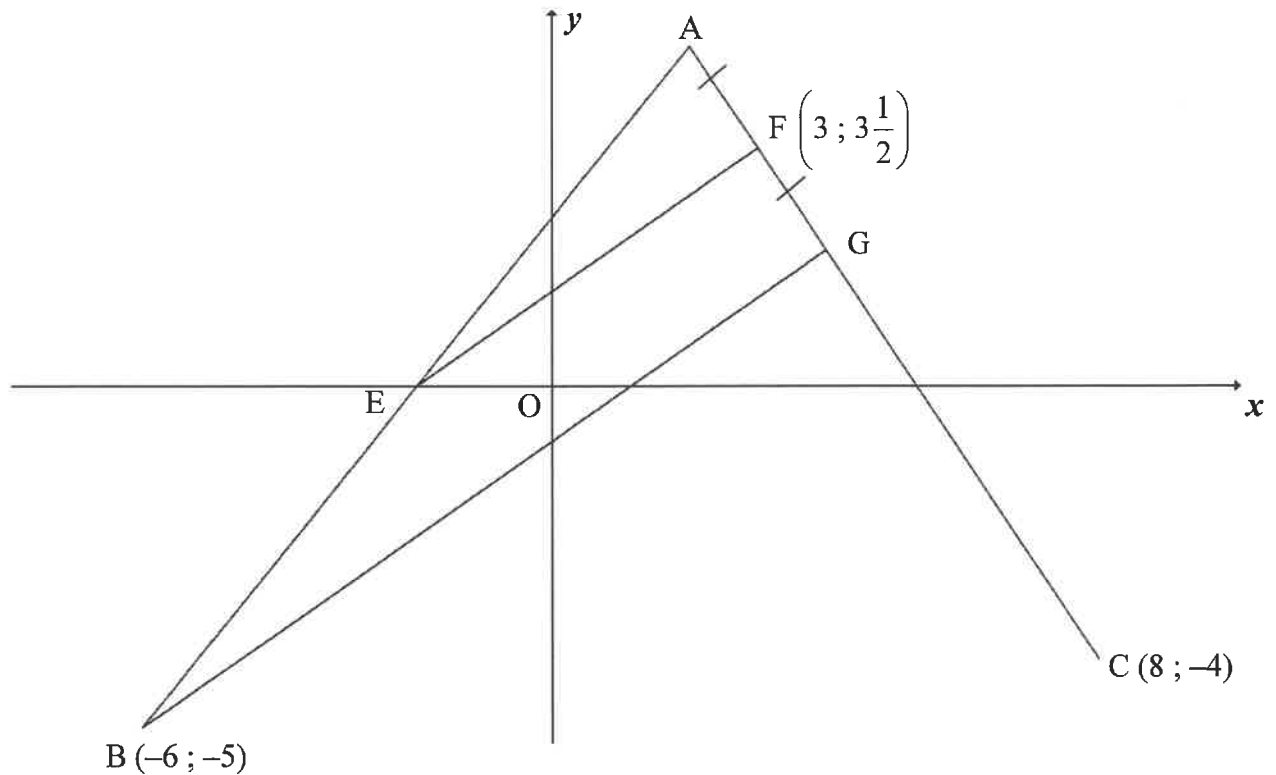
2.4.1 Watter van die twee groepe, meisies of seuns, het die laagste mediaantyd gehad om die kleure van die reghoeke korrek op te noem? (1)

2.4.2 Die eerste drie leiders wat die kleure van al 30 reghoeke in die kortste tyd korrek opnoem, sal 'n prys ontvang. Hoeveel seuns sal onder hierdie drie prysweners wees? Motiveer jou antwoord. (2)

[13]

VRAAG 3

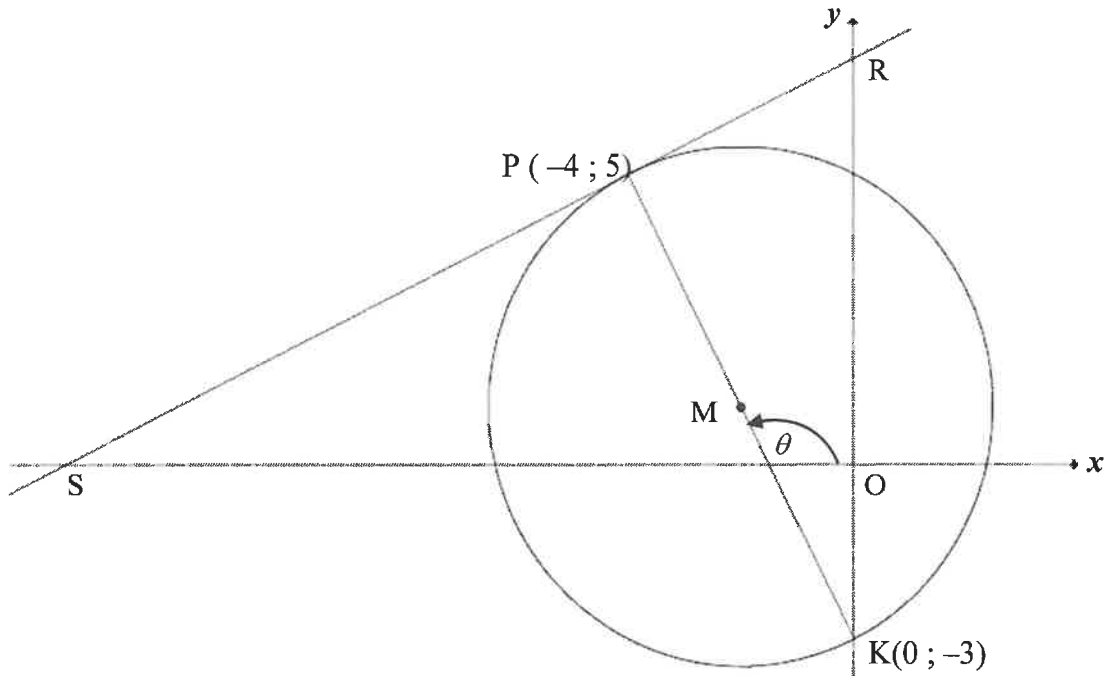
In die diagram is A, B(-6 ; -5) en C(8 ; -4) punte in die Cartesiese vlak. $F\left(3; 3\frac{1}{2}\right)$ en G is punte op lyn AC sodat $AF = FG$. E is die x-afsnit van AB.



- 3.1 Bereken:
- 3.1.1 Die vergelyking van AC in die vorm $y = mx + c$ (4)
- 3.1.2 Die koördinate van G indien $7x - 10y = 8$ die vergelyking van BG is (3)
- 3.2 Toon deur middel van berekening dat (2 ; 5) die koördinate van A is. (2)
- 3.3 Bewys dat $EF \parallel BG$. (4)
- 3.4 ABCD is 'n parallelogram met D in die eerste kwadrant. Bereken die koördinate van D. (4)
- [17]**

VRAAG 4

In die diagram is $P(-4 ; 5)$ en $K(0 ; -3)$ die eindpunte van die middellyn van 'n sirkel met middelpunt M . S en R is onderskeidelik die x - en y -afsnit van die raaklyn aan die sirkel by P . θ is die inklinasie van PK met die positiewe x -as.



- 4.1 Bepaal:
- 4.1.1 Die gradiënt van SR (4)
- 4.1.2 Die vergelyking van SR in die vorm $y = mx + c$ (2)
- 4.1.3 Die vergelyking van die sirkel in die vorm $(x - a)^2 + (y - b)^2 = r^2$ (4)
- 4.1.4 Die grootte van \hat{PKR} (3)
- 4.1.5 Die vergelyking van die raaklyn aan die sirkel by K in die vorm $y = mx + c$ (2)
- 4.2 Bepaal die waardes van t sodat die lyn $y = \frac{1}{2}x + t$ die sirkel by twee verskillende punte sny. (3)
- 4.3 Bereken die oppervlakte van $\triangle SMK$. (5)

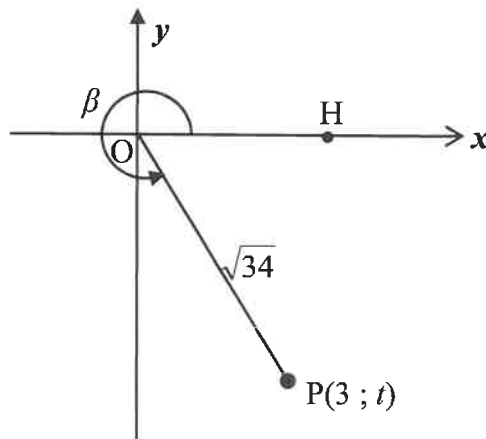
[23]

VRAAG 5

5.1 Gegee:
$$\frac{\sin(A - 360^\circ) \cdot \cos(90^\circ + A)}{\cos(90^\circ - A) \cdot \tan(-A)}$$

Vereenvoudig die uitdrukking tot 'n enkele trigonometriese verhouding. (6)

5.2 In die diagram is $P(3 ; t)$ 'n punt in die Cartesiese vlak. $OP = \sqrt{34}$ en $\widehat{HOP} = \beta$ is 'n inspringende (refleks-) hoek.



Sonder die gebruik van 'n sakrekenaar, bepaal die waarde van:

5.2.1 t (2)

5.2.2 $\tan \beta$ (1)

5.2.3 $\cos 2\beta$ (4)

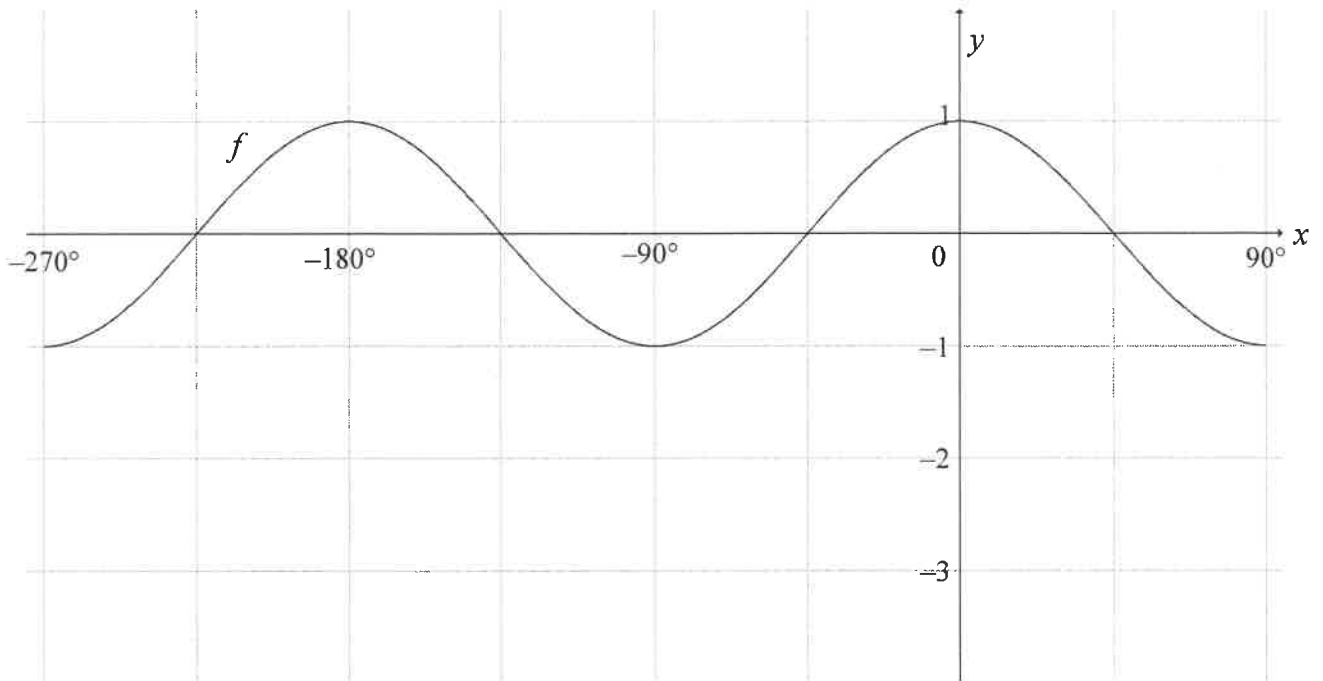
5.3 Bewys:

5.3.1 $\sin(A + B) - \sin(A - B) = 2 \cos A \cdot \sin B$ (2)

5.3.2 **Sonder die gebruik van 'n sakrekenaar, dat $\sin 77^\circ - \sin 43^\circ = \sin 17^\circ$** (4)
[19]

VRAAG 6

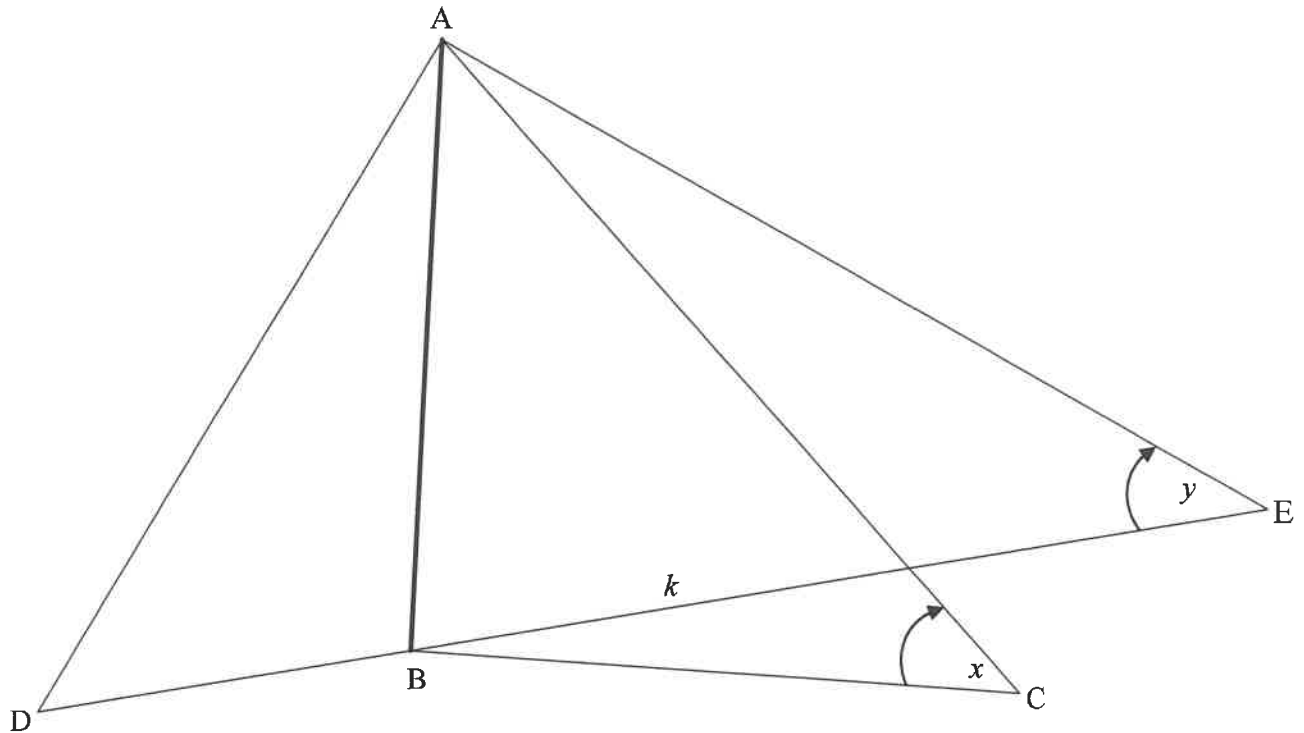
In die diagram is die grafiek van $f(x) = \cos 2x$ vir die interval $x \in [-270^\circ; 90^\circ]$ geskets.



