basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIORSERTIKAAT-EKSAMEN/
NASIONALE SENIORSERTIFIKAAT-EKSAMEN

WISKUNDE V2

MEI/JUNIE 2024

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 12 bladsye, 1 inligtingsblad en 'n antwoordeboek van 23 bladsye.

INSTRUKSIES EN INLIGTING

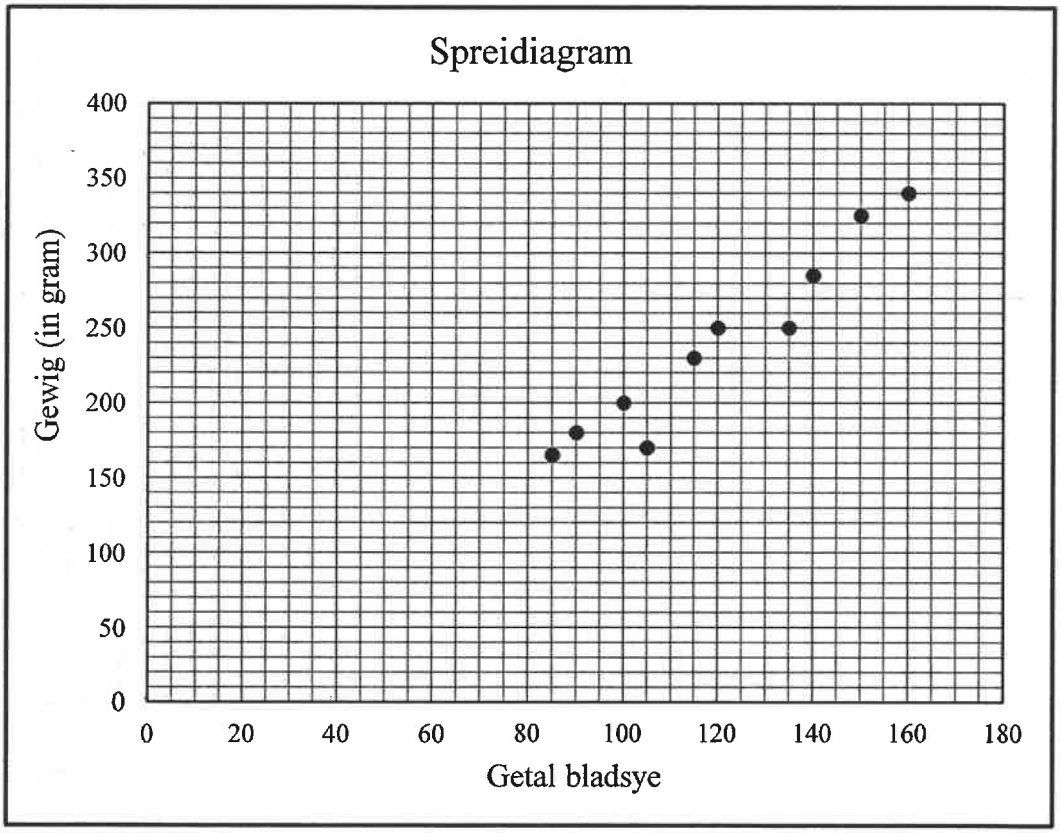
Lees die volgende instruksies aandagtig deur voordat jy die vrae beantwoord.

1. Hierdie vraestel bestaan uit 10 vrae.
2. Beantwoord AL die vrae in die SPESIALE ANTWOORDEBOEK wat verskaf word.
3. Dui ALLE berekeninge, diagramme, grafieke, ens. wat jy in die beantwoording van die vrae gebruik, duidelik aan.
4. Slegs antwoorde sal NIE noodwendig volpunte verdien NIE.
5. Jy mag 'n goedgekeurde wetenskaplike sakrekenaar gebruik (nieprogrammeerbaar en niegrafies), tensy anders vermeld.
6. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.
7. Diagramme is NIE noodwendig volgens skaal geteken NIE.
8. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
9. Skryf netjies en leesbaar.

VRAAG 1

Die getal bladsye in tien A4-boeke en hulle ooreenstemmende gewig (in gram) is in die tabel hieronder gegee. Die data word ook in die spreidiagram verteenwoordig.

Getal bladsye (x)	85	150	100	120	90	140	135	105	115	160
Gewig (in gram) (y)	165	325	200	250	180	285	250	170	230	340



1.1 Bepaal die vergelyking van die kleinstekwadrate-regressielyn. (3)

1.2 Trek die kleinstekwadrate-regressielyn op die spreidiagram in die ANTWOORDEBOEK. (2)

1.3 Voorspel die gewig van 'n A4-boek wat 110 bladsye bevat. (2)

1.4 Bereken die persentasie toename in gewig tussen 'n boek met 110 bladsye en 'n boek met 130 bladsye. (3)
[10]

VRAAG 2

Vyftig atlete moet toegang tot geskikte oefenfasiliteite hê. Die tabel hieronder toon die afstande, in km, wat hulle moet ry om toegang tot geskikte oefenfasiliteite te kry.

AFSTAND (x km)	**GETAL ATLETE**
$0 \leq x < 5$	3
$5 \leq x < 10$	7
$10 \leq x < 15$	20
$15 \leq x < 20$	12
$20 \leq x < 25$	5
$25 \leq x < 30$	3

2.1 Voltooi die kumulatiewefrekwensie-kolom wat in die tabel in die ANTWOORDEBOEK verskaf word. (2)

2.2 Op die rooster wat in die ANTWOORDEBOEK verskaf word, teken 'n kumulatiewefrekwensie-kromme (ogief) om die data hierbo te verteenwoordig. (3)

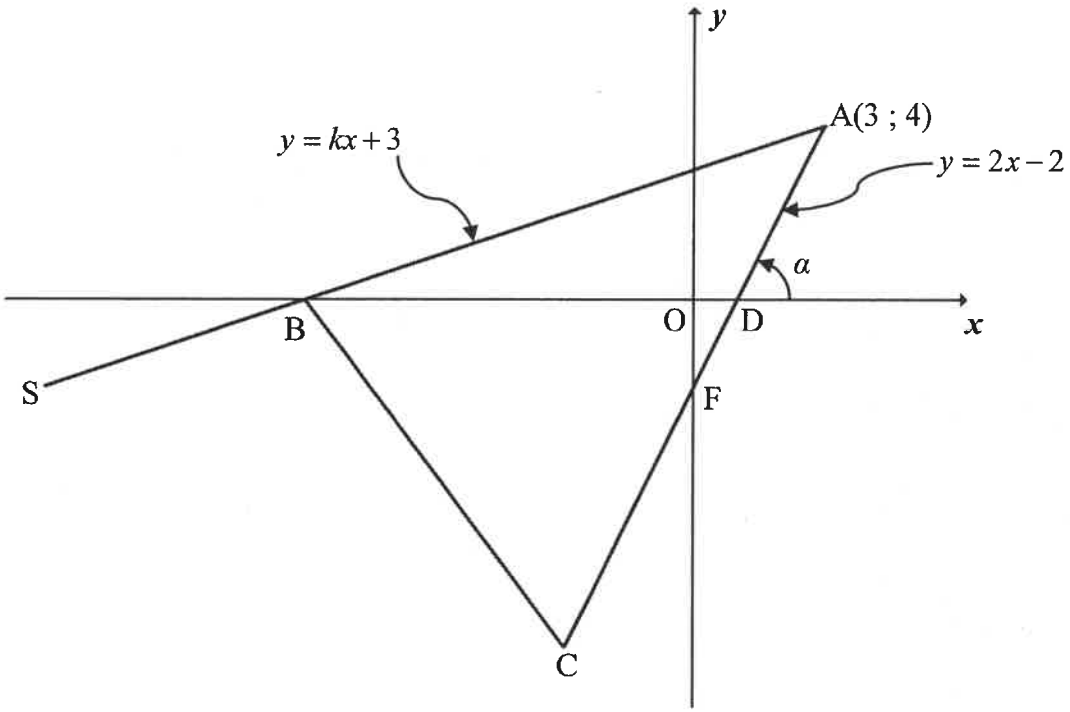
2.3 Bereken die interkwartielvariasiewydte (IKR) van die data hierbo. (2)

2.4 Die gesinne van 4 van die atlete hierbo wat tussen 15 en 20 km vanaf 'n geskikte oefenfasiliteit woon, besluit om 10 kilometer nader aan die fasiliteit te trek. In watter interval sal die getal atlete toeneem? (1)

2.5 Bereken die geskatte gemiddelde afstand wat die vyftig atlete moet ry nadat die 4 gesinne 10 kilometer nader aan die fasiliteit getrek het. Toon ALLE berekeninge duidelik. (3)
[11]

VRAAG 3

In die diagram is A(3 ; 4), B en C hoekpunte van ΔABC. AB is verleng na S. D en F is onderskeidelik die x- en y-afsnitte van AC. F is die middelpunt van AC en die inklinasiehoek van AC is α. Die vergelyking van AB is $y = kx + 3$ en die vergelyking van AC is $y = 2x - 2$.