- 6.1 Skets die grafiek van $g(x) = 2\sin x - 1$ vir die interval $x \in [-270^\circ; 90^\circ]$ op die rooster wat in jou ANTWOORDEBOEK verskaf word. Toon AL die afsnitte met die asse, asook die draaipunte. (4)
- 6.2 Gestel A is 'n snypunt van die grafieke van f en g . Toon dat die x -koördinaat van A die vergelyking $\sin x = \frac{-1 + \sqrt{5}}{2}$ bevredig. (4)
- 6.3 Bereken vervolgens die koördinate van die sny punte van grafieke van f en g vir die interval $x \in [-270^\circ; 90^\circ]$. (4)
- [12]**

VRAAG 7

AB stel 'n vertikale netbalpaal voor. Twee spelers word aan weerskante van die netbalpaal by punt D en E geplaas sodat D, B en E op dieselfde reguitlyn is. 'n Derde speler word by C geplaas. Die punte B, C, D en E is in dieselfde horisontale vlak. Die hoogtehoeke vanaf C na A en vanaf E na A is onderskeidelik x en y . Die afstand vanaf B na E is k .

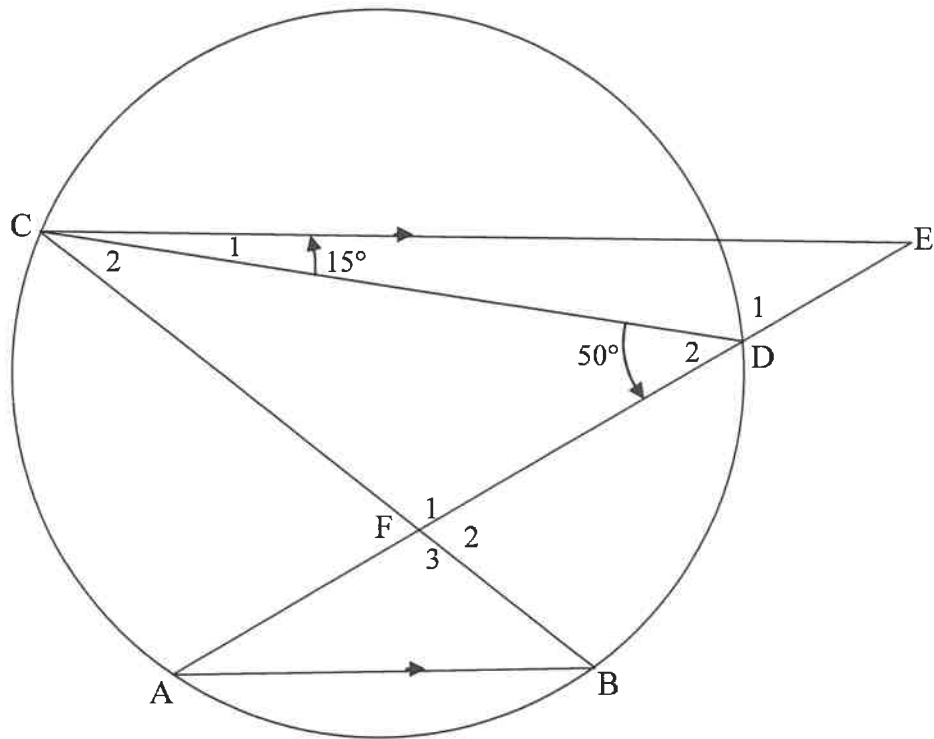


- 7.1 Skryf die grootte van \hat{ABC} neer. (1)
- 7.2 Toon dat $AC = \frac{k \cdot \tan y}{\sin x}$ (4)
- 7.3 Indien dit verder gegee word dat $\hat{DAC} = 2x$ en $AD = AC$, toon dat $2k \tan y$ die afstand DC tussen die spelers by D en C is. (5)
- [10]

Gee redes vir jou bewerings in VRAAG 8, 9, 10 en 11.

VRAAG 8

In die diagram lê punt A, B, D en C op 'n sirkel. $CE \parallel AB$ met E op AD verleng. Koord CB en AD sny mekaar in F. $\hat{D}_2 = 50^\circ$ en $\hat{C}_1 = 15^\circ$.



8.1 Bereken, met redes, die grootte van:

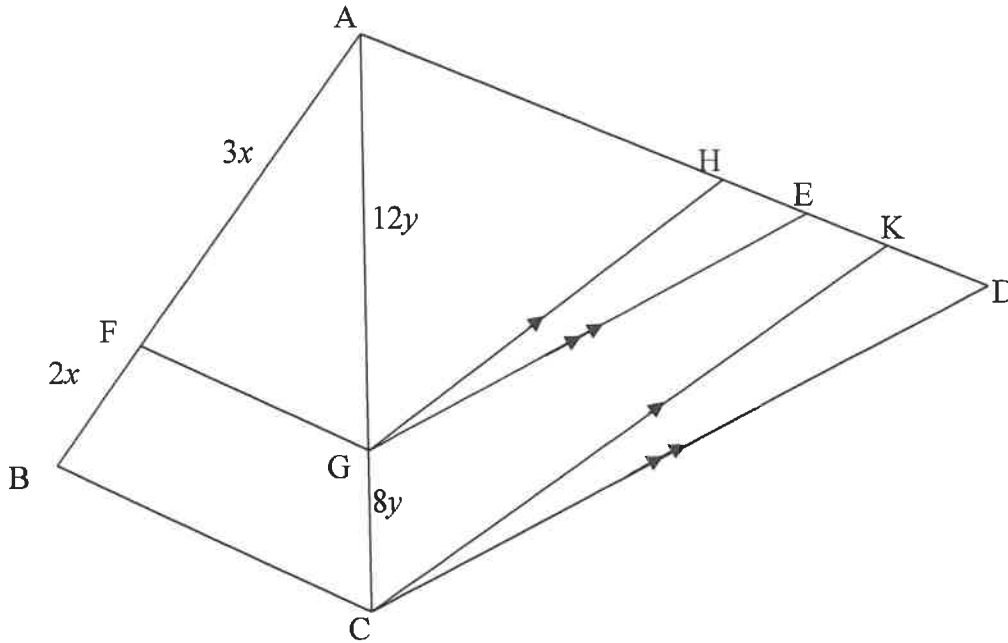
8.1.1 \hat{A} (3)

8.1.2 \hat{C}_2 (2)

8.2 Bewys, met 'n rede, dat CF 'n raaklyn aan die sirkel is wat deur punt C, D en E gaan. (2)
[7]

VRAAG 9

In die diagram is $\triangle ABC$ en $\triangle ACD$ geskets. F en G is punte op sy AB en AC onderskeidelik sodat $AF = 3x$, $FB = 2x$, $AG = 12y$ en $GC = 8y$. H, E en K is punte op sy AD sodat $GH \parallel CK$ en $GE \parallel CD$.



9.1 Bewys dat:

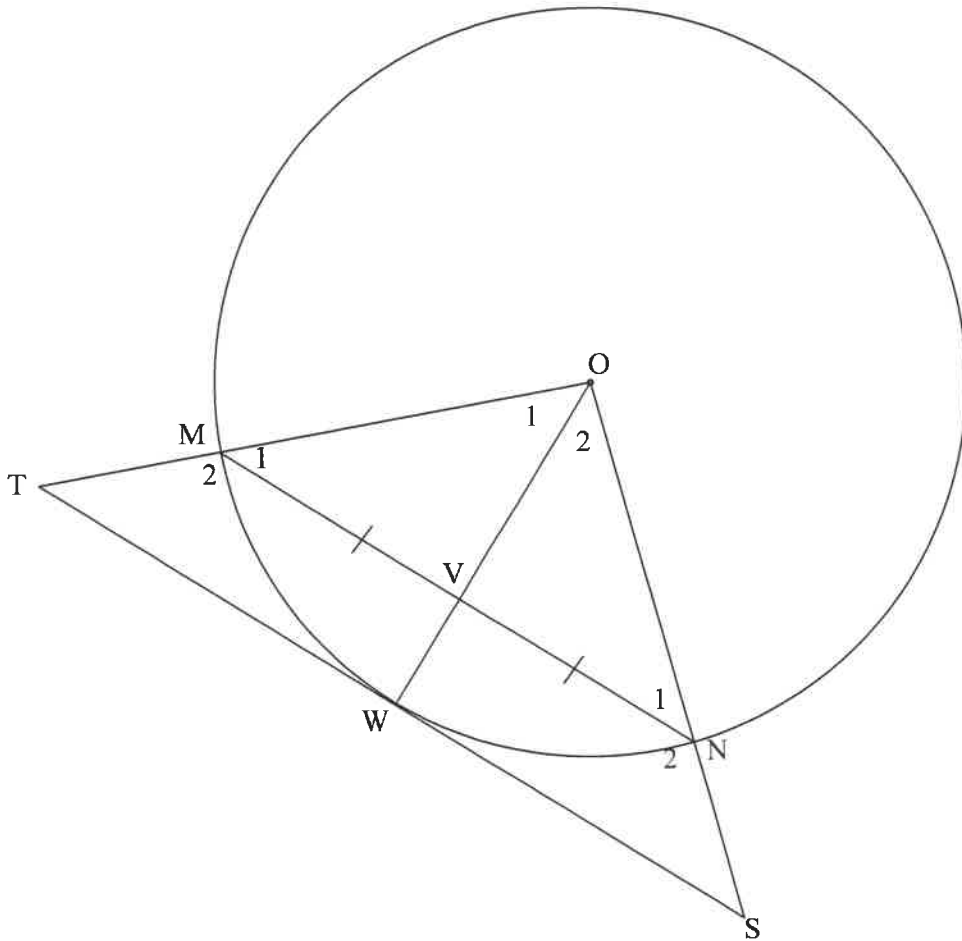
9.1.1 $FG \parallel BC$ (2)

9.1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (3)

9.2 As dit verder gegee word dat $AH = 15$ en $ED = 12$, bereken die lengte van EK . (5)
[10]

VRAAG 10

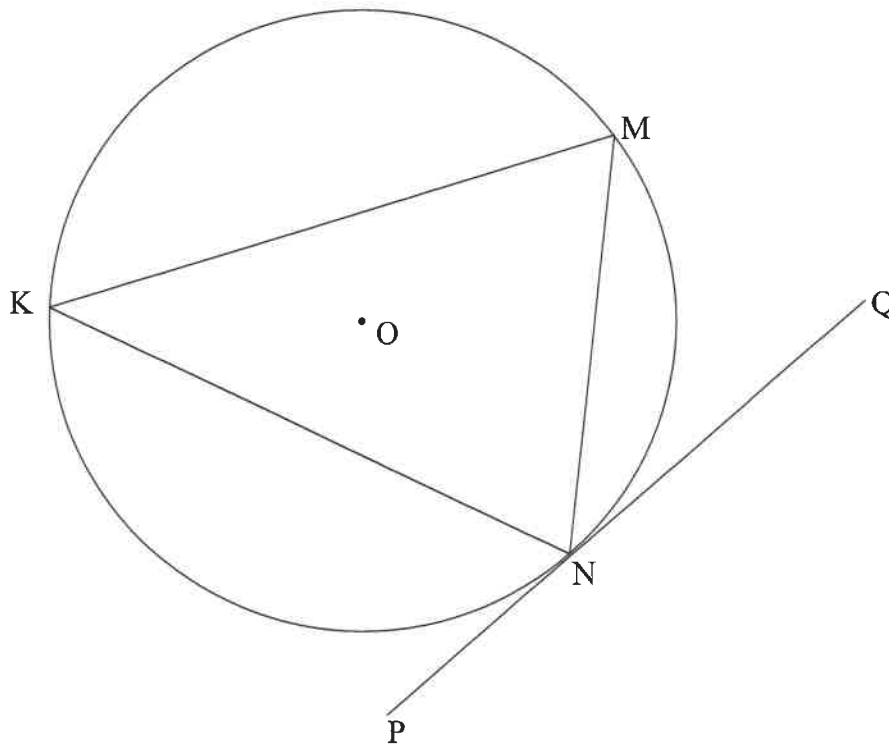
In die diagram is W 'n punt op die sirkel met middelpunt O. V is 'n punt op OW. Koord MN word getrek sodat $MV = VN$. Die raaklyn by W ontmoet OM verleng by T en ON verleng by S.



- 10.1 Gee 'n rede waarom $OV \perp MN$. (1)
 - 10.2 Bewys dat:
 - 10.2.1 $MN \parallel TS$ (2)
 - 10.2.2 $TMNS$ 'n koordevierhoek is (4)
 - 10.2.3 $OS \cdot MN = 2ON \cdot WS$ (5)
- [12]**

VRAAG 11

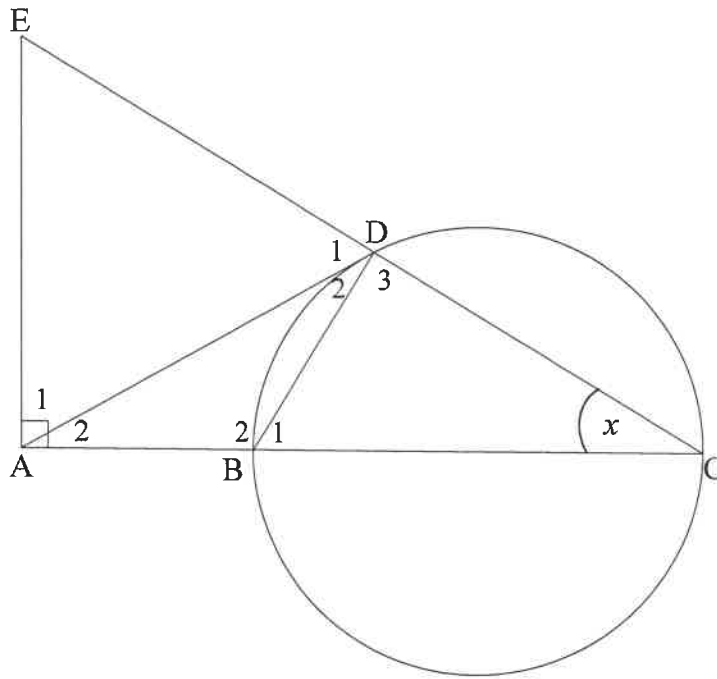
- 11.1 In die diagram word koord KM, MN en KN in die sirkel met middelpunt O getrek. PNQ is die raaklyn aan die sirkel by N.



Bewys die stelling wat beweer dat $\hat{M}N\hat{Q} = \hat{K}$.

(5)

- 11.2 In die diagram is BC 'n middellyn van die sirkel. Die raaklyn by punt D op die sirkel ontmoet CB verleng by A. CD word verleng na E sodat $EA \perp AC$. BD word getrek.
Laat $\hat{C} = x$.



11.2.1 Gee 'n rede waarom:

- (a) $\hat{D}_3 = 90^\circ$ (1)
- (b) ABDE 'n koordevierhoek is (1)
- (c) $\hat{D}_2 = x$ (1)

11.2.2 Bewys dat:

- (a) $AD = AE$ (3)
- (b) $\triangle ADB \parallel \triangle ACD$ (3)

11.2.3 Verder word gegee dat $BC = 2AB = 2r$.

- (a) Bewys dat $AD^2 = 3r^2$ (2)
- (b) Bewys vervolgens dat $\triangle ADE$ gelyksydig is. (4)

[20]

TOTAAL: 150

INLICHTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{oppervlakte } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$

FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG HIERDIE INSTRUKSIES NOUKEURIG
1. Clearly write your examination number and centre number in the spaces provided and attach your barcode label in the space provided.	1. Skryf jou eksamennommer en sentrumnummer duidelik in die ruimtes soos verskaf en plak jou stafieskodeplakker in die ruimte soos verskaf.
2. Remember that your own name (or the name of your school) may not appear anywhere on or in this answer book.	2. Onthou dat jou eie naam (of die naam van jou skool) nie op of in hierdie antwoordeboek mag voorkom nie.
3. No pages may be torn from this answer book.	3. Geen bladsye mag uit hierdie antwoordeboek geskeur word nie.
4. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	4. Lees die instruksies wat op jou eksamenrooster gedruk is, sorgvuldig deur, asook enige ander instruksies wat in elke eksamenvraestel gegee word.
5. Candidates may not retain any answer book or remove them from the examination room.	5. Geen antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word nie.
6. Answers must be written in black/blue ink as distinctly as possible.	6. Skryf die antwoorde so duidelik moontlik met swart/blou.
7. Draw a neat line through any work/rough work that must not be marked.	7. Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.