3.1 Toon dat $k = \frac{1}{3}$. (1)

3.2 Bereken die koördinate van B, die x-afsnit van lyn AS. (2)

3.3 Bereken die koördinate van C. (4)

3.4 Bepaal die vergelyking van die lyn parallel aan BC en wat deur S(–15 ; –2) gaan. Skryf jou antwoord in die vorm $y = mx + c$. (5)

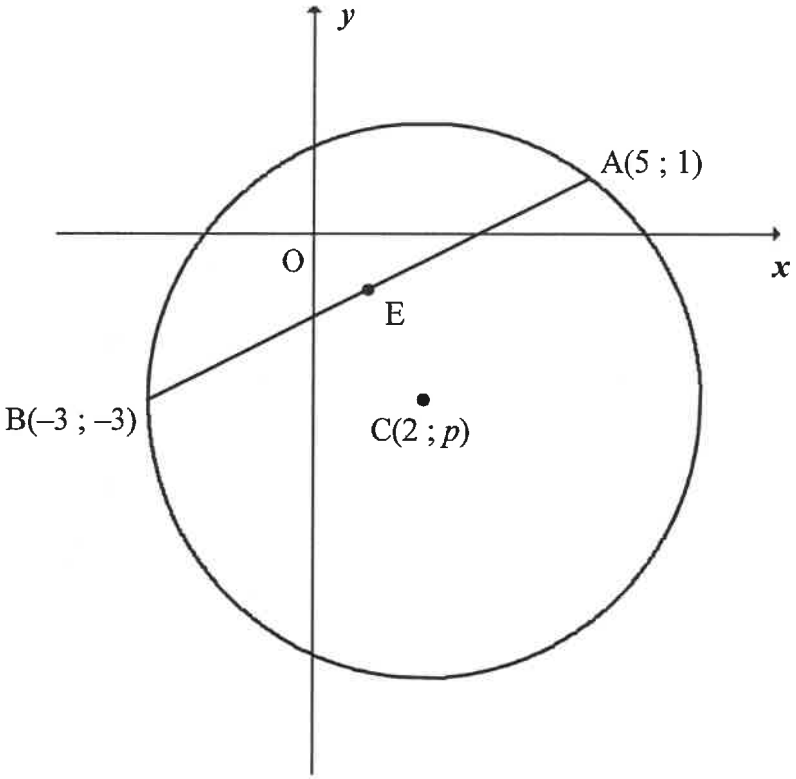
3.5 Bereken die grootte van $B\hat{A}C$. (5)

3.6 As dit verder gegee word dat AC se lengte $6\sqrt{5}$ eenhede is, bereken die waarde van $\frac{\text{Area van } \Delta ABD}{\text{Area van } \Delta ASC}$ (5)

[22]

VRAAG 4

In die diagram is die sirkel met middelpunt C(2 ; p) geteken. A(5 ; 1) en B$(-3;-3)$ is punte op die sirkel. E is die middelpunt van AB.



4.1 Bereken die koördinate van E, die middelpunt van AB. (2)

4.2 Bereken die lengte van AB. Laat jou antwoord in wortelvorm. (1)

4.3 Bepaal die vergelyking van die loodregte lyn wat AB halveer in die vorm $y = mx + c$. (4)

4.4 Toon dat $p = -3$. (1)

4.5 Toon, met berekeninge, dat die vergelyking van die sirkel $x^2 + y^2 - 4x + 6y - 12 = 0$ is. (4)

4.6 Bereken die waardes van t waarvoor die reguitlyn $y = tx + 8$ nie die sirkel sal sny nie. (6)

[18]

VRAAG 5

5.1 Indien $\sin 40° = p$, skryf ELK van die volgende in terme van p.

5.1.1 $\sin 220°$ (2)

5.1.2 $\cos^2 50°$ (2)

5.1.3 $\cos(-80°)$ (3)

5.2 Gegee: $\tan x(1-\cos^2 x)+\cos^2 x = \dfrac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$

5.2.1 Bewys die identiteit hierbo. (5)

5.2.2 Vir watter waardes van x, in die interval $x \in [-180° ; 180°]$, sal die identiteit ongedefinieerd wees? (3)

5.3 Gegee die uitdrukking: $\dfrac{\sin 150° + \cos^2 x - 1}{2}$

5.3.1 **Sonder die gebruik van 'n sakrekenaar**, vereenvoudig die uitdrukking hierbo gegee tot 'n enkele trigonometriese term in terme van $\cos 2x$. (6)

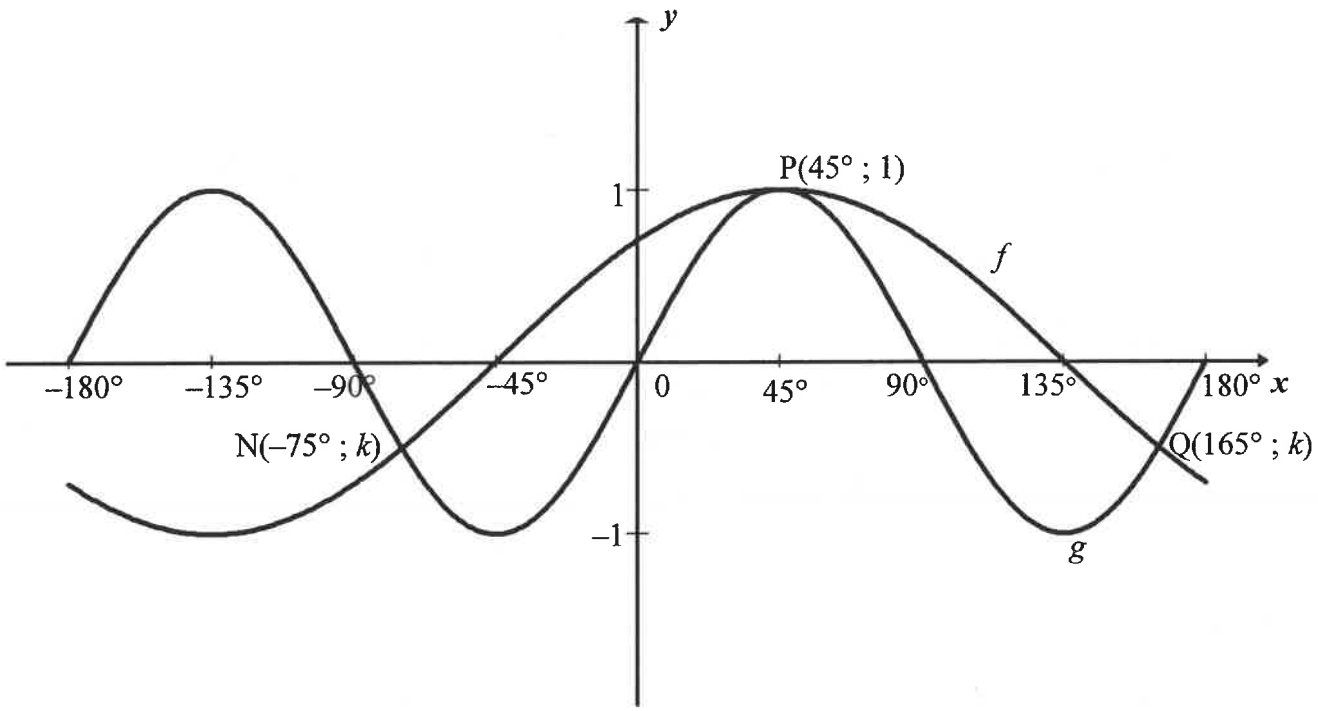
5.3.2 Bepaal gevolglik die algemene oplossing van

$$\frac{\sin 150° + \cos^2 x - 1}{2} = \frac{1}{25}$$ (5)

[26]

VRAAG 6

In die diagram is die grafieke van $f(x) = \cos(x + a)$ en $g(x) = \sin 2x$ vir die interval $x \in [-180°;180°]$ geskets. Die grafieke sny by N(–75° ; k), P(45° ; 1) en Q$(165°;k)$. P is ook 'n draaipunt van albei grafieke.