QUESTION/VRAAG 1

Time for 100 m sprint (in seconds) <i>Tyd vir 100 m-naelloop (in sekondes)</i>	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres) <i>Afstand van beste sprong in verspring (in meter)</i>	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

	Solution/Oplissing	Marks Punte
1.1		(3)
1.2		(2)
1.3		(2)
		[7]

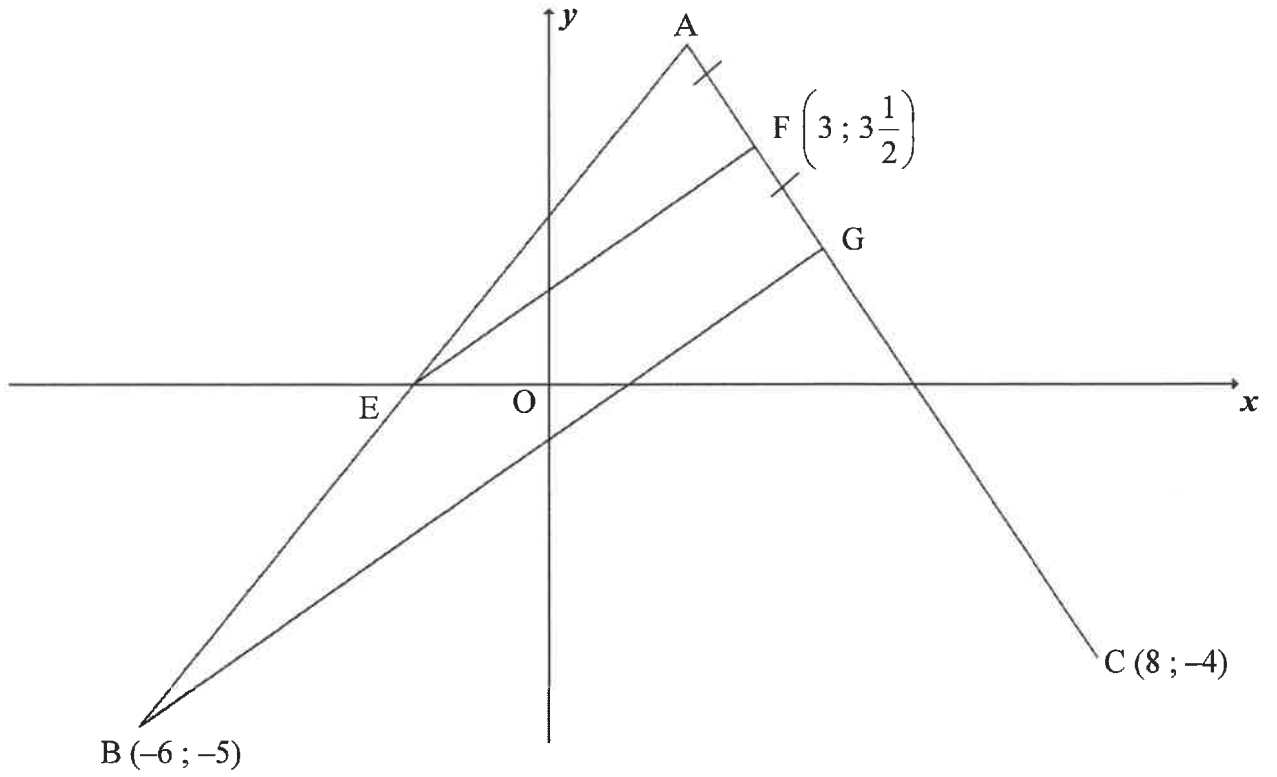
QUESTION/VRAAG 2

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

	Solution/Oplossing	Marks Punte
2.1.1	<hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	(2)
2.1.2	<hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	(3)
2.2	<hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/>	(2)
2.3	<hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <hr/> <p style="text-align: center;">10 12 14 16 18 20 22 24 26 28 30 32 34 36 38</p>	(3)

	Solution/<i>Oplossing</i>	Marks <i>Punte</i>
2.4.1		(1)
2.4.2		(2)
		[13]

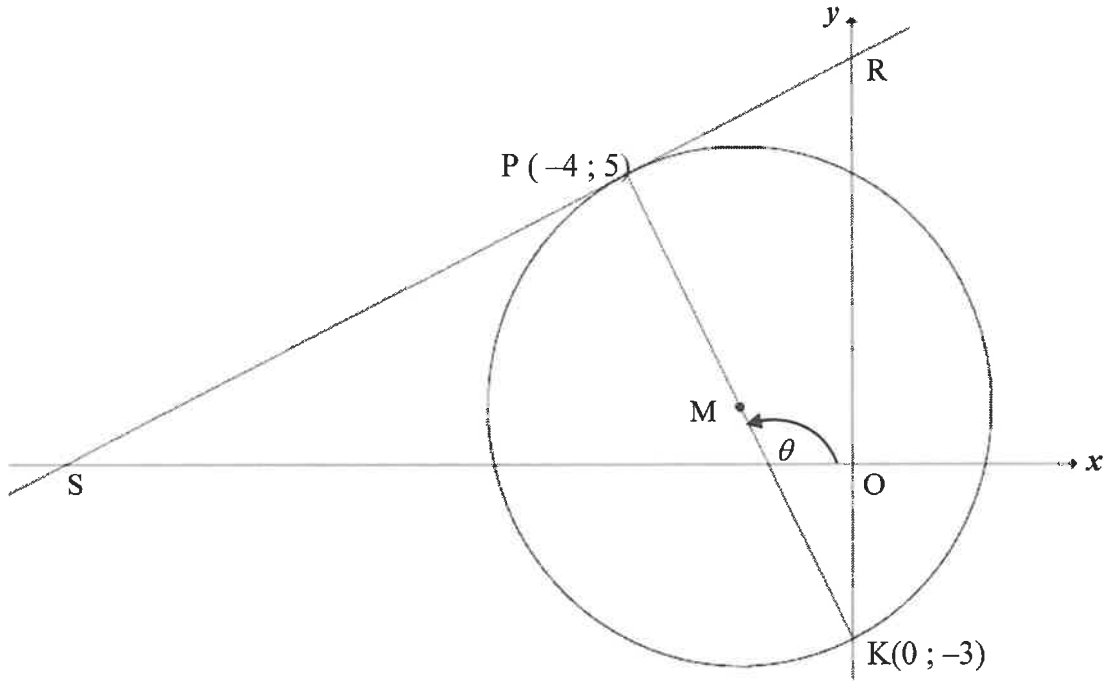
QUESTION/VRAAG 3



	Solution/Oplissing	Marks Punte
3.1.1		
		(4)
3.1.2		

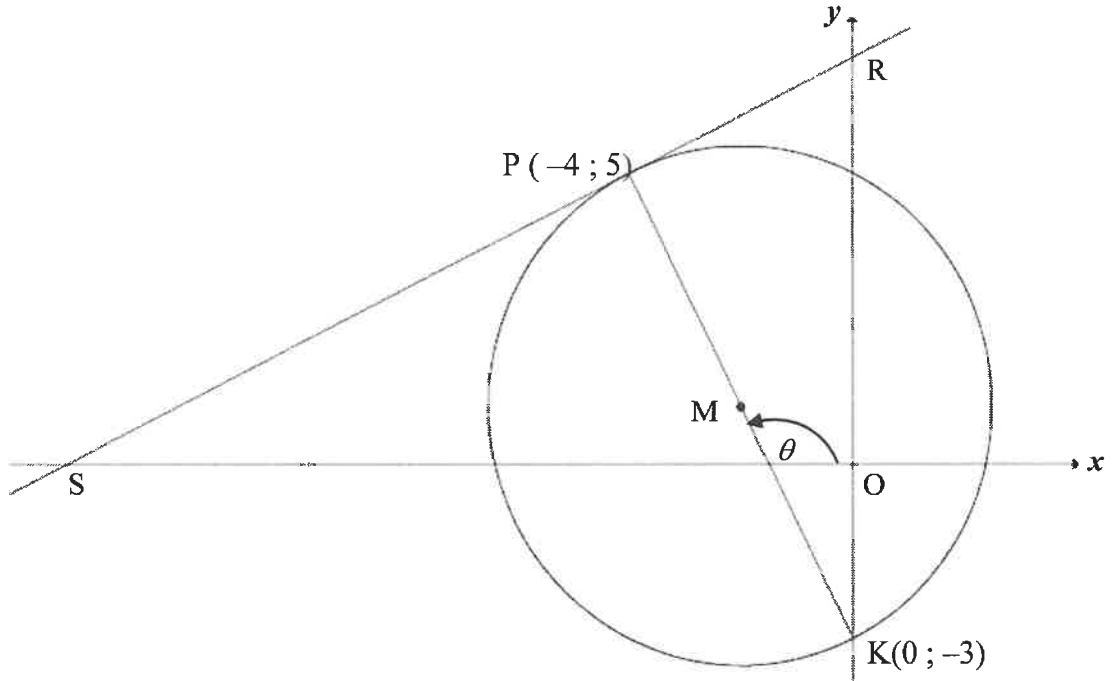
	Solution/Oplissing	Marks Punte
3.1.2 (cont./ vervolg)		(3)
3.2		(2)
3.3		(4)
3.4		(4)
		[17]

QUESTION/VRAAG 4



	Solution/Oplissing	Marks Punte
4.1.1		(4)
4.1.2		(2)

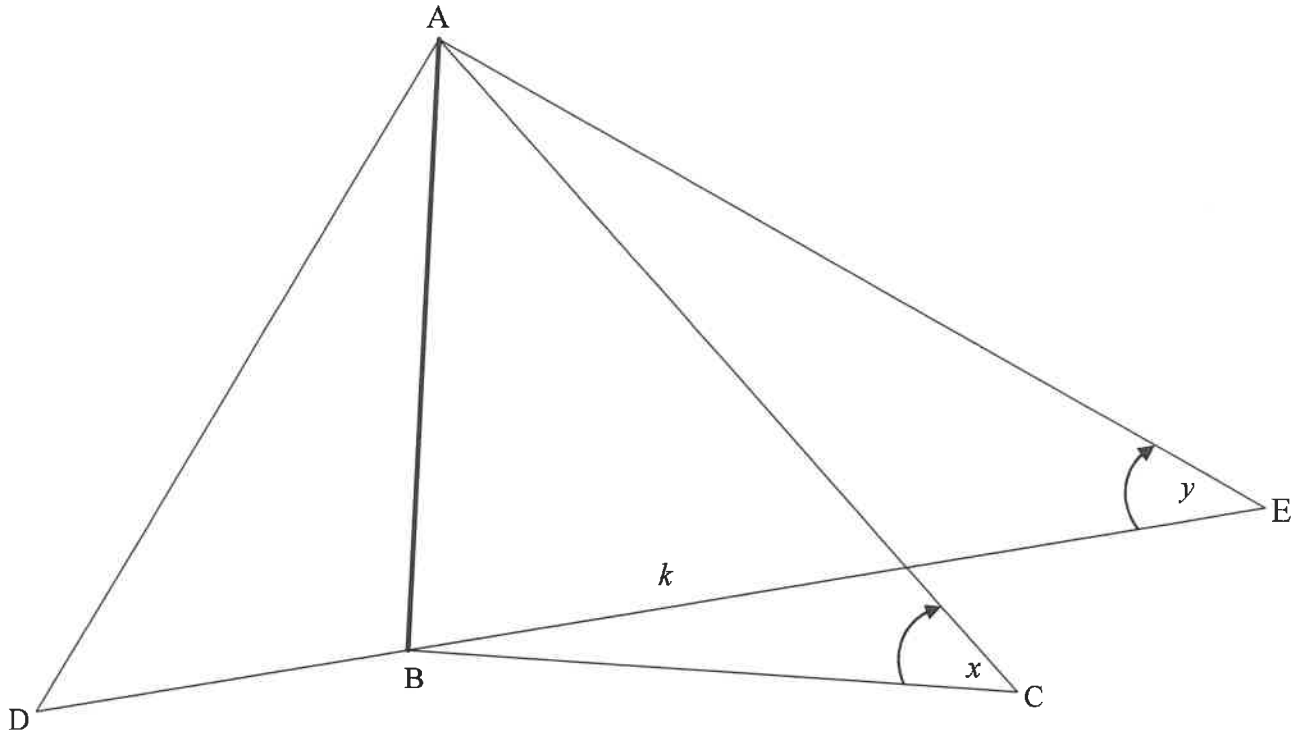
	Solution/<i>Oplossing</i>	Marks <i>Punte</i>
4.1.3		(4)
4.1.4		(3)
4.1.5		(2)
4.2		(3)



	Solution/Oplissing	Marks Punte
4.3		
		(5)
		[23]

	Solution/Oplissing	Marks Punte
5.2.2		(1)
5.2.3		(4)
5.3.1		(2)
5.3.2		(4)
		[19]

QUESTION/VRAAG 7

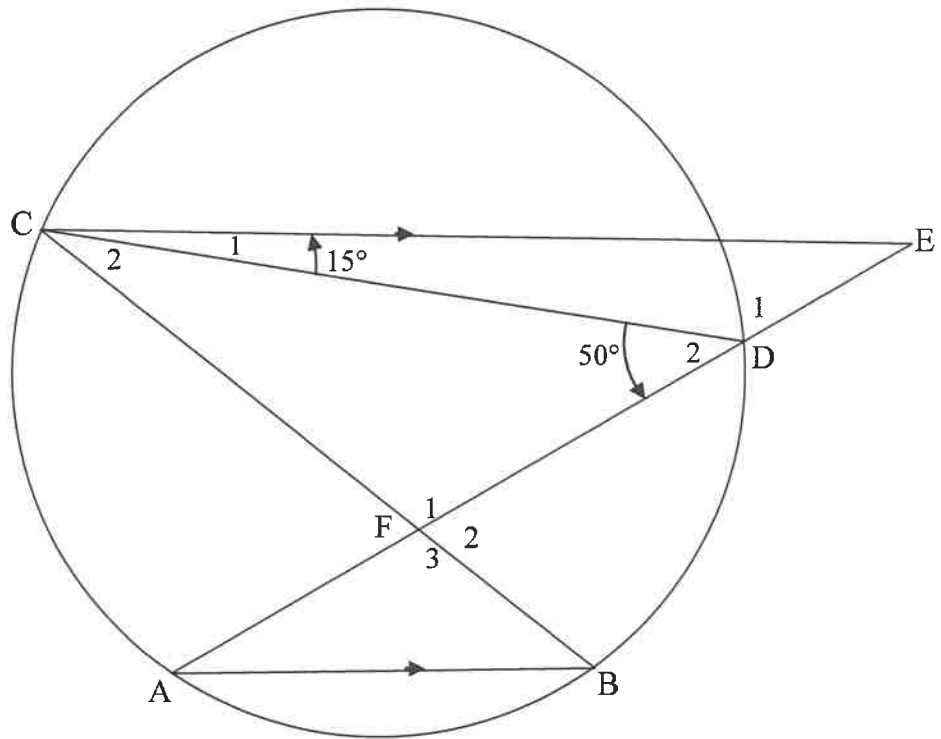


	Solution/Oplissing	Marks Punte
7.1		(1)
7.2		(4)

	Solution/<i>Oplossing</i>	Marks <i>Punte</i>
7.3		
		(5) [10]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.
Gee redes vir jou bewerings in VRAAG 8, 9, 10 en 11.