6.1 Skryf die periode van f neer. (1)

6.2 Skryf die amplitude van g neer. (1)

6.3 Skryf die waarde van a neer. (1)

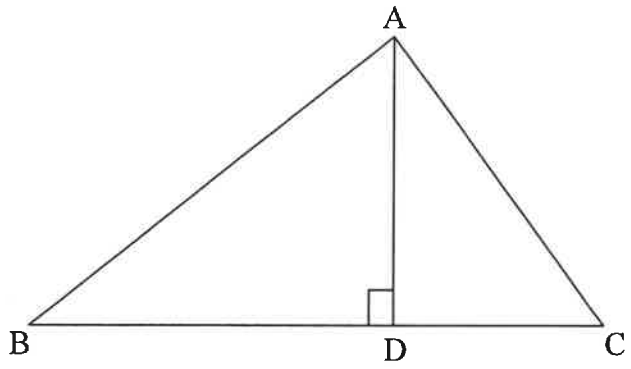
6.4 Bereken die waarde van k, die y-koördinaat van N en Q, **sonder die gebruik van 'n sakrekenaar**. (2)

6.5 Bereken die waarde van x as $g(x + 60°) = f(x + 60°)$ en $x \in [-45° ; 0°]$. (1)

6.6 **Sonder die gebruik van 'n sakrekenaar**, bepaal die aantal oplossings wat die vergelyking $\sqrt{2}\sin 2x = \sin x + \cos x$ in die interval $x \in [-90° ; 90°]$ het. Toon ALLE berekeninge duidelik. (4)
[10]

VRAAG 7

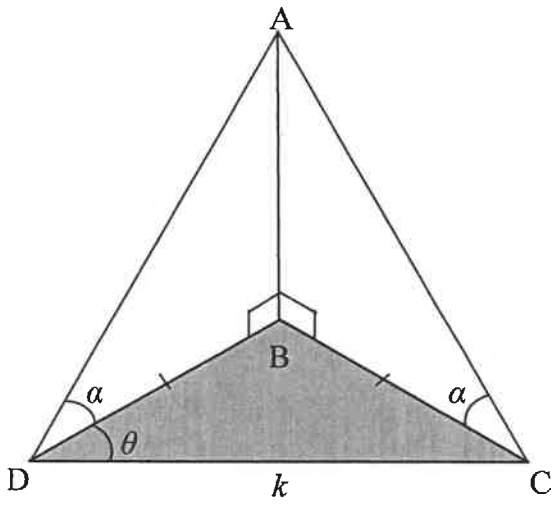
7.1 ΔABC is in die diagram hieronder geskets. AD is sodanig getrek dat $AD \perp BC$.



7.1.1 Gebruik die diagram hierbo om AD in terme van $\sin\hat{B}$ te bepaal. (2)

7.1.2 Bewys vervolgens dat die oppervlakte van $\Delta ABC = \frac{1}{2}(BC)(AB)\sin\hat{B}$ (1)

7.2 In die diagram lê punte B, C en D in dieselfde horisontale vlak.
$A\hat{D}B = A\hat{C}B = \alpha$, $C\hat{D}B = \theta$ en DC = k eenhede. BD = BC.



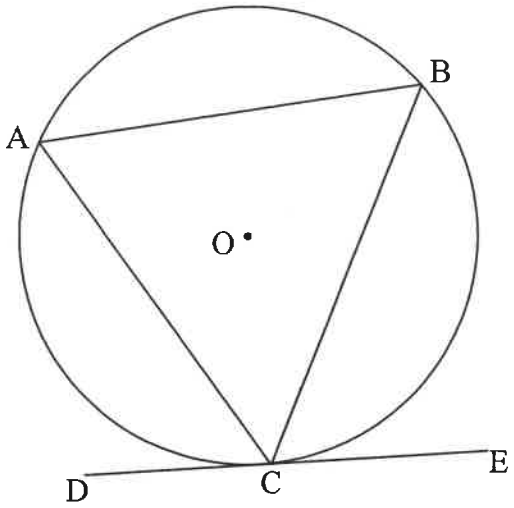
7.2.1 Bewys dat AD = AC (2)

7.2.2 Bewys dat $BD = \frac{k}{2\cos\theta}$ (3)

7.2.3 Bepaal die oppervlakte van ΔBCD in terme van k en as 'n enkele trigonometriese verhouding van θ. (3)
[11]

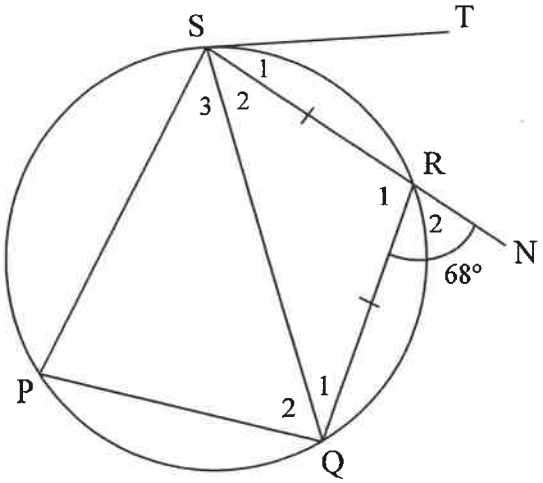
VRAAG 8

8.1 In die diagram is koorde AB, BC en AC getrek in die sirkel met middelpunt O. DCE is 'n raaklyn aan die sirkel by C.



Bewys die stelling wat sê dat die hoek tussen die raaklyn aan 'n sirkel en die koord getrek vanuit die raakpunt, gelyk is aan die hoek in die teenoorstaande segment, m.a.w. $B\hat{C}E = \hat{A}$. (5)

8.2 In die diagram is PQRS 'n koordevierhoek met RQ = RS. ST is 'n raaklyn aan die sirkel by S. SR is na N verleng. $\hat{R}_2 = 68°$.



Bepaal, met redes, die grootte van:

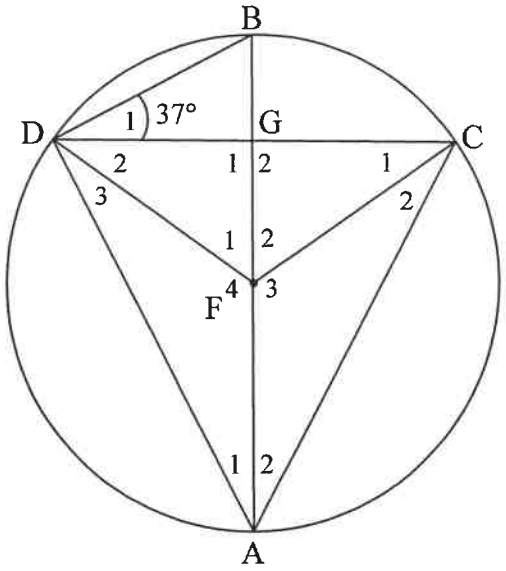
8.2.1 $\hat{P}$ (2)

8.2.2 $\hat{Q}_1$ (2)

8.2.3 $\hat{S}_1$ (2)
[11]

VRAAG 9

In die diagram is AB 'n middellyn van die sirkel met middelpunt F. AB en CD sny by G. FD en FC is getrek. BA halveer $C\hat{A}D$ en $\hat{D}_1 = 37°$.



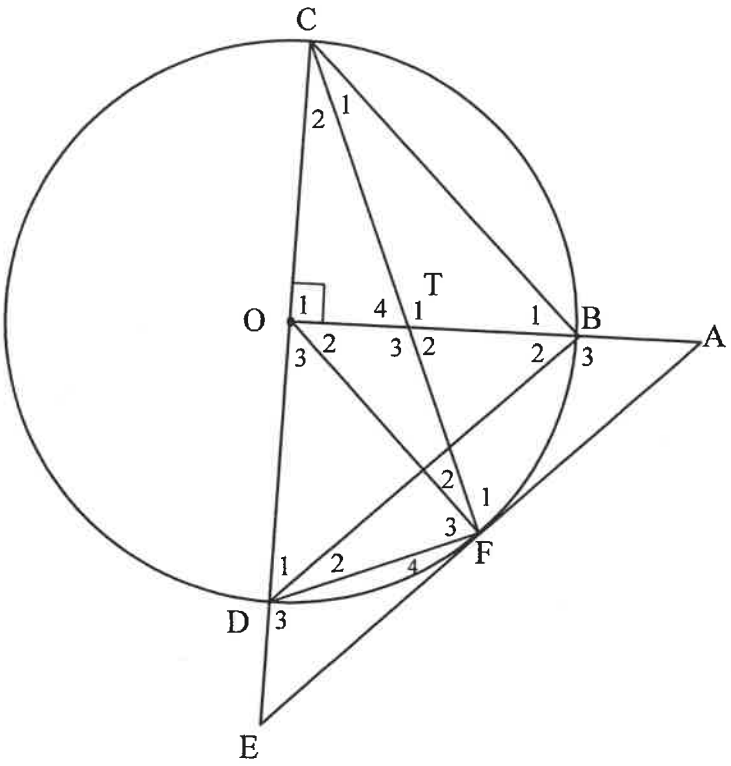
9.1 Bepaal, met redes, enige drie ander hoeke gelyk aan $\hat{D}_1$. (4)

9.2 Toon dat DG = GC. (4)

9.3 Indien dit verder gegee word dat die radius van die sirkel 20 eenhede is, bereken die lengte van BG. (4)
[12]

VRAAG 10

In die diagram is COD die middellyn van die sirkel met middelpunt O. EA is 'n raaklyn aan die sirkel by F. AO ⊥ CE. Middellyn COD is verleng om die raaklyn aan die sirkel by E te sny. OB is verleng en sny die raaklyn aan die sirkel by A. CF sny OB by T. CB, BD, OF en FD is getrek.



Bewys, met redes, dat:

10.1 TODF 'n koordevierhoek is (4)

10.2 $\hat{D}_3 = \hat{T}_1$ (3)

10.3 $\Delta TFO \,|||\, \Delta DFE$ (5)

10.4 Indien $\hat{B}_2 = \hat{E}$, bewys dat DB || EA. (2)

10.5 Bewys dat $DO = \dfrac{TO.FE}{AB}$ (5)
[19]

TOTAAL: 150

INLIGTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

InΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta\text{ABC} = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$\text{P(A)} = \frac{n(\text{A})}{n(\text{S})} \qquad \text{P(A of B)} = \text{P(A)} + \text{P(B)} - \text{P(A en B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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Candidate Barcode label/Stafieskodeplakker

SC/NSC Answer Book
SS/NSS-antwoordeboek

Senior Certificate/National Senior Certificate/*Senior Sertifikaat/Nasionale Senior Sertifikaat* (Grade 12/*Graad 12*) 2024

CENTRE NUMBER / *SENTRUMNOMMER*	
EXAMINATION NUMBER / *EKSAMENNOMMER*	
DATE / *DATUM*	
BOOK NUMBER / *BOEKNOMMER*	OF / *VAN* BOOKS / *BOEKE*
SUBJECT CODE / *VAKKODE*	
PAPER NUMBER / *VRAESTELNOMMER*	2
SUBJECT NAME / *VAKNAAM*	MATHEMATICS/*WISKUNDE*

MARKER/*NASIENER*				MODERATOR'S INITIALS IN RELEVANT BLOCK / *MODERATOR SE VOORLETTERS IN RELEVANTE BLOKKIE*													
Question / *Vraag*	Marks / *Punte*			Marker's Code & Initials / *Nasiener se Kode & Voorletters*	Marks / *Punte*			SM	Marks / *Punte*			DCM / *AHN*	Marks / *Punte*			CM / *HN*	Marks / *Punte*			IM	Marks / *Punte*			EM
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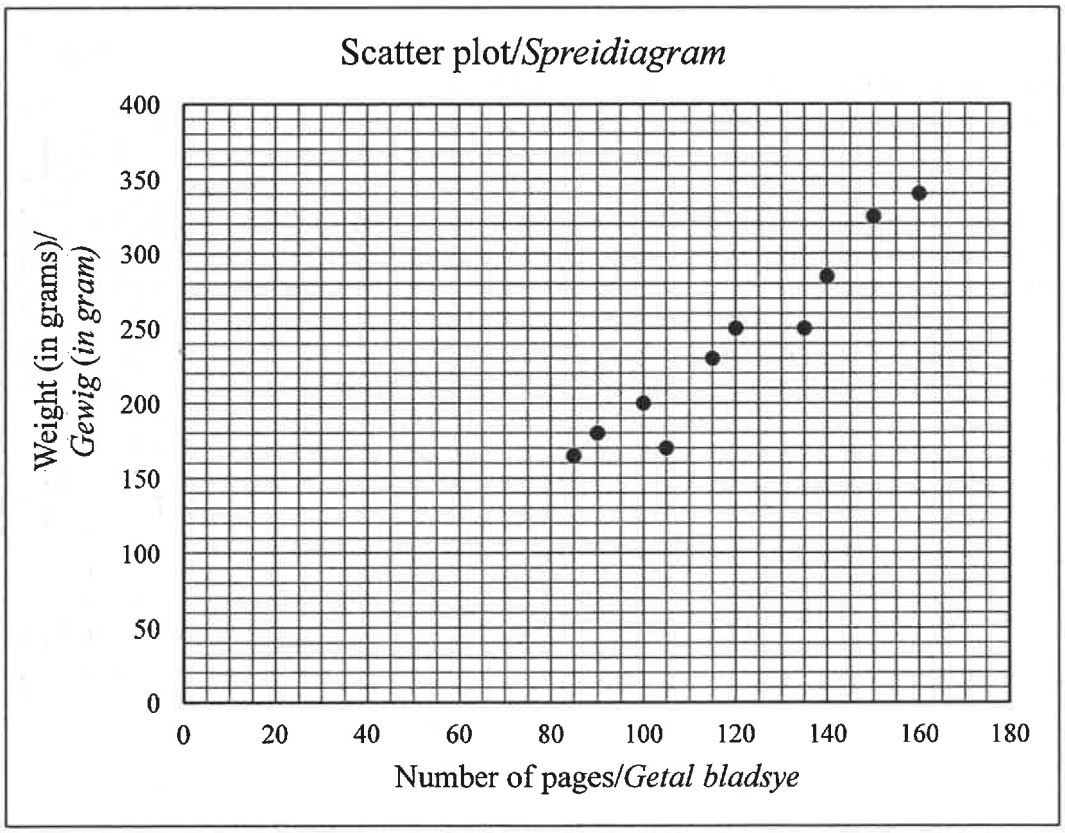
CONTROLLED AND CERTIFIED CORRECT (SURNAME AND INITIALS OF EA) / *GEKONTROLEER EN AS KORREK GESERTIFISEER (VAN EN VOORLETTERS VAN EA)*		READ INSTRUCTIONS ON THE NEXT PAGE. / *LEES INSTRUKSIES OP VOLGENDE BLADSY.*

This answer book consists of 23 pages./*Hierdie antwoordeboek bestaan uit 23 bladsye.*

PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	*VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG*
1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.	1. *Skryf jou eksamennommer en sentrumnommer duidelik in die ruimtes soos verskaf en plak jou stafieskodeplakker in die ruimte soos verskaf.*
2. Remember that your own name (or the name of your school) may not appear anywhere on or in this answer book.	2. *Onthou dat jou eie naam (of die naam van jou skool) nie op of in hierdie antwoordeboek mag voorkom nie.*
3. Answer ALL questions in the spaces provided.	3. *Beantwoord ALLE vrae in die ruimtes wat voorsien is.*
4. No pages may be torn from this answer book.	4. *Geen bladsye mag uit hierdie antwoordeboek geskeur word nie.*
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	5. *Lees die instruksies wat op jou eksamenrooster gedruk is, sorgvuldig deur, asook enige ander instruksies wat op elke eksamenvraestel gegee word.*
6. Candidates may not retain an answer book or remove it from the examination room.	6. *Geen antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word nie.*
7. Answers must be written in black/blue ink as distinctly as possible. Do not write in the margins.	7. *Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.*
8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.	8. *Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.*
9. If you require additional space for your answers: 9.1 Use the additional space provided at the end of the answer book. 9.2 When answering a question in the additional space, indicate clearly the question number in the column on the LHS. 9.3 Rule off after each answer.	9. *In geval jy bykomende ruimte benodig vir jou antwoorde:* *9.1 Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek voorsien word.* *9.2 As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan.* *9.3 Trek 'n lyn na elke antwoord.*
10. Draw a neat line through any work/rough work that must not be marked.	10. *Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.*

QUESTION/*VRAAG 1*

Number of pages/ *Getal bladsye* (x)	85	150	100	120	90	140	135	105	115	160
Weight (in grams)/ *Gewig (in gram)* (y)	165	325	200	250	180	285	250	170	230	340

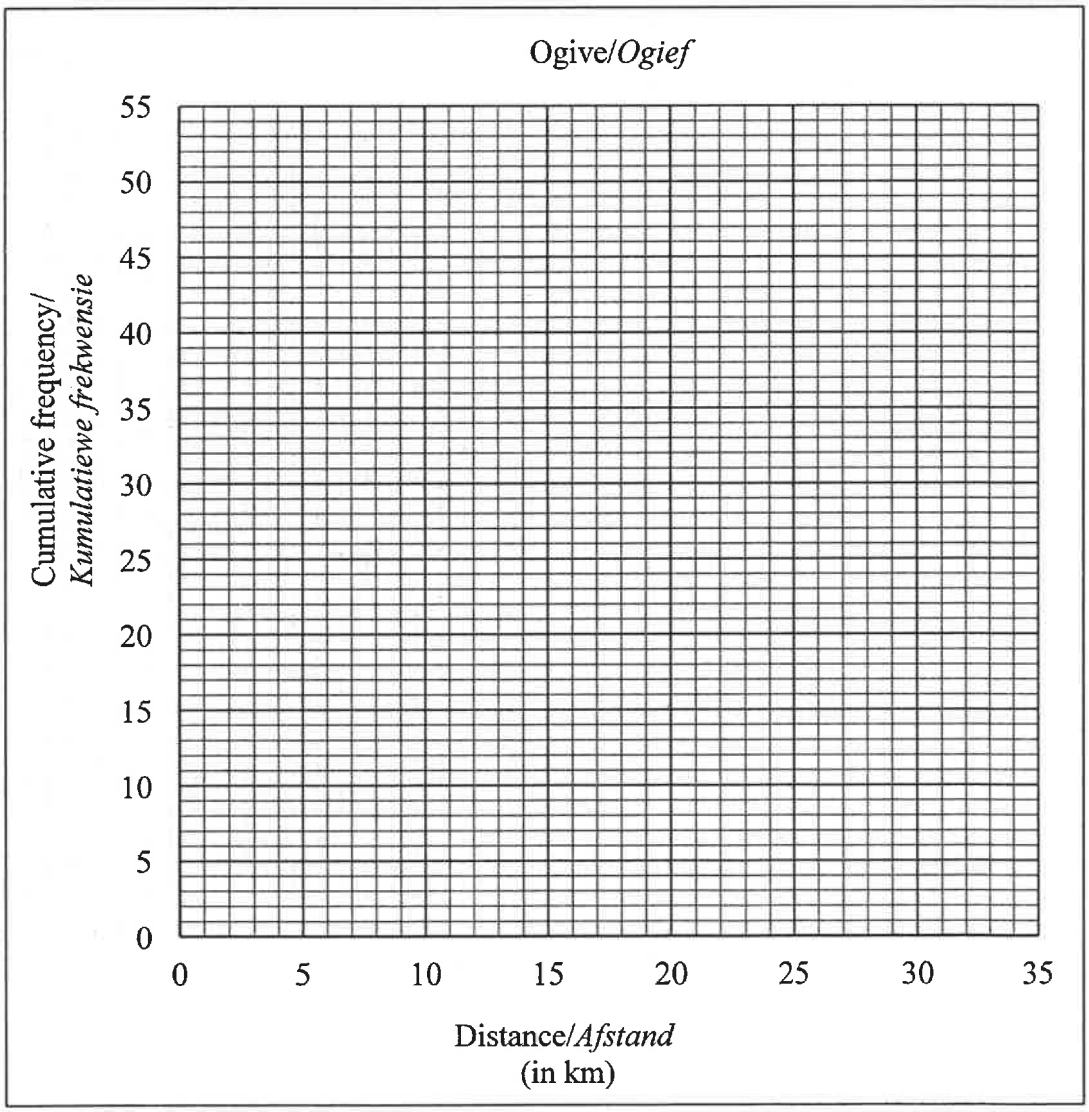
	Solution/*Oplossing*	**Marks *Punte***
1.1		(3)
1.2		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
1.3		(2)
1.4		(3)
		[10]