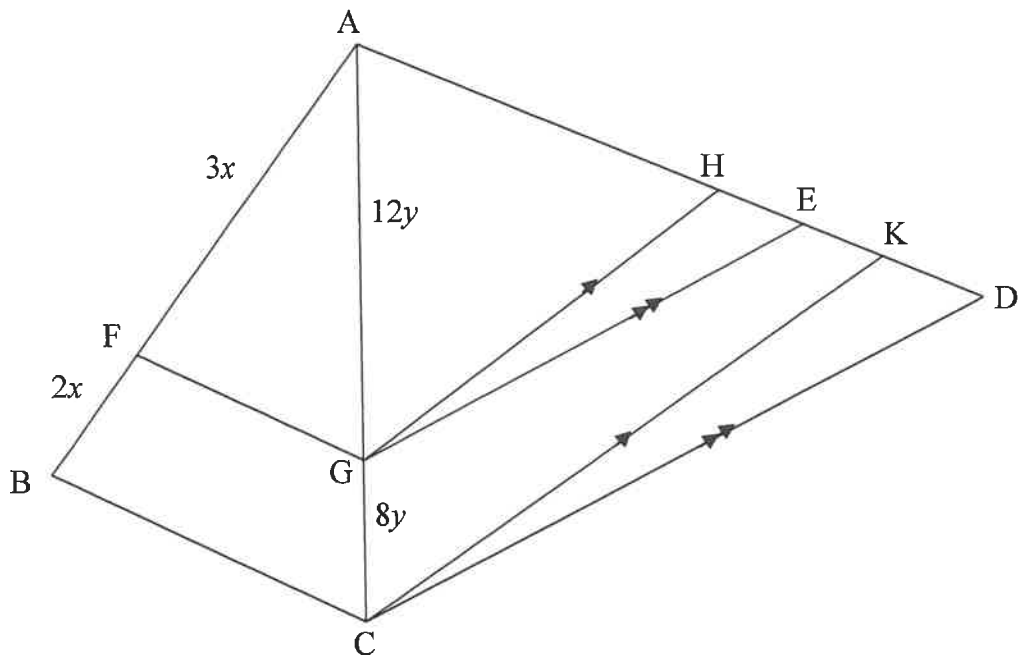
QUESTION/VRAAG 8



	Solution/Oplissing	Marks Punte
8.1.1	<hr/> <hr/> <hr/> <hr/> <hr/>	(3)

	Solution/<i>Oplossing</i>	Marks <i>Punte</i>
8.1.2		(2)
8.2		(2)
		[7]

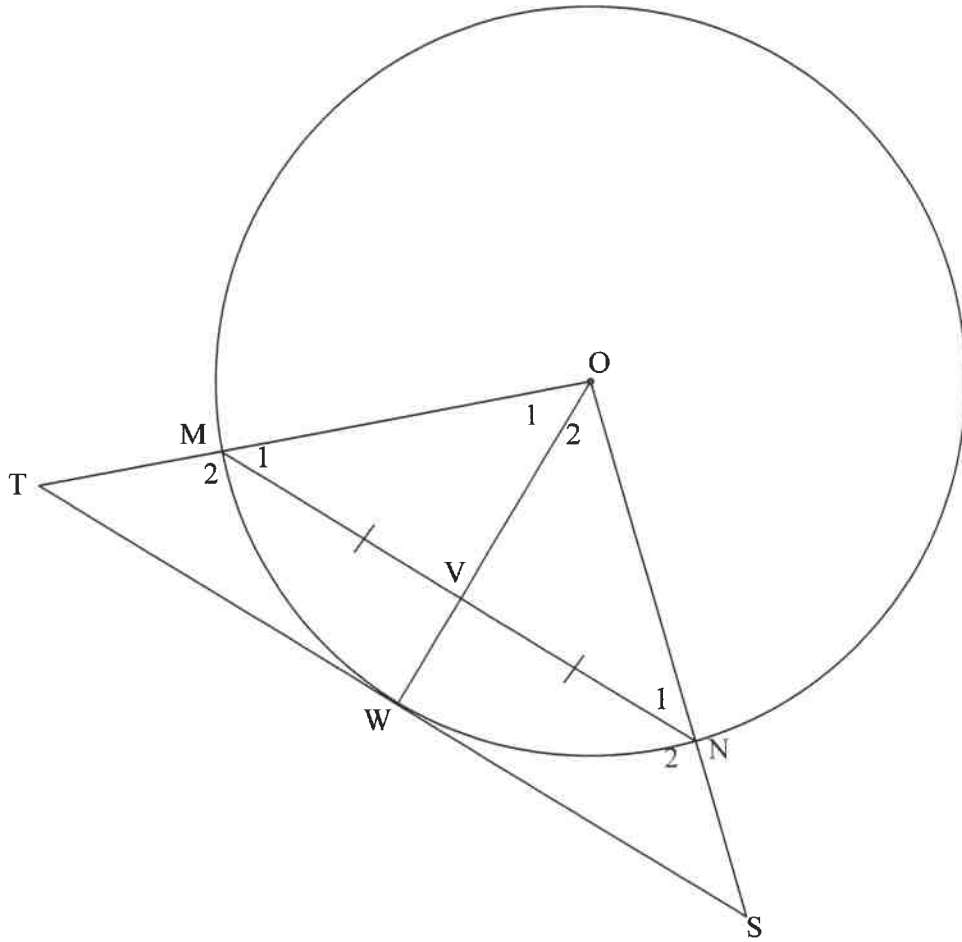
QUESTION/VRAAG 9



	Solution/Oplissing	Marks Punte
9.1.1		
		(2)
9.1.2		
		(3)

	Solution/<i>Oplossing</i>	Marks <i>Punte</i>
9.2		
		(5) [10]

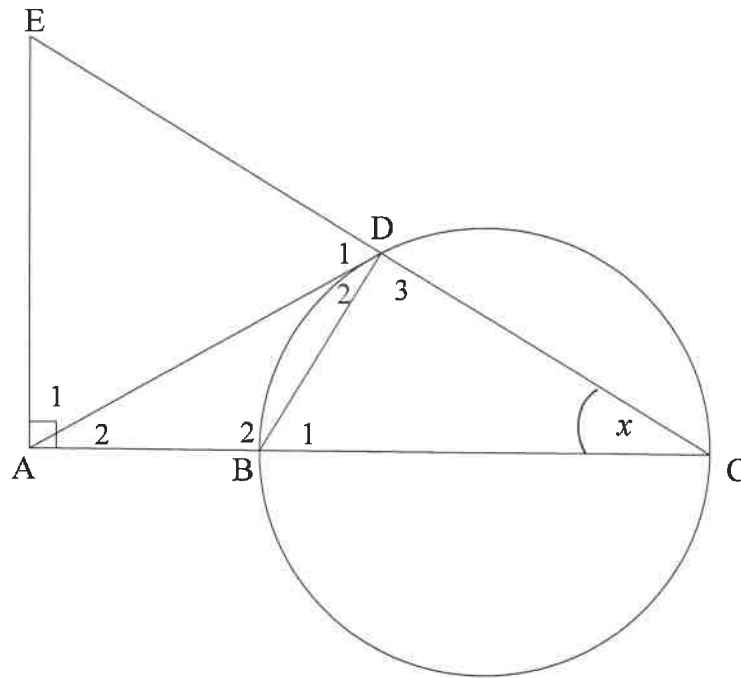
QUESTION/VRAAG 10



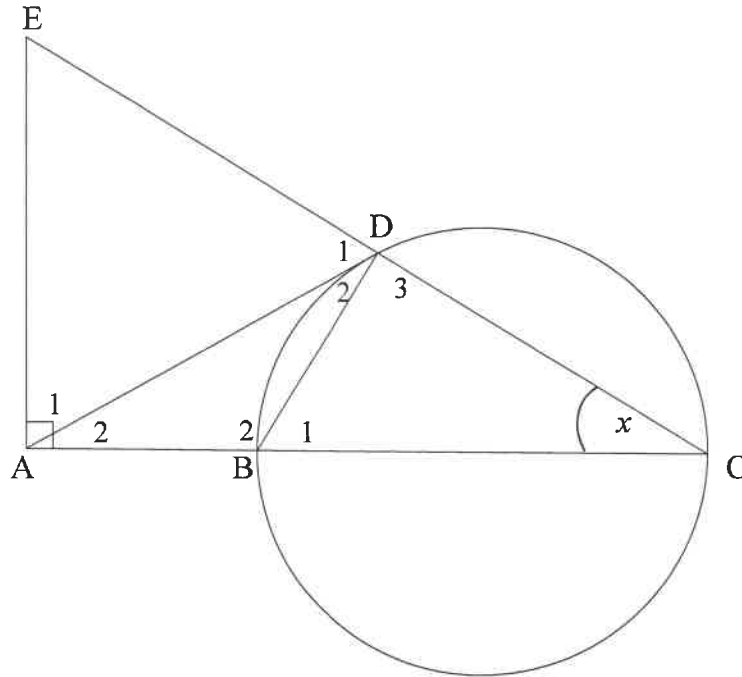
	Solution/Oplissing	Marks Punte
10.1		(1)
10.2.1		(2)

	Solution/Oplissing	Marks Punte
10.2.2		(4)
10.2.3		(5) [12]

11.2



	Solution/Oplissing	Marks Punte
11.2.1(a)		(1)
11.2.1(b)		(1)
11.2.1(c)		(1)
11.2.2(a)		(3)



	Solution/Oplissing	Marks Punte
11.2.2(b)		
		(3)
11.2.3(a)		
		(2)

	Solution/Oplossing	Marks Punte
11.2.3(b)		
		(4)
		[20]

	Additional space/Bykomende ruimte	Marks Punte



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE
*NASIONALE SENIOR
SERTIFIKAAT***

GRADE 12/*GRAAD 12*

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2017

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

**These marking guidelines consist of 29 pages.
*Hierdie nasienriglyne bestaan uit 28 bladsye.***

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

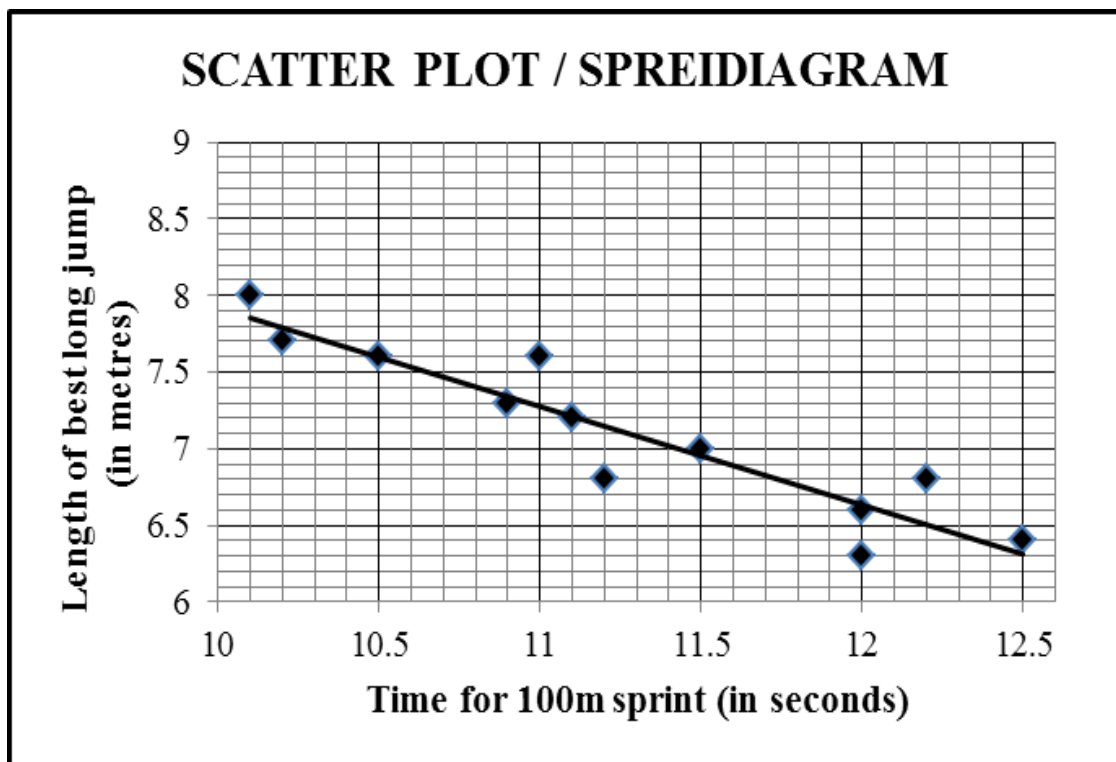
NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	A mark for a correct statement (A statement mark is independent of a reason.)
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede.)
R	A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is.)
S/R	Award a mark if the statement AND reason are both correct.
	Ken 'n punt toe as beide die bewering EN rede korrek is.

QUESTION/VRAAG 1

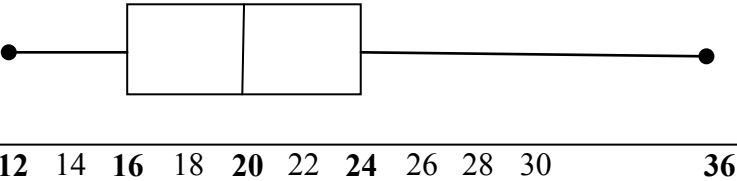
Time for 100 m sprint (in seconds) <i>Tyd vir 100 m-naelloop (in sekondes)</i>	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres) <i>Afstand van beste sprong in verspring (in meter)</i>	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4



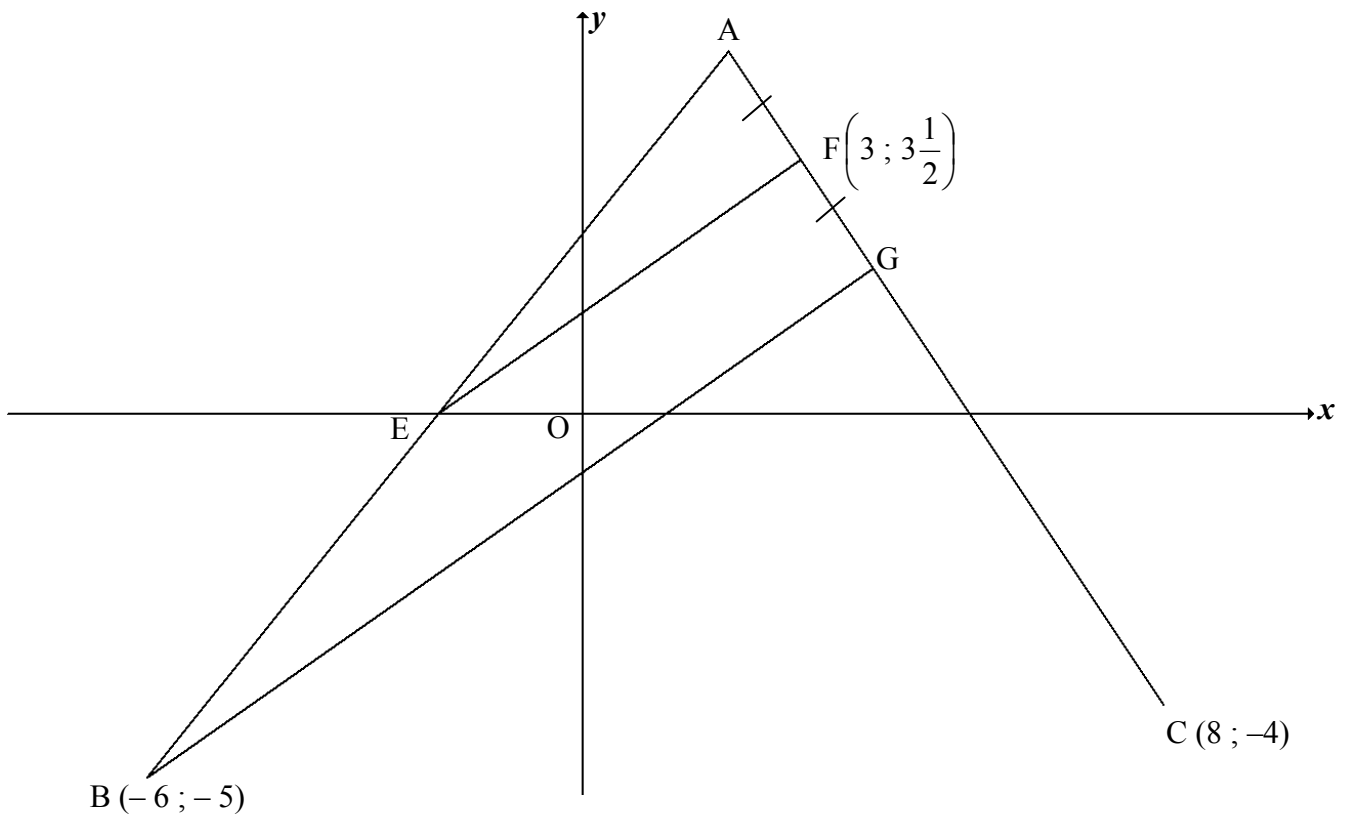
1.1	$a = 14,343... = 14,34$ $b = -0,642... = -0,64$	✓✓ value of a ✓ value of b (3)
1.2	$y = 14,34 - 0,64(11,7)$ $= 6,85$ OR/OF $y = 6,83$ (calculator / <i>sakrekenaar</i>)	✓ substitution correctly ✓ answer ✓✓ answer (2) (2)
1.3	The gradient increases / <i>Die gradient neem toe</i> The point (12,3 ; 7,6) lies some distance above the current data. <i>/Die punt (12,3 ; 7,6) lê bokant die huidige data.</i>	✓ increases/ <i>neem toe</i> ✓ reasoning in words/ <i>redenasië in woorde</i> (2) [7]

QUESTION/VRAAG 2

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1.1	$\bar{x} = \frac{472}{23}$ $\bar{x} = 20,52 \text{ seconds / sekonde}$	✓ $\frac{472}{23}$ ✓ answer (2)
2.1.2	$Q_1 = 16$ $Q_3 = 24$ $IQR/IKO = Q_3 - Q_1$ $= 24 - 16 = 8$	✓ Q_1 ✓ Q_3 ✓ answer (3)
2.2	$20,52 + 5,94 = 26,46$ $\therefore > 26,46$ $\therefore 4 \text{ girls/dogters}$	✓ 26,46 ✓ answer (2)
2.3	 <p>12 14 16 18 20 22 24 26 28 30 36</p>	✓ whiskers ending at 12 & 36 ✓ $Q_1 = 16$ & $Q_3 = 24$ (box) ✓ $Q_2 = 20$ (3)
2.4.1	Girls / Meisies	✓ answer (1)
2.4.2	Five-number summary of boys: (15 ; 21 ; 23,5 ; 26 ; 38) None of the boys / Nie een van die seuns nie 5 girls completed in less than 15 seconds which was the minimum time taken by the boys. <i>5 meisies voltooi in minder as 15 sekondes, wat die minimumtyd is wat die seuns geneem het.</i>	✓ answer ✓ reason/rede (2) [13]

QUESTION/VRAAG 3



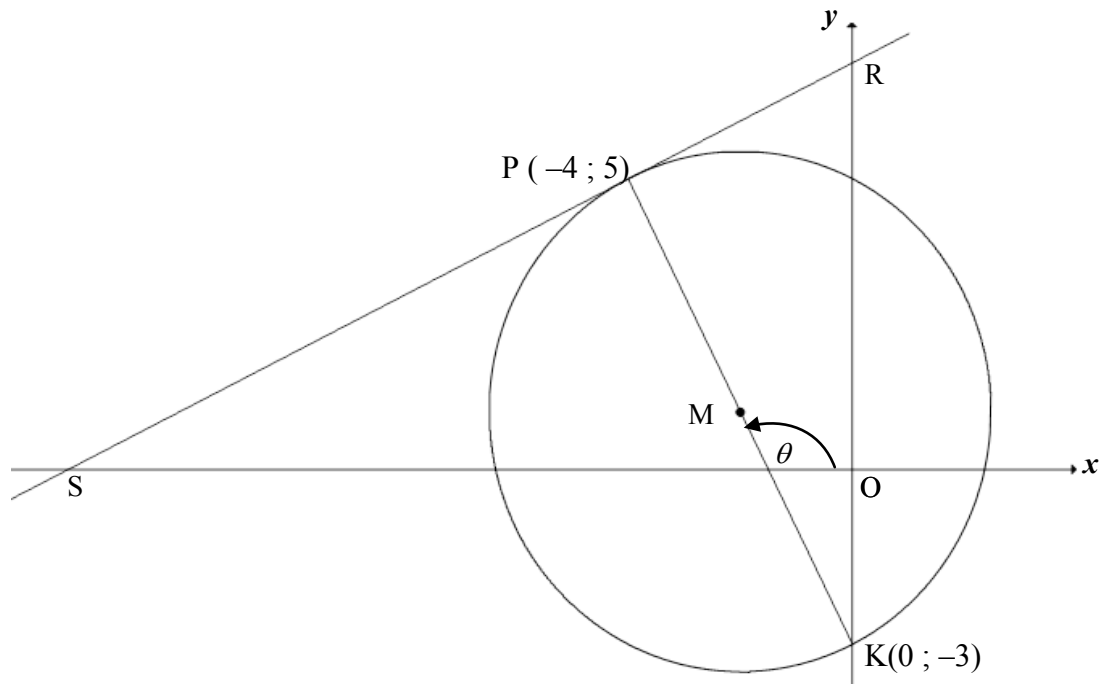
<p>3.1.1</p>	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{3\frac{1}{2} - (-4)}{3 - 8}$ $= -\frac{3}{2}$ <p>$y = mx + c$ $y - y_1 = m(x - x_1)$</p> $y = -\frac{3}{2}x + c$ $-4 = -\frac{3}{2}(8) + c \quad \text{OR/OF} \quad (y - (-4)) = -\frac{3}{2}(x - 8)$ $c = 8$ $y = -\frac{3}{2}x + 8$ <p>OR/OF</p> $y + 4 = -\frac{3}{2}x + 12$ $y = -\frac{3}{2}x + 8$	<ul style="list-style-type: none"> ✓ substitution of (8 ; -4) & $(3; 3\frac{1}{2})$ ✓ gradient ✓ substitution of m and (8 ; -4) ✓ equation of AC <p style="text-align: right;">(4)</p>
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	$m_{FC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4) - \left(3\frac{1}{2}\right)}{8 - 3}$ $= -\frac{3}{2}$ $y = mx + c$ $3\frac{1}{2} = -\frac{3}{2}(3) + c$ $c = 8$ $y = -\frac{3}{2}x + 8$ $y - y_1 = m(x - x_1)$ $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}(x - 3)$ <p style="text-align: center;">OR/OF</p> $\left(y - 3\frac{1}{2}\right) = -\frac{3}{2}x + \frac{9}{2}$ $y = -\frac{3}{2}x + 8$	<ul style="list-style-type: none"> ✓ substitution of $(8 ; -4)$ & $\left(3 ; 3\frac{1}{2}\right)$ ✓ gradient ✓ substitution of m and $\left(3 ; 3\frac{1}{2}\right)$ ✓ equation of AC <p style="text-align: right;">(4)</p>
<p>3.1.2</p>	<p>AC: $3x + 2y = 16$ and BG: $7x - 10y = 8$</p> $15x + 10y = 80$ $\underline{7x - 10y = 8}$ $22x = 88$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ <p>∴ G(4 ; 2)</p> <p>OR/OF</p> <p>BG: $7x - 10y = 8$ ∴ $y = \frac{7}{10}x - \frac{8}{10}$</p> $\therefore \frac{7}{10}x - \frac{8}{10} = -\frac{3}{2}x + 8 \quad [\text{CA from 3.1.1}]$ $\frac{11}{5}x = \frac{44}{5}$ $x = 4$ $3(4) + 2y = 16$ $y = 2$ <p>∴ G(4 ; 2)</p>	<ul style="list-style-type: none"> ✓ method /metode: solving simultaneously / los gelyktydig op ✓ x coordinate ($x > 0$) ✓ y coordinate <p style="text-align: right;">(3)</p> <ul style="list-style-type: none"> ✓ method: equating metode: stel vgl's gelyk ✓ x coordinate ($x > 0$) ✓ y coordinate <p style="text-align: right;">(3)</p>
<p>3.2</p>	$\frac{x_A + 4}{2} = 3 \quad \text{and} \quad \frac{y_A + 2}{2} = 3\frac{1}{2}$ <p>∴ A(2 ; 5)</p> <p>OR/OF by translation/deur translasie:</p> $x_A = 3 - (4 - 3) = 2$ $y_A = 3\frac{1}{2} + (3\frac{1}{2} - 2) = 5$ <p>∴ A(2 ; 5)</p>	<ul style="list-style-type: none"> ✓ equation ito x ✓ equation ito y <p style="text-align: right;">(2)</p> <ul style="list-style-type: none"> ✓ equation ito x ✓ equation ito y <p style="text-align: right;">(2)</p>

<p>3.3</p>	<p>The coordinates of the midpt of AB / <i>Die koordinaat van midpt van AB is:</i></p> $\left(\frac{2+(-6)}{2}; \frac{5+(-5)}{2}\right) = (-2; 0)$ <p>But the y-coordinate of E is 0 ∴ E(-2 ; 0) is the midpoint of AB ∴ EF BG [midpoint theorem/<i>middelpuntst</i> OR/OF line divides 2 sides of Δ in prop/lyn <i>verdeel 2 sye van Δ in dies verh</i>]</p> <p>OR/OF The coordinates of the midpt of AB / <i>Die koordinaat van midpt van AB is:</i></p> $\left(\frac{2+(-6)}{2}; \frac{5+(-5)}{2}\right) = (-2; 0)$ $AE = \sqrt{(-2-2)^2 + (0-5)^2} = \sqrt{41}$ $EB = \sqrt{(-2-(-6))^2 + (0-(-5))^2} = \sqrt{41}$ <p>∴ In ΔABG: AE = EB and AF = FG ∴ EF BG [midpoint theorem/<i>middelpuntst</i>]</p> <p>OR/OF Equation of AB:</p> $y - (-5) = \left(\frac{5-(-5)}{2-(-6)}\right)(x - (-6))$ $y + 5 = \frac{10}{8}x + \frac{15}{2} \quad \therefore y = \frac{5}{4}x + \frac{5}{2}$ <p>x-intercept of AB:</p> $0 = \frac{5}{4}x + \frac{5}{2} \quad \therefore x = -2$ <p>∴ E(-2 ; 0)</p> $m_{EF} = \frac{3\frac{1}{2} - 0}{3 - (-2)} = \frac{7}{10}$ $m_{EF} = m_{BG} = \frac{7}{10}$ <p>∴ EF BG</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>BG: $7x - 10y = 8$ $\therefore y = \frac{7}{10}x - \frac{8}{10}$ $\therefore m_{BG} = \frac{7}{10}$</p> </div>	<p>✓ subst A & B into midpt formula ✓ y coordinate = 0</p> <p>✓ E = midpt ✓ Reason (4)</p> <p>✓ subst A & B into midpt formula ✓ lengths of AE & EB</p> <p>✓ AE = EB or E = midpt ✓ Reason (4)</p> <p>✓ equation of AB</p> <p>✓ coordinates of E</p> <p>✓ gradient of EF ✓ gradient EF = gradient BG (4)</p>
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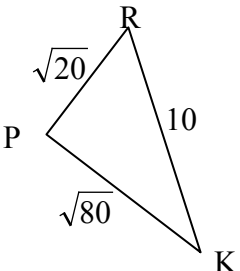
3.4	<p>Midpoint of AC = $\left(5; \frac{1}{2}\right)$</p> $\frac{x_D + (-6)}{2} = 5 \quad \text{and} \quad \frac{y_D + (-5)}{2} = \frac{1}{2}$ <p>$\therefore D(16; 6)$</p> <p>OR/OF by translation/<i>dmv translasië</i>: D(16; 6)</p> <p>OR/OF</p> $m_{BC} = \frac{-5 - (-4)}{-6 - 8} = \frac{1}{14} \quad \text{and} \quad m_{AB} = \frac{5 - (-5)}{2 - (-6)} = \frac{5}{4}$ <p>AD: $y - 5 = \frac{1}{14}(x - 2) \Rightarrow y = \frac{1}{14}x + \frac{34}{7}$</p> <p>CD: $y + 4 = \frac{5}{4}(x - 8) \Rightarrow y = \frac{5}{4}x - 14$</p> $\frac{5}{4}x - 14 = \frac{1}{14}x + \frac{34}{7}$ <p>$\therefore \quad x = 16$ $\quad \quad y = 6$</p>	<p>$\checkmark\checkmark \left(5; \frac{1}{2}\right)$</p> <p>$\checkmark$ x value \checkmark y value (4)</p> <p>\checkmark method finding x \checkmark method finding y \checkmark x value \checkmark y value (4)</p> <p>$\checkmark\checkmark$ equating \checkmark x value \checkmark y value (4)</p> <p>[17]</p>
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QUESTION/VRAAG 4



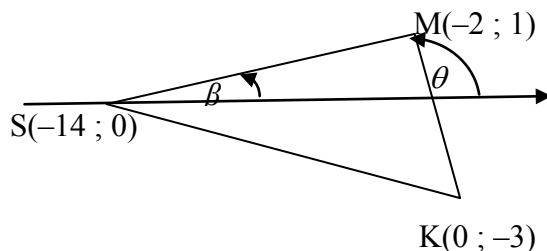
<p>4.1.1</p>	$m_{PK} = \frac{5 - (-3)}{-4 - 0}$ $= -2$ <p>$PK \perp SR$ [radius \perp tangent/raaklyn]</p> $\therefore m_{PK} \times m_{RS} = -1$ $\therefore m_{RS} = \frac{1}{2}$	<ul style="list-style-type: none"> ✓ substitution P & K into gradient formula ✓ gradient of PK ✓ $PK \perp SR$ OR $r \perp$ tangent ✓ answer <p style="text-align: right;">(4)</p>
<p>4.1.2</p>	$y = \frac{1}{2}x + c$ $5 = \frac{1}{2}(-4) + c \quad \mathbf{OR/OR} \quad (y - 5) = \frac{1}{2}(x - (-4))$ $c = 7 \quad (y - 5) = \frac{1}{2}x + 2$ $y = \frac{1}{2}x + 7 \quad y = \frac{1}{2}x + 7$	<ul style="list-style-type: none"> ✓ substitution of m and P ✓ equation <p style="text-align: right;">(2)</p>

<p>4.1.3</p>	$M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right)$ $\therefore M(-2; 1)$ $r^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ $r^2 = (-2 + 4)^2 + (1 - 5)^2$ $\therefore r^2 = 20$ $\therefore (x + 2)^2 + (y - 1)^2 = 20 \text{ or } (\sqrt{20})^2$ <p>OR/OF</p> $M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right) \therefore M(-2; 1)$ $(x + 2)^2 + (y - 1)^2 = r^2$ $(-4 + 2)^2 + (5 - 1)^2 = r^2$ $\therefore r^2 = 20$ $\therefore (x + 2)^2 + (y - 1)^2 = 20 \text{ or } (\sqrt{20})^2$ <p>OR/OF</p> $M\left(\frac{-4+0}{2}; \frac{5+(-3)}{2}\right) \therefore M(-2; 1)$ $PK = \sqrt{(-4 - 0)^2 + (5 - (-3))^2} = \sqrt{80}$ $r = \frac{\sqrt{80}}{2} = \sqrt{20}$ $\therefore (x + 2)^2 + (y - 1)^2 = 20 \text{ or } (\sqrt{20})^2$	<p>✓ x value of M ✓ y value of M</p> <p>✓ $r^2 = 20$</p> <p>✓ equation (4)</p> <p>✓✓ M (- 2 ; 1)</p> <p>$r^2 = 20$</p> <p>✓ equation (4)</p> <p>✓✓ M (- 2 ; 1)</p> <p>$r^2 = 20$</p> <p>✓ equation (4)</p>
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<p>4.1.4</p>	<p> $\tan \theta = m_{PK} = -2$ $\therefore \theta = 180^\circ - 63,43^\circ$ $= 116,57^\circ$ $\hat{P}KR = 116,57^\circ - 90^\circ$ [ext \angle of ΔMOK] $= 26,57^\circ$ OR/OF </p>  <p> <u>In ΔRPK:</u> $PK = \sqrt{(0 - (-4))^2 + (-3 - 5)^2} = \sqrt{80}$ $PR = \sqrt{(-4 - 0)^2 + (5 - 7)^2} = \sqrt{20}$ $RK = 10$ $\cos \hat{P}KR = \frac{PK^2 + KR^2 - PR^2}{2 \cdot PK \cdot KR} = \frac{(\sqrt{80})^2 + (10)^2 - (\sqrt{20})^2}{2(\sqrt{80})(10)}$ $= \frac{2\sqrt{5}}{5}$ $\hat{P}KR = 26,57^\circ$ OR/OF </p> <p> $\sin \hat{P}KR = \frac{\sqrt{20}}{10}$ OR/OF $\cos \hat{P}KR = \frac{\sqrt{80}}{10}$ $\hat{P}KR = 26,57^\circ$ $\hat{P}KR = 26,57^\circ$ </p> <p> OR/OF $\tan \hat{P}KR = \frac{\sqrt{20}}{\sqrt{80}}$ $\hat{P}KR = 26,57^\circ$ </p>	<p> $\checkmark \tan \theta = -2$ \checkmark size of θ \checkmark answer (3) </p> <p> \checkmark lengths of PK, PR & RK \checkmark correct values into cos rule \checkmark answer (3) </p> <p> \checkmark lengths of sides \checkmark ratio \checkmark answer (3) </p> <p> \checkmark lengths of sides \checkmark ratio \checkmark answer (3) </p>
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4.1.5	<p>RS tangent at K(0 ; - 3)</p> $\therefore m_{PS} = m_{\text{tang}} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$ <p>OR/OF</p> $m_{PK} = \frac{1-5}{-2+4} = -2$ $m_{PK} \times m_{\text{tang}} = -1 \quad [\text{radius } \perp \text{ tangent/raaklyn}]$ $\therefore m_{\text{tang}} = \frac{1}{2}$ $\therefore y = \frac{1}{2}x - 3$	<p>✓ gradient</p> <p>✓ equation (2)</p> <p>✓ gradient</p> <p>✓ equation (2)</p>
4.2	<p>$t \in (-3 ; 7)$</p> <p>OR/OF</p> $-3 < t < 7$	<p>✓ - 3 (A)</p> <p>✓ 7 (CA from 4.1.2)</p> <p>✓ correct inequality (3)</p> <p>✓ - 3 (A)</p> <p>✓ 7 (CA from 4.1.2)</p> <p>✓ correct inequality (3)</p>
4.3	<p>RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14 ; 0)$</p> $SP = \sqrt{(-14 - (-4))^2 + (0 - 5)^2} = \sqrt{100 + 25} = \sqrt{125}$ $\text{Area } \triangle SMK = \frac{1}{2} \cdot MK \cdot SP$ $= \frac{1}{2} (\sqrt{20})(\sqrt{125})$ $= 25 \text{ square units}$	<p>✓ coordinates of S</p> <p>✓ length of SP</p> <p>✓ correct base & height into Area rule</p> <p>✓ correct substitution</p> <p>✓ answer (5)</p>