QUESTION/*VRAAG* 2

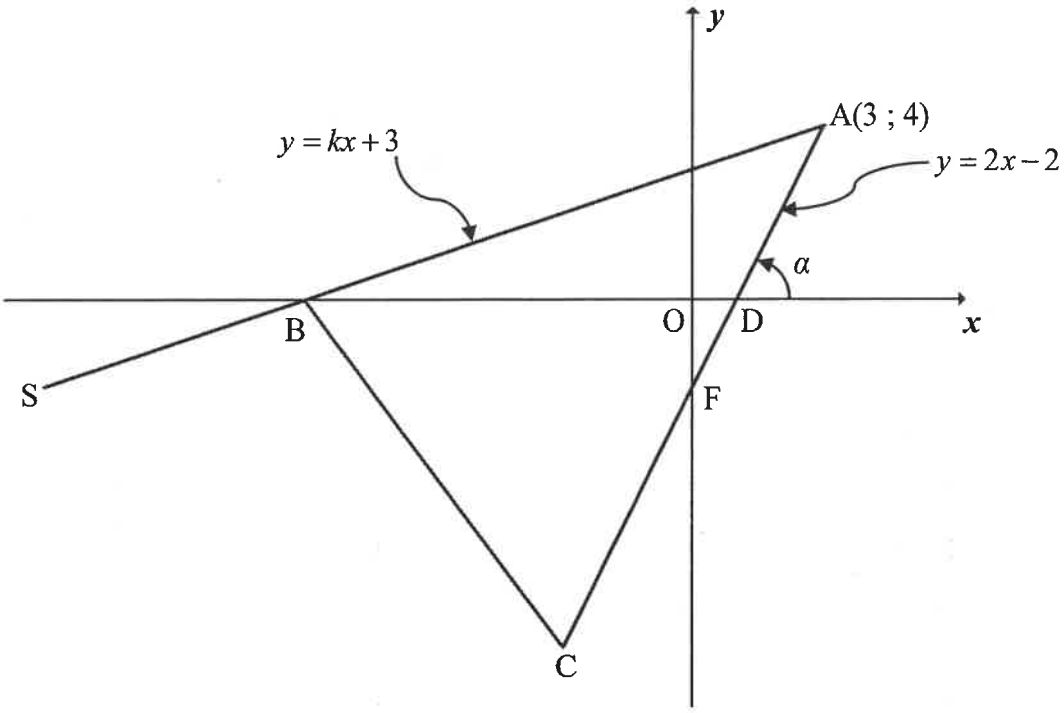
	Solution/*Oplossing*	**Marks** ***Punte***
2.1		(2)
2.2		(3)

Distance/*Afstand* **(*x* km)**	**Number of athletes/** ***Getal atlete***	**Cumulative frequency/** ***Kumulatiewe frekwensie***
$0 \leq x < 5$	3	
$5 \leq x < 10$	7	
$10 \leq x < 15$	20	
$15 \leq x < 20$	12	
$20 \leq x < 25$	5	
$25 \leq x < 30$	3	



	Solution/*Oplossing*	**Marks** ***Punte***
2.3		(2)
2.4		(1)
2.5		(3)
		[11]

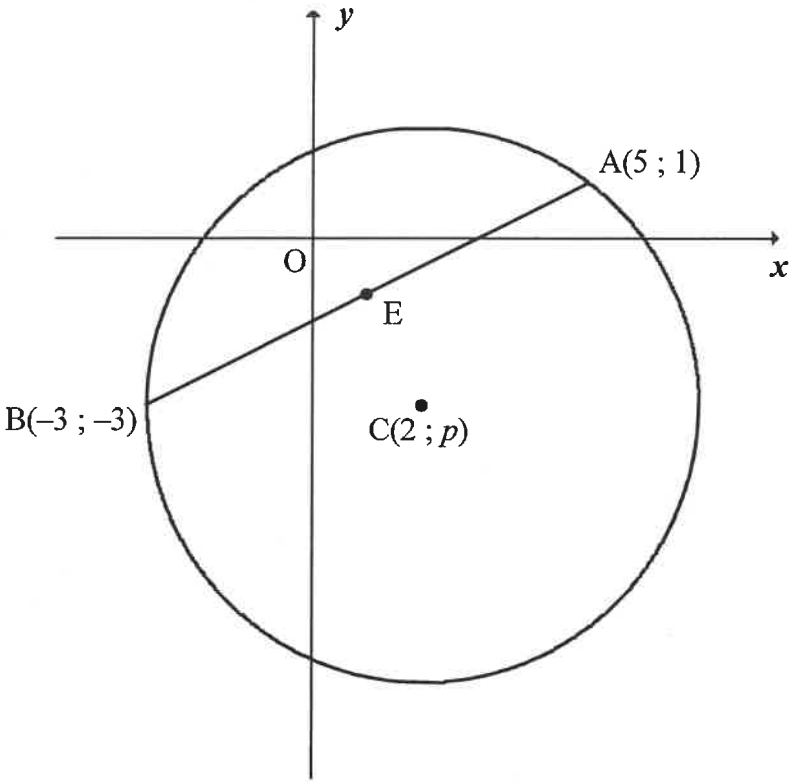
QUESTION/*VRAAG* 3



	Solution/*Oplossing*	**Marks** ***Punte***
3.1		(1)
3.2		(2)
3.3		(4)

	Solution/*Oplossing*	**Marks** ***Punte***
3.4		(5)
3.5		(5)
3.6		(5)
		[22]

QUESTION/*VRAAG* 4



	Solution/*Oplossing*	**Marks** ***Punte***
4.1		(2)
4.2		(1)
4.3		(4)

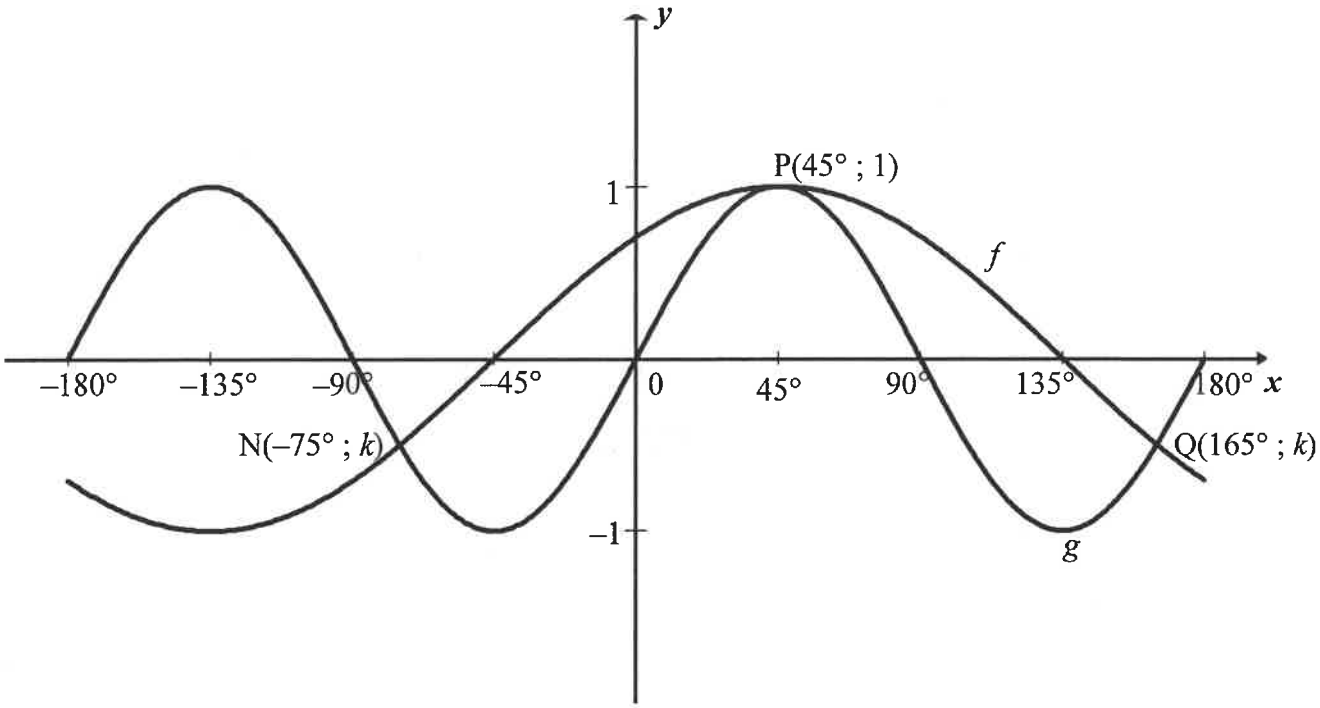
	Solution/*Oplossing*	**Marks** ***Punte***
4.4		(1)
4.5		(4)
4.6		(6)
		[18]