OR/OF



Let β = inclination of SM/ *inklinasie van SM*

RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14; 0)$

$SM = \sqrt{(-14 - (-2))^2 + (0 - 1)^2} = \sqrt{145}$

$\tan \beta = \frac{1 - 0}{-2 - (-14)} = \frac{1}{12} \quad \therefore \beta = 4,76^\circ$

$\therefore \hat{SMK} = 116,57^\circ - 4,76^\circ \quad [\text{ext } \angle \text{ of } \Delta]$
 $= 111,81^\circ$

Area $\Delta SMK = \frac{1}{2}(SM)(MK) \cdot \sin \hat{SMK}$
 $= \frac{1}{2}(\sqrt{145})(\sqrt{20}) \cdot \sin 111,81^\circ$
 $= 24,9985 = 25 \text{ square units}$

✓ coordinates of S

✓ length of SM

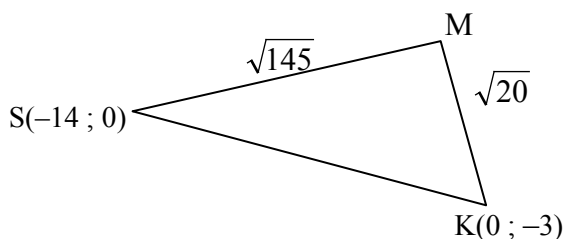
✓ size of/grootte v \hat{SMK}

✓ correct substitution into area rule

✓ answer

(5)

OR/OF



RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14; 0)$

$SK = \sqrt{(-14 - 0)^2 + (0 + 3)^2} = \sqrt{205}$

$\cos \hat{SMK} = \frac{(\sqrt{145})^2 + (\sqrt{20})^2 - (\sqrt{205})^2}{2(\sqrt{145})(\sqrt{20})} = -\frac{2\sqrt{29}}{29}$

$\hat{SMK} = 111,80^\circ$

Area $\Delta SMK = \frac{1}{2}(SM)(MK) \cdot \sin \hat{SMK}$
 $= \frac{1}{2}(\sqrt{145})(\sqrt{20}) \cdot \sin 111,81^\circ$
 $= 24,9985 = 25 \text{ square units}$

✓ coordinates of S

✓ length of SK

✓ size of/grootte v \hat{SMK}

✓ correct substitution into area rule

✓ answer

(5)

	<p>OR/OF</p> <p>Produce KS to T</p> <p>RS: $y = \frac{1}{2}x + 7 \quad \therefore S(-14; 0)$</p> <p>$SK = \sqrt{(-14 - 0)^2 + (0 + 3)^2} = \sqrt{205}$</p> <p>$SM = \sqrt{(-14 - (-2))^2 + (0 - 1)^2} = \sqrt{145}$</p> <p>$m_{SK} = -\frac{3}{14} \Rightarrow T\hat{S}O = 167,91^\circ$</p> <p>$m_{SM} = \frac{1}{12} \Rightarrow M\hat{S}O = 4,76^\circ$</p> <p>$M\hat{S}K = 180^\circ - 167,91^\circ + 4,76^\circ = 16,85^\circ$</p> <p>Area $\Delta SMK = \frac{1}{2}(SM)(SK) \cdot \sin M\hat{S}K$</p> <p>$= \frac{1}{2}(\sqrt{145})(\sqrt{205}) \cdot \sin 16,85^\circ$</p> <p>$= 24,9985 = 25 \text{ square units}$</p>	<p>✓ coordinates of S</p> <p>✓ length of SK & SM</p> <p>✓ size of /grootte v $M\hat{S}K$</p> <p>✓ correct substitution into area rule</p> <p>✓ answer</p> <p>(5)</p>
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QUESTION/VRAAG 5

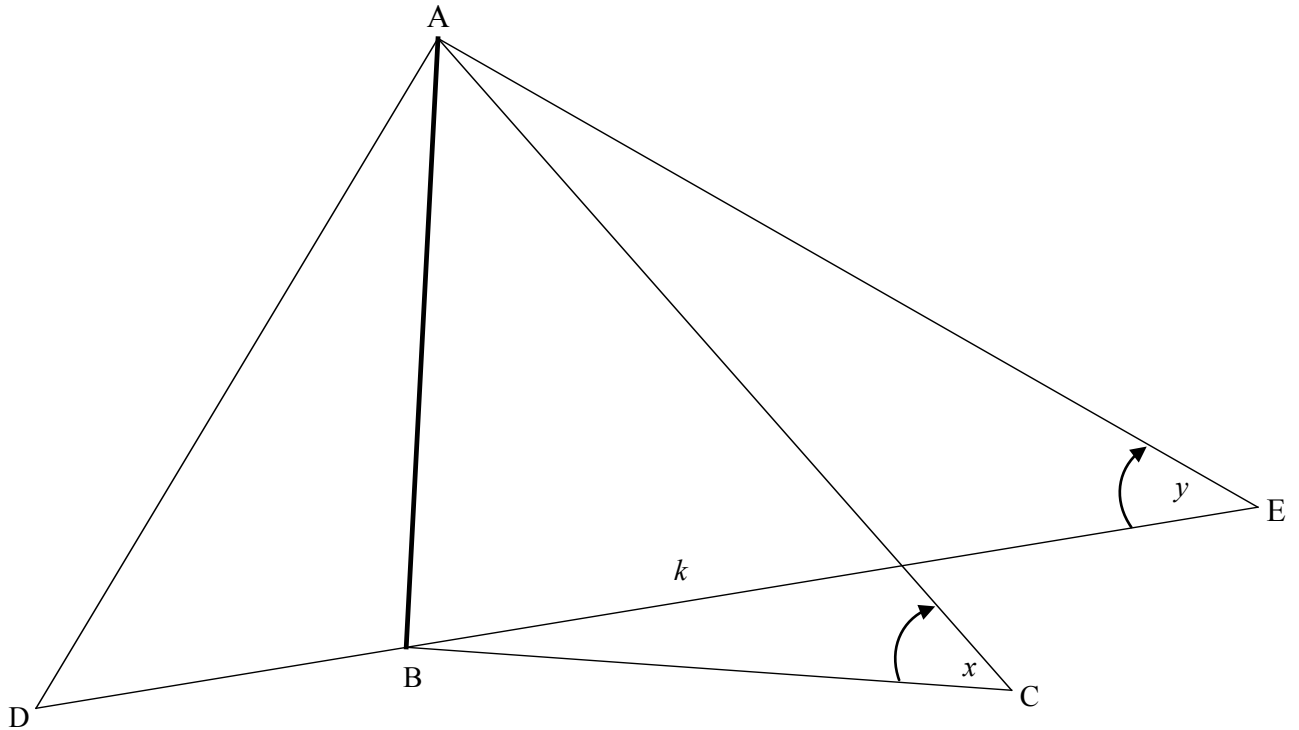
5.1	$\frac{\sin(A - 360^\circ) \cdot \cos(90^\circ + A)}{\cos(90^\circ - A) \cdot \tan(-A)}$ $= \frac{\sin A(-\sin A)}{\sin A(-\tan A)}$ $= \frac{\sin A}{\left(\frac{\sin A}{\cos A}\right)}$ $= \cos A$	<ul style="list-style-type: none"> ✓ sin A ✓ -sin A ✓ sin A ✓ -tan A ✓ $\tan A = \frac{\sin A}{\cos A}$ ✓ answer <p style="text-align: right;">(6)</p>
5.2.1	$t^2 = (\sqrt{34})^2 - (3)^2$ $\therefore t = -5$	<ul style="list-style-type: none"> ✓ substitution ✓ answer <p style="text-align: right;">(2)</p>
5.2.2	$\tan \beta = \frac{-5}{3}$	<ul style="list-style-type: none"> ✓ correct ratio <p style="text-align: right;">(1)</p>
5.2.3	$\cos 2\beta = 2 \cos^2 \beta - 1$ $= 2 \left(\frac{3}{\sqrt{34}} \right)^2 - 1$ $= 2 \left(\frac{9}{34} \right) - 1$ $= -\frac{16}{34} \text{ OR } -\frac{8}{17}$ <p>OR/OF</p> $\cos 2\beta = 1 - 2 \sin^2 \beta$ $= 1 - 2 \left(-\frac{5}{\sqrt{34}} \right)^2$ $= 1 - 2 \left(\frac{25}{34} \right)$ $= -\frac{16}{34} \text{ OR } -\frac{8}{17}$ <p>OR/OF</p> $\cos 2\beta = \cos^2 \beta - \sin^2 \beta$ $= \left(\frac{3}{\sqrt{34}} \right)^2 - \left(-\frac{5}{\sqrt{34}} \right)^2$ $= \frac{9}{34} - \frac{25}{34}$ $= -\frac{16}{34} \text{ OR } -\frac{8}{17}$	<ul style="list-style-type: none"> ✓ compound formula ✓ substitution ✓ simplification ✓ answer <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓ compound formula ✓ substitution ✓ simplification ✓ answer <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓ compound formula ✓ substitution ✓ simplification ✓ answer <p style="text-align: right;">(4)</p>

5.3.1	$\begin{aligned} \text{LHS} &= \sin(A + B) - \sin(A - B) \\ &= \sin A \cdot \cos B + \cos A \cdot \sin B - (\sin A \cdot \cos B - \cos A \cdot \sin B) \\ &= \sin A \cdot \cos B + \cos A \cdot \sin B - \sin A \cdot \cos B + \cos A \cdot \sin B \\ &= 2\cos A \cdot \sin B \\ &= \text{RHS} \end{aligned}$	✓ compound formula ✓ compound formula (2)
5.3.2	$\begin{aligned} \sin 77^\circ - \sin 43^\circ &= \sin(60^\circ + 17^\circ) - \sin(60^\circ - 17^\circ) \\ &= 2\cos 60^\circ \cdot \sin 17^\circ \\ &= 2 \times \frac{1}{2} \times \sin 17^\circ \\ &= \sin 17^\circ \end{aligned}$ <p>OR/OF</p> $\begin{aligned} \sin 77^\circ - \sin 43^\circ &= \sin(60^\circ + 17^\circ) - \sin(60^\circ - 17^\circ) \\ &= (\sin 60^\circ \cos 17^\circ + \cos 60^\circ \sin 17^\circ) - \\ &\quad (\sin 60^\circ \cos 17^\circ - \cos 60^\circ \sin 17^\circ) \\ &= \frac{\sqrt{3}}{2} \cos 17^\circ + \frac{1}{2} \sin 17^\circ - \frac{\sqrt{3}}{2} \cos 17^\circ + \frac{1}{2} \sin 17^\circ \\ &= \sin 17^\circ \end{aligned}$	✓ $60^\circ + 17^\circ$ ✓ $60^\circ - 17^\circ$ ✓ simplify ✓ $\frac{1}{2}$ (4) ✓ $60^\circ + 17^\circ$ ✓ $60^\circ - 17^\circ$ ✓ expansion ✓ $\frac{1}{2}$ (4) [19]

QUESTION/VRAAG 6

<p>6.1</p>		<ul style="list-style-type: none"> ✓ $(-90^\circ ; -3)$ ✓ $(0 ; -1)$ ✓ x – intercepts: -210° & 30° ✓ shape <p style="text-align: right;">(4)</p>
<p>6.2</p>	$\cos 2x = 2 \sin x - 1$ $1 - 2 \sin^2 x = 2 \sin x - 1$ $2 \sin^2 x + 2 \sin x - 2 = 0$ $\sin^2 x + \sin x - 1 = 0$ $\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2(1)}$ $\sin x = \frac{-1 + \sqrt{5}}{2}, \text{ since } \sin x = \frac{-1 - \sqrt{5}}{2} < -1 \text{ has no solution}$	<ul style="list-style-type: none"> ✓ $\cos 2x = 1 - 2 \sin^2 x$ ✓ standard form ✓ using quadratic formula ✓ substitution into quadratic formula <p style="text-align: right;">(4)</p>
<p>6.3</p>	$\sin x = \frac{-1 + \sqrt{5}}{2} = 0,618\dots$ <p>Reference $\angle = 38,17^\circ$</p> $\therefore x = 38,17^\circ + k \cdot 360^\circ \text{ or } x = 141,83^\circ + k \cdot 360^\circ ; k \in \mathbb{Z}$ $\therefore x = 38,17^\circ \text{ or } -218,17^\circ$ $y = 0,24$ <p>\therefore Points of intersection/snyppunte: $(38,17^\circ ; 0,24)$ and $(-218,17^\circ ; 0,24)$</p>	<ul style="list-style-type: none"> ✓ $38,17^\circ$ ✓ $141,83^\circ$ ✓ $-218,17^\circ$ ✓ $0,24$ <p style="text-align: right;">(4) [12]</p>

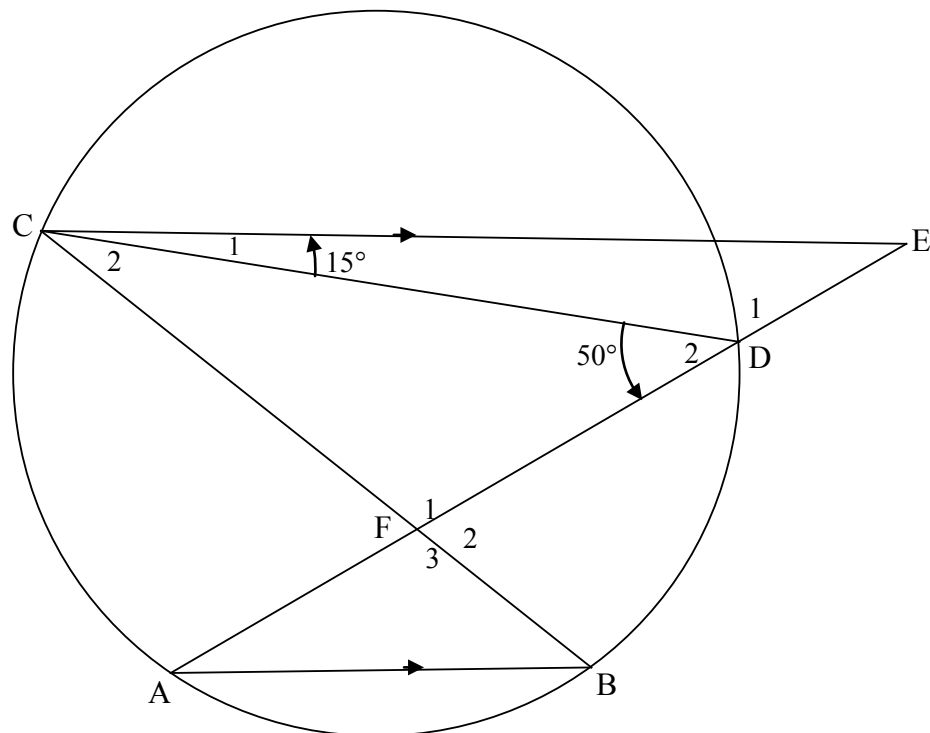
QUESTION/VRAAG 7



7.1	$\hat{A}BC = 90^\circ$	✓ answer (1)
7.2	<p>In $\triangle ABE$:</p> $\frac{AB}{BE} = \tan y$ $AB = k \tan y$ <p>In $\triangle ABC$:</p> $\frac{AB}{AC} = \sin x$ $AC = \frac{AB}{\sin x}$ $= \frac{k \tan y}{\sin x}$	<p>✓ correct ratio</p> <p>✓ value AB</p> <p>✓ correct ratio</p> <p>✓ AC as subject and substitution (4)</p>