QUESTION/*VRAAG* 5

	Solution/*Oplossing*	**Marks** ***Punte***
5.1.1		(2)
5.1.2		(2)
5.1.3		(3)
5.2.1		(5)

	Solution/*Oplossing*	**Marks** ***Punte***
5.2.2		(3)
5.3.1		(6)
5.3.2		(5)
		[26]

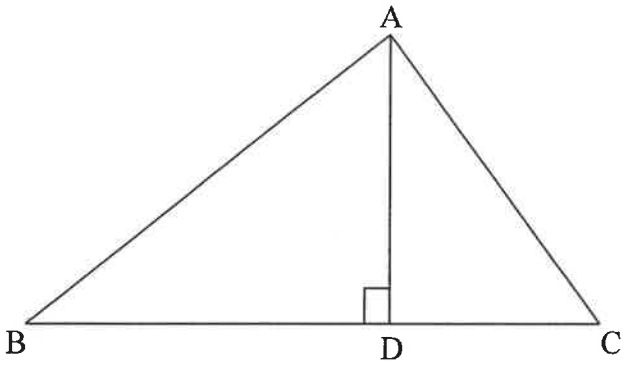
QUESTION/*VRAAG* 6



	Solution/*Oplossing*	**Marks** ***Punte***
6.1		(1)
6.2		(1)
6.3		(1)
6.4		(2)

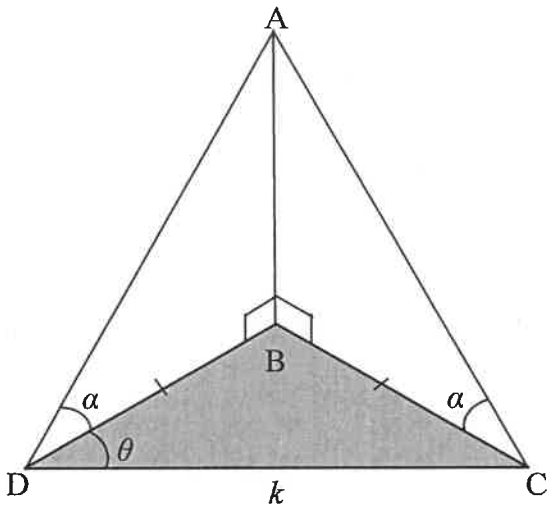
	Solution/*Oplossing*	**Marks** ***Punte***
6.5		(1)
6.6		(4)
		[10]

QUESTION/*VRAAG* 7



	Solution/*Oplossing*	**Marks** ***Punte***
7.1.1		(2)
7.1.2		(1)

7.2

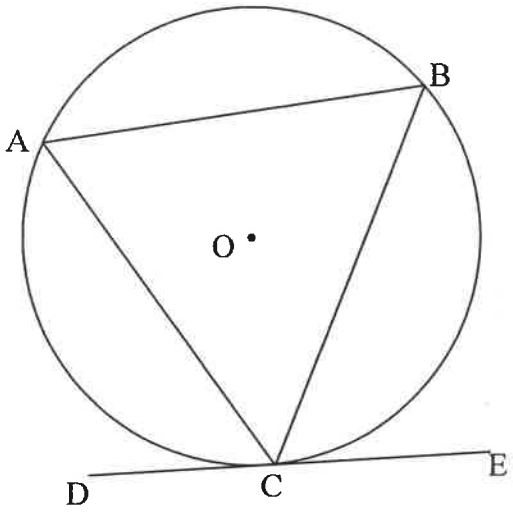


	Solution/*Oplossing*	**Marks** ***Punte***
7.2.1		(2)
7.2.2		(3)
7.2.3		(3)
		[11]

Provide reasons for your statements in QUESTIONS 8, 9 and 10.
Verskaf redes vir jou bewerings in VRAAG 8, 9 en 10.

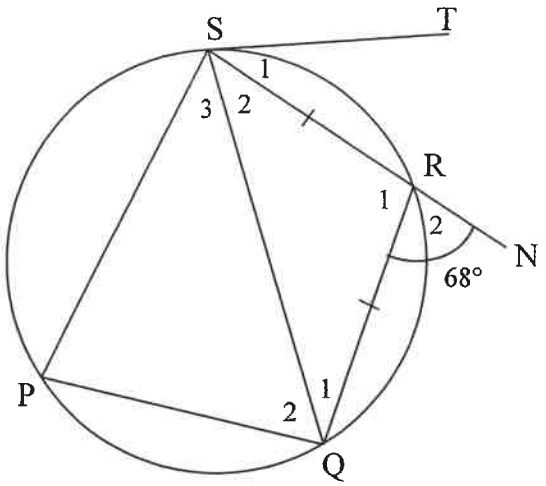
QUESTION/*VRAAG* 8

8.1



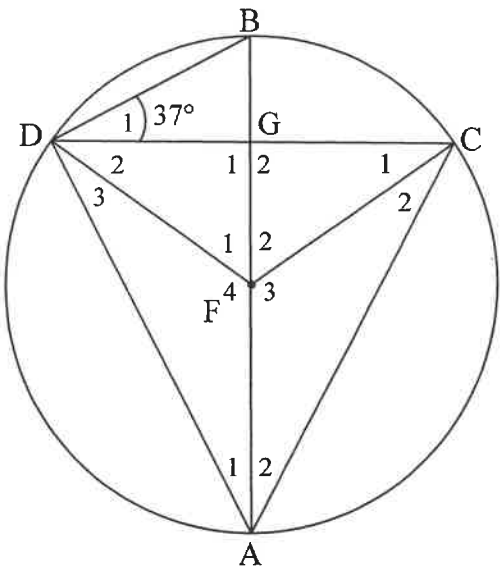
	Solution/*Oplossing*	**Marks** ***Punte***
8.1		(5)

8.2



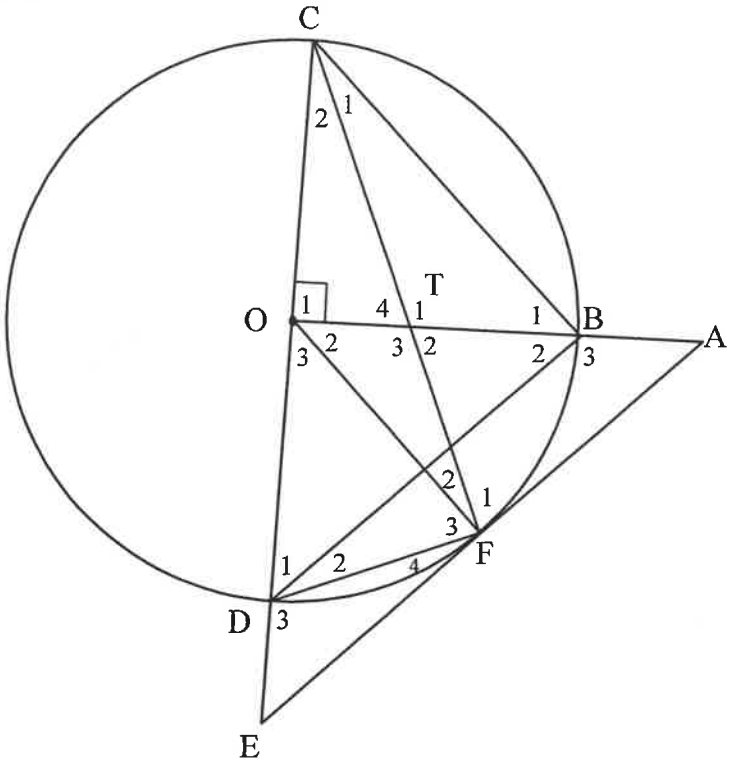
	Solution/*Oplossing*	**Marks** ***Punte***
8.2.1		(2)
8.2.2		(2)
8.2.3		(2)
		[11]

QUESTION/*VRAAG* 9



	Solution/*Oplossing*	**Marks** ***Punte***
9.1		(4)
9.2		(4)
9.3		(4)
		[12]

QUESTION/*VRAAG* 10



	Solution/*Oplossing*	**Marks** ***Punte***
10.1		(4)
10.2		(3)

	Solution/*Oplossing*	**Marks** ***Punte***
10.3		(5)
10.4		(2)
10.5		(5)
		[19]

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

TOTAL/*TOTAAL*: 150

RE-MARK/RE-CHECK *HERMERK/HERSIEN*				
Question *Vraag*	Marks *Punte*			Initials *Voorletters*
TOTAL *TOTAAL*				
HASH TOTAL *KAF-TOTAAL*				

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SENIOR CERTIFICATE EXAMINATIONS/
SENIORSERTIFIKAAT-EKSAMEN
NATIONAL SENIOR CERTIFICATE EXAMINATIONS/
NASIONALE SENIORSERTIFIKAAT-EKSAMEN

MATHEMATICS P2/*WISKUNDE V2*

MARKING GUIDELINES/*NASIENRIGLYNE*

MAY/JUNE/*MEI/JUNIE* 2024

MARKS: 150
PUNTE: 150

These marking guidelines consist of 26 pages./
Hierdie nasienriglyne bestaan uit 26 bladsye.

NOTE:

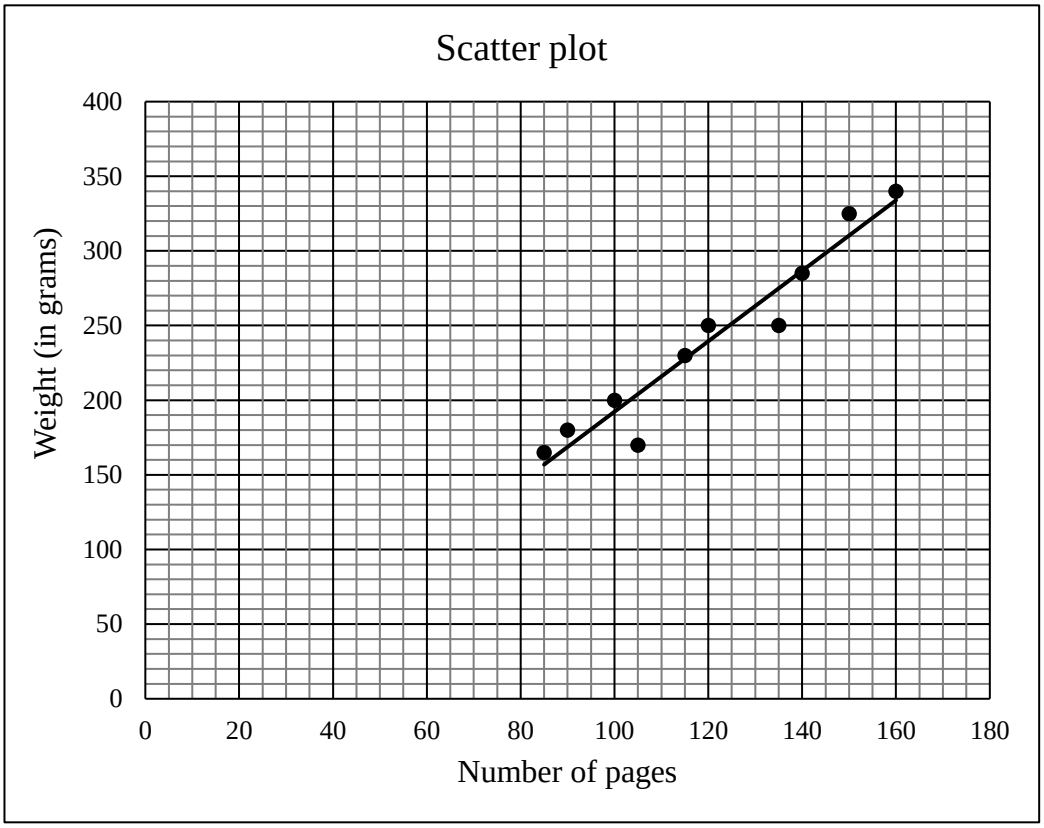
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and did not redo the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the Marking Guidelines. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord op 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die Nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde/waardes om 'n probleem op te los, word NIE toegelaat nie.*

GEOMETRY	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering **('n Punt vir 'n bewering is onafhanklik van die rede)**
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede **('n Punt word slegs vir die rede toegeken as die bewering korrek is)**
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

QUESTION/*VRAAG* 1

1.1	$a = -43,72$ $b = 2,36$ $y = -43,72 + 2,36x$	✓ $a = -43,72$ ✓ $b = 2,36$ ✓ equation (3)
1.2		✓ any correct two points ✓ straight line joining the points for $x \in [85 ; 160]$ (2)
1.3	$y = -43,72 + 2,36(110)$ $y = 215,88$ **OR** y = 215,90 (calculator)	✓ substitution ✓ answer (2) ✓✓ answer (2)
1.4	$y = -43,72 + 2,36(130)$ $y = 263,08$ Percentage increase in weight $= \frac{263,08 - 215,88}{215,88} \times 100$ $= 21,86\%$ **OR** $y = 263,08$ Percentage $= \frac{263,08}{215,88} \times 100$ $= 121,86\,\%$ Percentage increase in weight $= 121,86 - 100 = 21,86$	✓ y -value ✓ difference between y-values ✓ +ve answer (3) ✓ y -value ✓ difference between % ✓ +ve answer (3)
		[10]

QUESTION/*VRAAG* 2

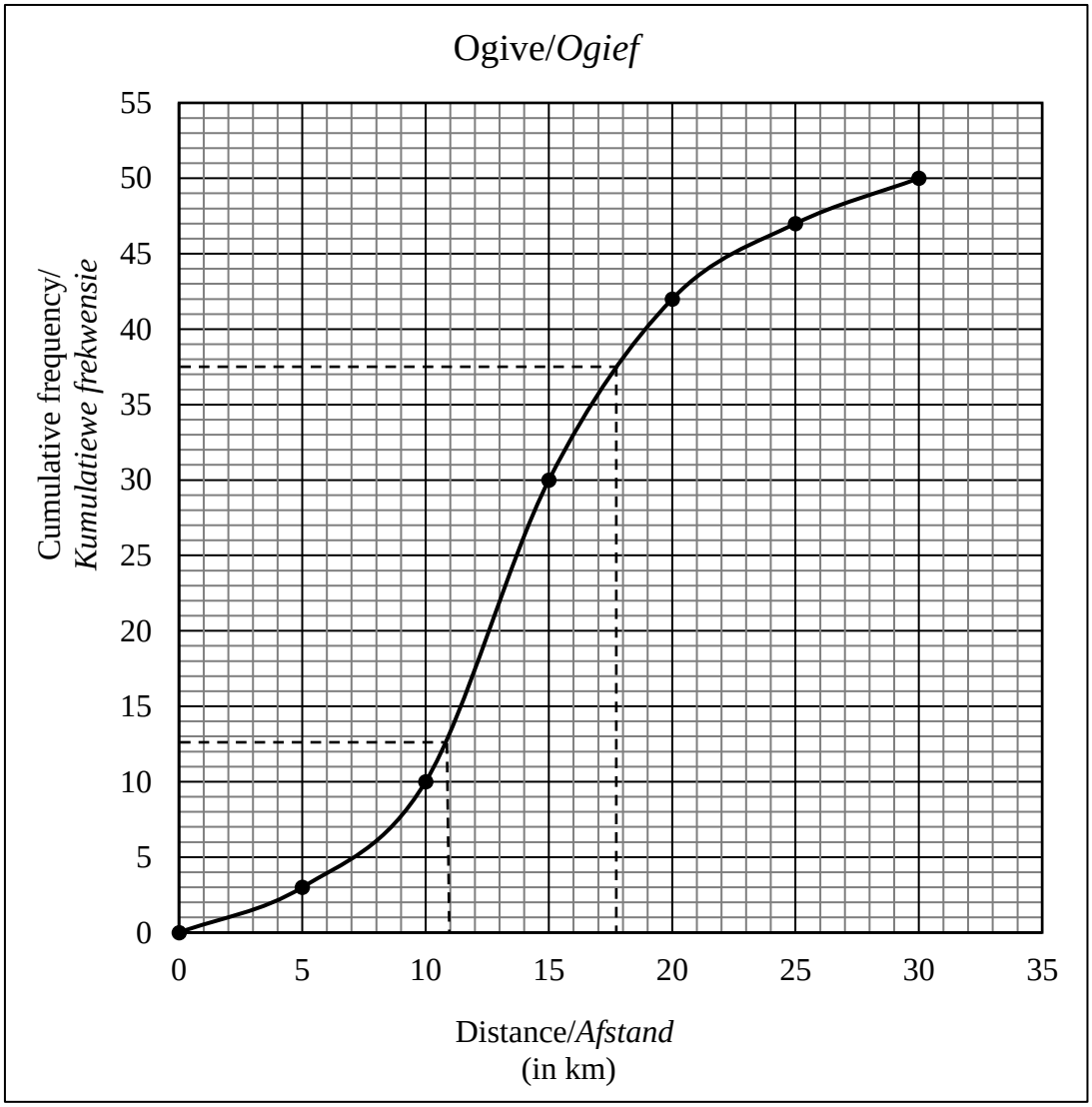
2.1

Distance (x km)	**Frequency**	**Cumulative frequency**
$0 \le x < 5$	3	3
$5 \le x < 10$	7	10
$10 \le x < 15$	20	30
$15 \le x < 20$	12	42
$20 \le x < 25$	5	47
$25 \le x < 30$	3	50