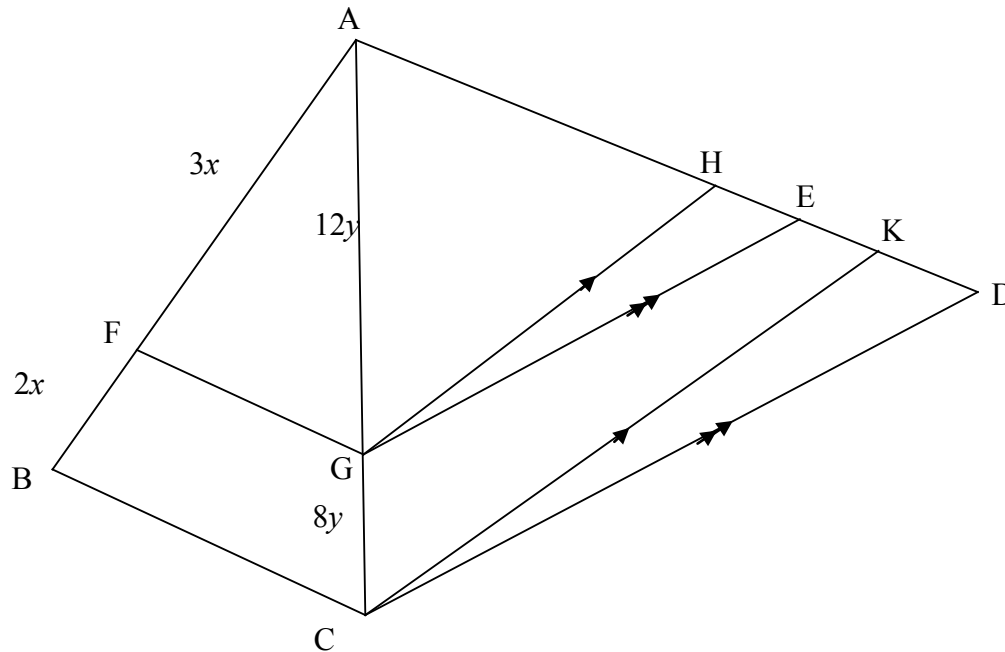
<p>7.3</p>	$\hat{A}DC = \hat{A}CD = \frac{180^\circ - 2x}{2} = 90^\circ - x$ $\frac{DC}{\sin 2x} = \frac{AC}{\sin(90^\circ - x)}$ $\frac{DC}{2 \sin x \cos x} = \frac{AC}{\cos x}$ $DC = \frac{AC(2 \sin x \cos x)}{\cos x}$ $= \frac{k \tan y}{\sin x} \cdot \frac{2 \sin x \cos x}{\cos x}$ $= 2k \tan y$ <p>OR/OF</p> $DC^2 = AD^2 + AC^2 - 2AD \cdot AC \cos 2x$ $= AC^2 + AC^2 - 2AC^2 \cos 2x$ $= 2AC^2(1 - \cos 2x)$ $= 2AC^2(1 - 1 + \sin^2 x)$ $= 4AC^2 \sin^2 x$ $DC = 2AC \cdot \sin x$ $= 2 \left(\frac{k \cdot \tan y}{\sin x} \right) \cdot \sin x$ $= 2k \cdot \tan y$ <p>OR/OF</p> $DC^2 = AD^2 + AC^2 - 2AD \cdot AC \cos 2x$ $= 2 \left(\frac{k \tan y}{\sin x} \right)^2 - 2 \left(\frac{k \tan y}{\sin x} \right)^2 \cos 2x$ $= \frac{2k^2 \tan^2 y}{\sin^2 x} - \frac{2k^2 \tan^2 y}{\sin^2 x} (1 - 2 \sin^2 x)$ $= \frac{2k^2 \tan^2 y}{\sin^2 x} - \frac{2k^2 \tan^2 y}{\sin^2 x} + 4k^2 \tan^2 y$ $DC = \sqrt{4k^2 \tan^2 y}$ $= 2k \tan y$	<ul style="list-style-type: none"> ✓ $90^\circ - x$ ✓ subst into sine rule ✓ $2 \sin x \cos x$ ✓ $\cos x$ ✓ substitution (5) ✓ substitution into cos rule ✓ factorisation ✓ $1 - 2 \sin^2 x$ ✓ DC ito AC and $\sin x$ ✓ substitution (5) ✓ correct cos rule ✓ substitution ✓ $1 - 2 \sin^2 x$ ✓ squaring and multiplication ✓ $\sqrt{4k^2 \tan^2 y}$ (5) <p style="text-align: right;">[10]</p>
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QUESTION/VRAAG 8

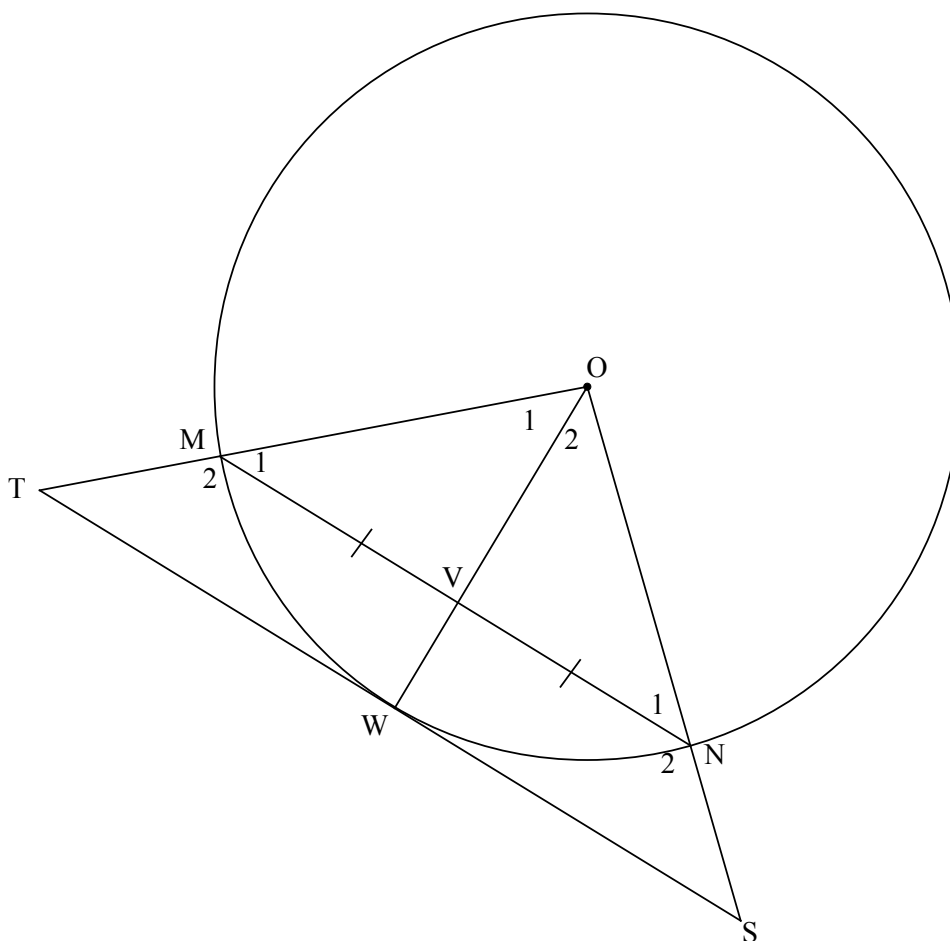


<p>8.1.1</p>	<p>$\hat{E} = 50^\circ - 15^\circ = 35^\circ$ [ext \angle of Δ/buite \angle van Δ] $\hat{A} = 35^\circ$ [alt \angles / verwiss \anglee; CE AB]</p> <p>OR/OF $\hat{E} = 180^\circ - (130^\circ + 15^\circ) = 35^\circ$ [str line; \angles of Δ/rt lyn; \anglee van Δ] $\hat{A} = 35^\circ$ [alt \angles / verwiss \anglee; CE AB]</p> <p>OR/OF $\hat{B} = 50^\circ$ [\angles in same segment/\anglee in dieselfde segment] $\hat{C}_2 + 15^\circ = 50^\circ$ [alt \angles / verwiss \anglee; CE AB] $\therefore \hat{C}_2 = 35^\circ$ $\hat{A} = 35^\circ$ [\angles in same segment/\anglee in dieselfde segment]</p>	<p>✓ S ✓ S ✓ R (3)</p> <p>✓ S ✓ S ✓ R (3)</p> <p>✓ S ✓ S ✓ R (3)</p>
<p>8.1.2</p>	<p>$\hat{C}_2 = 35^\circ$ [\angles in same segment/\anglee in dieselfde segment]</p>	<p>✓ S ✓ R (2)</p>
<p>8.2</p>	<p>$\hat{C}_2 = \hat{E}$ [from 8.1.1 and 8.1.2] \therefore CF is a tangent to the circle [converse tan chord theorem] \therefore CF is 'n raaklyn aan die sirkel [omgekeerde raakl koordst]</p>	<p>✓ S ✓ R (2) [7]</p>

QUESTION/VRAAG 9



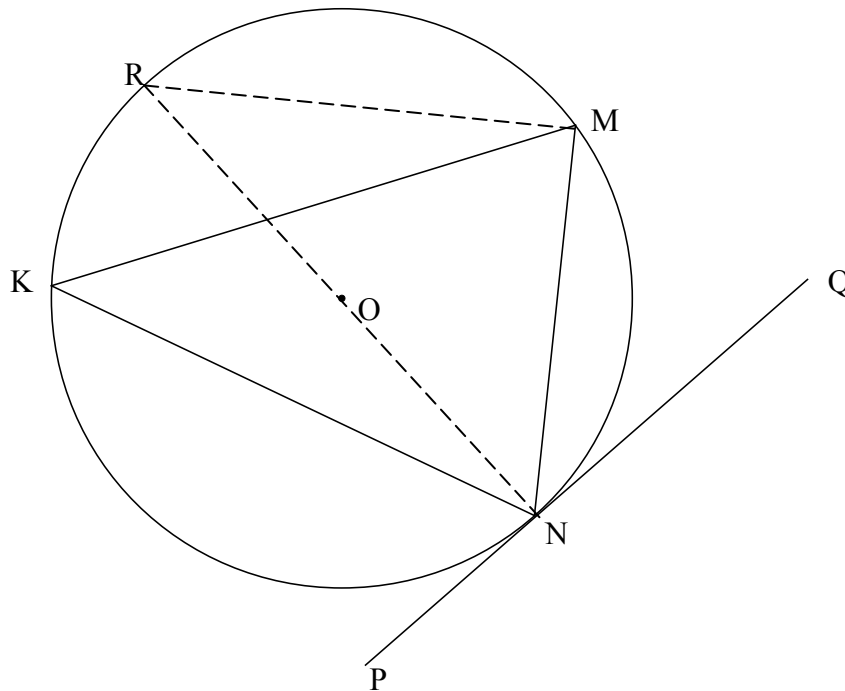
<p>9.1.1</p>	$\frac{AF}{BF} = \frac{3x}{2x} = \frac{3}{2} \quad \& \quad \frac{AG}{GC} = \frac{12y}{8y} = \frac{3}{2}$ $\therefore \frac{AF}{BF} = \frac{AG}{GC}$ <p>$\therefore FG \parallel BC$ [conv prop th/omg eweredigh st. OR line divides 2 sides of Δ in prop/lyn verdeel 2 sye v Δ in dies verh]</p>	<p>$\checkmark \frac{AF}{BF} = \frac{AG}{GC}$</p> <p>$\checkmark R$</p> <p style="text-align: right;">(2)</p>
<p>9.1.2</p>	$\frac{AG}{GC} = \frac{AH}{HK} \quad \text{[prop theorem/eweredigh st; } \underline{GH \parallel CK} \text{ OR line } \parallel \text{ to 1 side of } \Delta \text{/lyn } \parallel \text{ 1 sy van } \Delta]$ $\frac{AG}{GC} = \frac{AE}{ED} \quad \text{[prop theorem/eweredigh st; } \underline{GE \parallel CD}]$ $\therefore \frac{AH}{HK} = \frac{AE}{ED}$	<p>$\checkmark S \checkmark R$</p> <p>$\checkmark S$</p> <p style="text-align: right;">(3)</p>
<p>9.2</p>	$\frac{AE}{ED} = \frac{3}{2} \quad \text{and} \quad \frac{AH}{HK} = \frac{3}{2}$ $\frac{AE}{12} = \frac{3}{2} \quad \text{and} \quad \frac{15}{HK} = \frac{3}{2}$ <p>$\therefore AE = 18$ and $HK = 10$</p> <p>$\therefore HE = AE - AH$ $= 18 - 15$ $= 3$</p> <p>$\therefore EK = HK - HE$ $= 10 - 3$ $= 7$</p> <p style="text-align: center;">OR/OF</p> $AD = 30$ $KD = AD - AH - HK$ $= 30 - 15 - 10$ $= 5$ $EK = ED - KD$ $= 12 - 5$ $= 7$	<p>\checkmark use of ratios</p> <p>$\checkmark AE = 18$</p> <p>$\checkmark HK = 10$</p> <p>$\checkmark HE = 3$ or $KD = 5$</p> <p>$\checkmark EK = 7$</p> <p style="text-align: right;">(5) [10]</p>

QUESTION/VRAAG 10

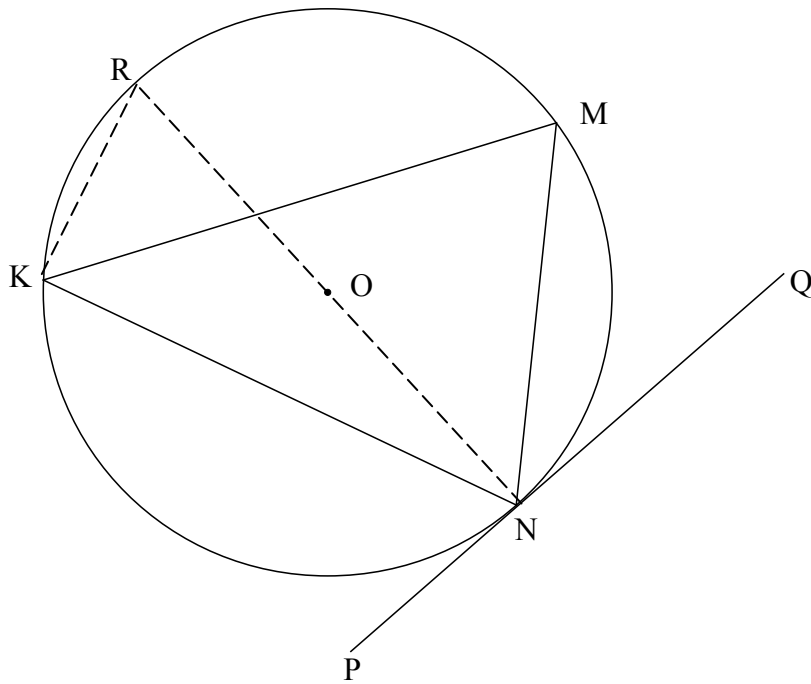
10.1	Line from centre to midpoint of chord/ <i>lyn vanaf midpt na midpt van koord</i>	✓ R	(1)
10.2.1	$\hat{O}\hat{W}T = \hat{O}\hat{W}S = 90^\circ$ [radius \perp tangent/ <i>raaklyn</i>] $\therefore MN \parallel TS$ [corresp \angle s =/ooreenkomstige \angle e = OR co-int \angle s 180° /ko-binne \angle e 180° OR alternate \angle s/ <i>verwiss</i> \angle e]	✓ R ✓ R	(2)
10.2.2	$\hat{M}_1 = \hat{N}_1$ [\angle s opp = sides/ \angle e teenoor = <i>sye</i>] $\hat{M}_1 = \hat{T}$ [corresp \angle s/ooreenk \angle e; $MN \parallel TS$] $\therefore \hat{N}_1 = \hat{T}$ \therefore TMNS is a cyclic quadrilateral [conv: ext \angle cyclic quad] TMNS is 'n koordevierhoek [<i>omgek: buite \angle kdvh</i>] OR/OF $\hat{M}_1 = \hat{N}_1$ [\angle s opp = sides/ \angle e teenoor = <i>sye</i>] $\hat{N}_1 = \hat{S}$ [corresp \angle s/ooreenk \angle e; $MN \parallel TS$] $\therefore \hat{S} = \hat{M}_1$ \therefore TMNS is a cyclic quadrilateral [conv: ext \angle cyclic quad] TMNS is 'n koordevierhoek [<i>omgek: buite \angle kdvh</i>]	✓ S ✓ S ✓ S ✓ R ✓ S ✓ S ✓ S ✓ R	(4) (4)

<p>10.2.3</p> <p>In $\triangle OVN$ and $\triangle OWS$</p> <p>$\hat{O}_2 = \hat{O}_2$ [common/<i>gemeenskaplik</i>]</p> <p>$O\hat{V}N = O\hat{W}S = 90^\circ$ [from 10.1]</p> <p>$O\hat{N}V = O\hat{S}W$ [sum $\angle s \triangle / som \angle e \triangle$]</p> <p>$\therefore \triangle OVN \parallel \triangle OWS$ [\angle, \angle, \angle]</p> <p>$\therefore \frac{VN}{WS} = \frac{ON}{OS}$</p> <p>But $VN = \frac{1}{2} MN$ [given]</p> <p>$\therefore \frac{\frac{1}{2} MN}{WS} = \frac{ON}{OS}$</p> <p>$\therefore OS \cdot MN = 2ON \cdot WS$</p> <p>OR/OF</p> <p>In $\triangle OVM$ and $\triangle OWS$</p> <p>$O\hat{V}M = O\hat{W}S = 90^\circ$ [from 10.1]</p> <p>$O\hat{M}V = O\hat{S}W$ [sum $\angle s \triangle / som \angle e \triangle$]</p> <p>$\therefore \triangle OVM \parallel \triangle OWS$ [\angle, \angle, \angle]</p> <p>$\therefore \frac{OM}{OS} = \frac{VM}{WS}$</p> <p>But $VN = \frac{1}{2} MN$ [given]</p> <p>$\therefore \frac{\frac{1}{2} MN}{WS} = \frac{OM}{OS}$</p> <p>$\therefore OS \cdot MN = 2ON \cdot WS$ [VM = VN]</p> <p>OR/OF</p> <p>If any other 2 $\triangle s$ are used, first need to prove that $TW = WS$ by proving $\triangle OWT \equiv \triangle OWS$</p> <p>In $\triangle OVM$ and $\triangle OWT$</p> <p>$\hat{O}_1 = \hat{O}_1$ [common/<i>gemeenskaplik</i>]</p> <p>$O\hat{V}M = O\hat{W}T = 90^\circ$ [from 10.1]</p> <p>$O\hat{M}V = O\hat{T}W$ [sum $\angle s \triangle / som \angle e \triangle$]</p> <p>$\therefore \triangle OVM \parallel \triangle OWT$ [\angle, \angle, \angle]</p> <p>$\therefore \frac{VM}{WT} = \frac{OM}{OT}$</p> <p>But $VN = VM = \frac{1}{2} MN$ [given]</p> <p>and $WT = WS$ and $OT = OS$ [$\triangle OWT \equiv \triangle OWS$]</p> <p>$\therefore \frac{\frac{1}{2} MN}{WS} = \frac{ON}{OS}$</p> <p>$\therefore OS \cdot MN = 2ON \cdot WS$</p>	<p>\checkmark S; S; S OR S; S; R</p> <p>$\checkmark \triangle OVN \parallel \triangle OWS$ $\checkmark \frac{VN}{WS} = \frac{ON}{OS}$ $\checkmark VN = \frac{1}{2} MN$</p> <p>$\checkmark$ substitution</p> <p>(5)</p> <p>\checkmark S; S; S OR S; S; R</p> <p>$\checkmark \triangle OVM \parallel \triangle OWS$ $\checkmark \frac{OM}{OS} = \frac{VM}{WS}$ $\checkmark VN = \frac{1}{2} MN$</p> <p>$\checkmark$ substitution</p> <p>(5)</p> <p>$\checkmark \checkmark$ similarity $\checkmark \checkmark$ congruency</p> <p>$\checkmark VN = VM = \frac{1}{2} MN$</p> <p>(5)</p> <p>[12]</p>
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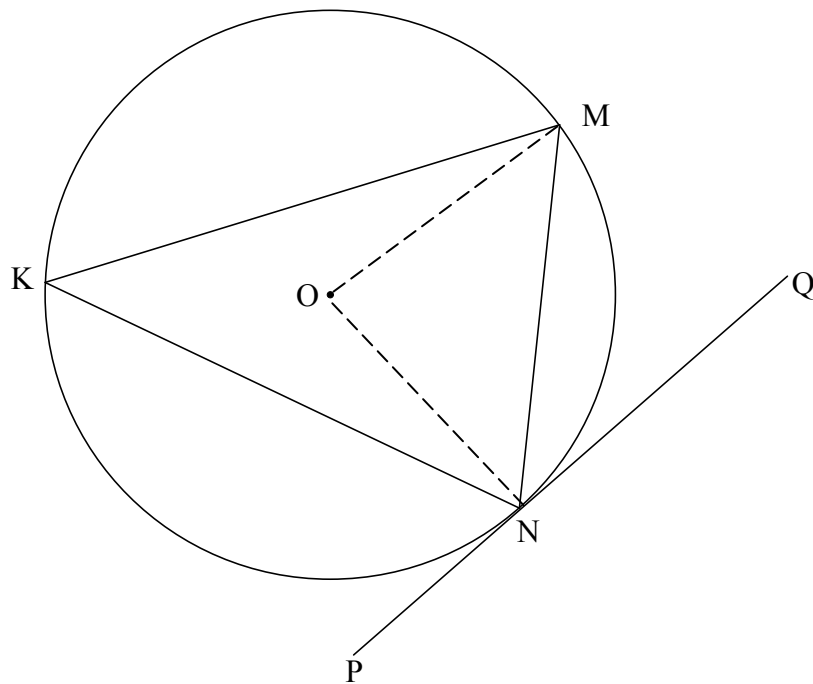
QUESTION/VRAAG 11



<p>11.1</p>	<p>Construction: Draw diameter NR and draw RM <i>Konstruksie: Trek middellyn NR en verbind RM</i> $\widehat{ONM} + \widehat{MNQ} = 90^\circ$ [radius \perp tangent/raaklyn] $\widehat{NMR} = 90^\circ$ [\angle in semi circle/semi-sirkel] $\therefore \widehat{MRN} = 180^\circ - (90^\circ + 90^\circ - \widehat{MNQ})$ [sum \angles Δ] $= \widehat{MNQ}$ but $\widehat{MRN} = \widehat{MKN}$ [\angles same segment/\anglee dieselfde segment] $\therefore \widehat{MNQ} = \widehat{MKN}$ OR/OF</p>	<p>✓ construction ✓ S / R ✓ S / R ✓ S ✓ S / R (5)</p>
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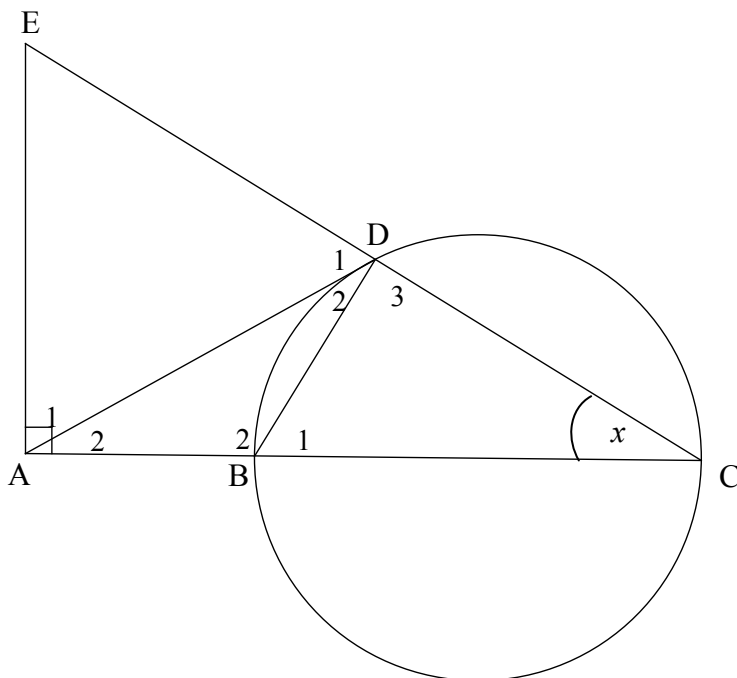


<p>11.1</p>	<p>Construction: Draw diameter NR and draw RK <i>Konstruksie: Trek middellyn NR en verbind RK</i> $M\hat{N}Q + R\hat{N}M = 90^\circ$ [radius \perp tangent/raaklyn] $N\hat{K}R = 90^\circ$ [\angle in semicircle/semi-sirkel] $\therefore M\hat{K}N = 90^\circ - R\hat{K}M$ $= 90^\circ - R\hat{N}M$ [\angles same segment/\anglee dieselfde segment] $\therefore M\hat{N}Q = \hat{K}$</p>	<p>✓ construction ✓ S / R ✓ S / R ✓ S ✓ S / R (5)</p>
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<p>11.1</p>	<p>Construction: Draw radii ON and OM <i>Konstruksie: Trek radiusse ON en OM</i> $\widehat{M\hat{O}N} = 2\hat{K}$ [\angle at centre = $2\angle$ at circumf/midpts $\angle = 2$ omtreks \angle] $\widehat{O\hat{N}M} + \widehat{O\hat{M}N} = 180^\circ - 2\hat{K}$ [\angles of Δ/ \anglee van Δ] $\widehat{O\hat{N}M} = \widehat{O\hat{M}N} = \frac{180^\circ - 2\hat{K}}{2} = 90^\circ - \hat{K}$ [\angles opp = sides/ \anglee teenoor = sye] $\widehat{O\hat{N}Q} = 90^\circ$ [radius \perp tangent/ radius \perp raaklyn] $\therefore \widehat{M\hat{N}Q} = \hat{K}$</p>	<p>✓ construction ✓ S / R ✓ S ✓ S / R ✓ S / R (5)</p>
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11.2



11.2.1(a)	Angle in a semi circle/ <i>Hoek in halfsirkel</i>	✓ R (1)
11.2.1(b)	Exterior \angle of quad = opp interior \angle / <i>Buite \angle van vierh = teenoorst binne \angle</i> OR/OF Opp \angle s of quad supplementary/ <i>Teenoorst \anglee van vierh is supplementêr</i>	✓ R (1)
11.2.1(c)	tangent chord theorem/ <i>raaklyn koord stelling</i>	✓ R (1)
11.2.2(a)	In $\triangle AEC$ $\hat{E} = 180^\circ - (90^\circ + x)$ [sum \angle s \triangle] $= 90^\circ - x$ $\hat{D}_1 = 180^\circ - (90^\circ + x)$ [\angle s on a straight line] $= \hat{E} = 90^\circ - x$ $\therefore AD = AE$ [sides opp = \angle s/ <i>syte teenoor = \anglee</i>]	✓ S ✓ S ✓ R (3)
11.2.2(b)	In $\triangle ADB$ and $\triangle ACD$ $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\hat{B}_2 = \hat{D}_2 + \hat{D}_3$ [sum \angle \triangle] $\therefore \triangle ADB \parallel \triangle ACD$ OR/OF In $\triangle ADB$ and $\triangle ACD$ $\hat{A}_2 = \hat{A}_2$ [common] $\hat{D}_2 = \hat{C}$ [proven] $\therefore \triangle ADB \parallel \triangle ACD$ [\angle , \angle , \angle]	✓ S ✓ S ✓ S (3) ✓ S ✓ S ✓ R (3)

<p>11.2.3(a)</p>	$\frac{AD}{AC} = \frac{AB}{AD} \quad [\Delta s]$ $AD^2 = AC \cdot AB$ $= 3r \times r$ $= 3r^2$	<p>✓ ratio</p> <p>✓ substitution</p> <p>(2)</p>
<p>11.2.3(b)</p>	<p>$AD = AE = \sqrt{3}r$ [from 11.2.2(a) & 11.2.3(a)]</p> <p>$AB = r$ and $BC = 2r \therefore AC = 3r$</p> <p><u>In ΔACE:</u></p> $\tan \hat{E} = \frac{AC}{AE}$ $= \frac{3r}{\sqrt{3}r} = \sqrt{3}$ <p>$\therefore \hat{E} = 60^\circ$</p> <p>$\therefore \hat{D}_1 = 60^\circ$ [from 11.2.2(a)]</p> <p>$\therefore \hat{A}_1 = 60^\circ$ [\angles of $\Delta = 180^\circ$]</p> <p>$\therefore \Delta ADE$ is equilateral/<i>is gelyksydig</i></p> <p>OR/OF</p> $\frac{AD}{AC} = \frac{DB}{CD} \quad [\Delta s]$ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$ $\tan x = \frac{1}{\sqrt{3}}$ <p>\therefore In ΔBDC: $x = 30^\circ$</p> <p>$\therefore \hat{E} = 60^\circ$</p> <p>$\therefore \hat{D}_1 = 60^\circ$ [from 11.2.2(a)]</p> <p>$\therefore \hat{A}_1 = 60^\circ$ [\angles of $\Delta = 180^\circ$]</p> <p>$\therefore \Delta ADE$ is equilateral/<i>is gelyksydig</i></p> <p>OR/OF</p> $\frac{AD}{AC} = \frac{DB}{CD} \quad [\Delta s]$ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD} \quad \therefore BD = \frac{CD}{\sqrt{3}}$ $DC^2 = BC^2 - DB^2$ $= 4r^2 - \frac{CD^2}{3}$ $3DC^2 = 12r^2 - CD^2$ $4CD^2 = 12r^2$ $DC = \sqrt{3}r$	<p>✓ AC ito r</p> <p>✓ trig ratio</p> <p>✓ simplification</p> <p>✓ all 3 \angles = 60°</p> <p>(4)</p> <p>✓ $\frac{\sqrt{3}r}{3r} = \frac{DB}{CD}$</p> <p>✓ $\frac{1}{\sqrt{3}} = \tan x$</p> <p>✓ $x = 30^\circ$</p> <p>✓ all 3 \angles = 60°</p> <p>(4)</p> <p>✓ $BD = \frac{CD}{\sqrt{3}}$</p> <p>✓ $DC = \sqrt{3}r$</p>

	$EC^2 = EA^2 + AC^2$ $= 3r^2 + 9r^2$ $EC = 2\sqrt{3}r$ $\therefore ED = EC - DC$ $= \sqrt{3}r$ $\therefore ED = EA = AD$ $\therefore \triangle ADE \text{ is equilateral/is gelyksydig}$	$\checkmark EC = 2\sqrt{3}r$ $\checkmark ED = EA = AD$ <p style="text-align: right;">(4) [20]</p>
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TOTAL/TOTAAL: 150