✓ 10
✓ all values correct

(2)

2.2



✓ grounding
✓ plotting a min of 3 points (cf at upper limits)
✓ smooth, increasing curve

(3)

2.3

$Q_3 = 17{,}8$
$Q_1 = 11$

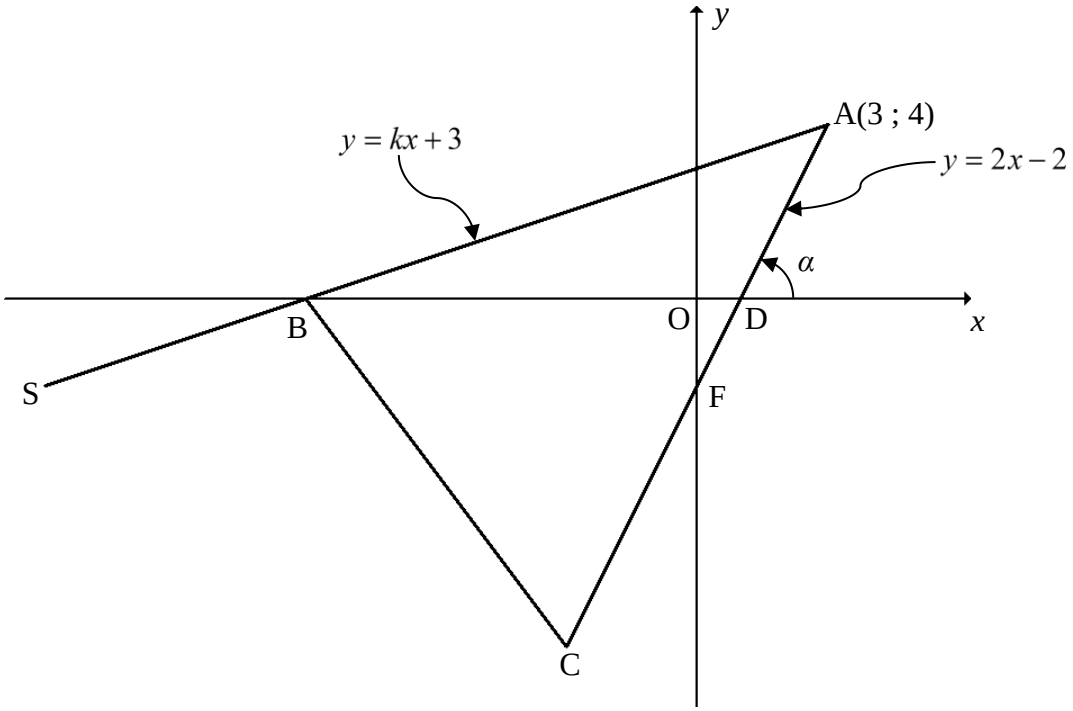
IQR = 6,8

✓ Q_3 (accept between 17-19) and Q_1 (accept between 10-12,5)
✓ answer (accept 5-9)

(2)

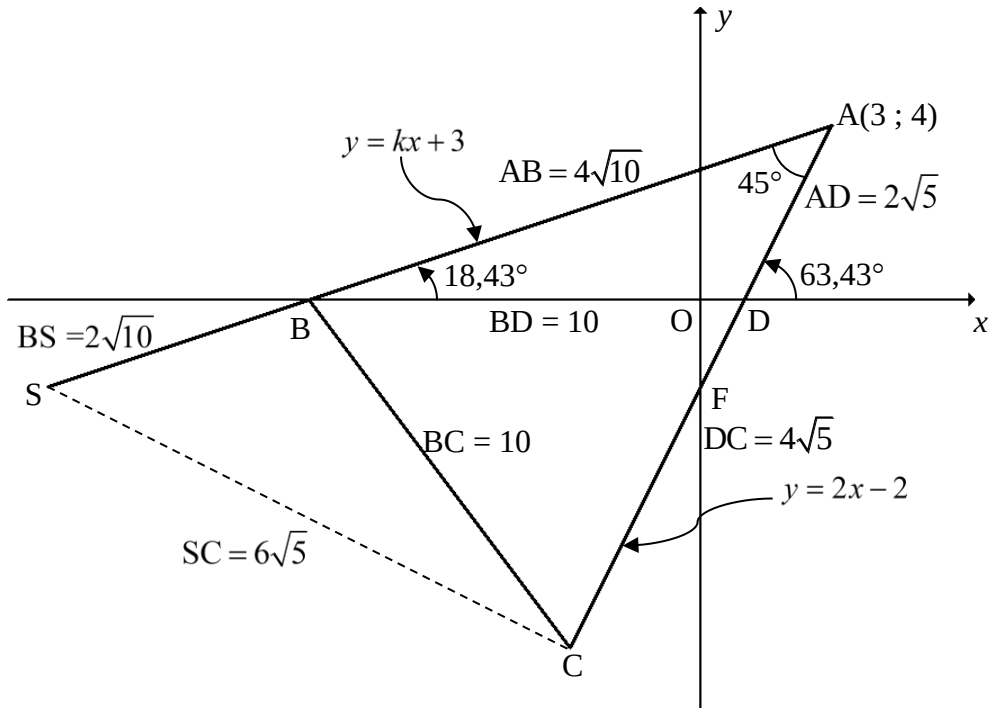
2.4	$5 \le x < 10$	✓ $5 \le x < 10$ (1)
2.5	Estimated mean $= \dfrac{2,5(3)+7,5(11)+12,5(20)+17,5(8)+22,5(5)+27,5(3)}{50}$ $= \dfrac{675}{50}$ $= 13,5$ km	✓ new frequencies ✓ $\sum fx$ ✓ answer (3)
		[11]

QUESTION/*VRAAG* 3



3.1	$y = kx + 3$ $4 = k(3) + 3$ $3k = 1$ $\therefore k = \frac{1}{3}$ **OR** *y*-intercept of AB: (0 ; 3) $m_{AB} = \frac{4-3}{3-0}$ $= \frac{1}{3}$ $\therefore k = \frac{1}{3}$	✓ substitution (3 ; 4) (1) ✓ substitution (0 ; 3) (1)
3.2	$0 = \frac{1}{3}x + 3$ $-3 = \frac{1}{3}x$ $x = -9$ $B(-9\ ;\ 0)$	✓ $y = 0$ ✓ answer (2)

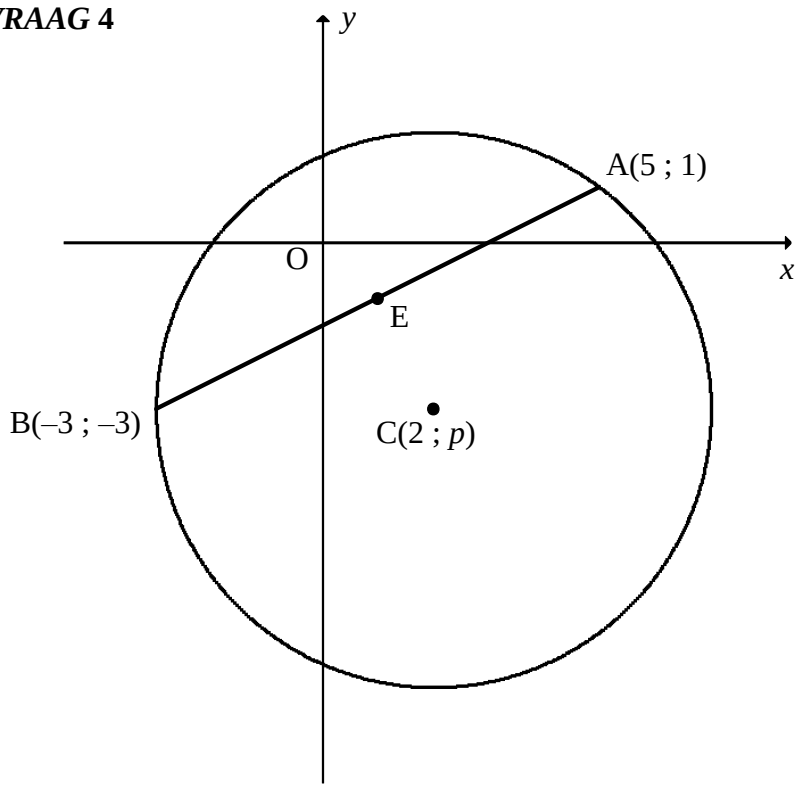
3.3	$\text{F}(0\,;-2)$ $\text{F}\left(\frac{x+3}{2}\,;\,\frac{y+4}{2}\right)$ $\frac{x+3}{2}=0 \qquad \frac{y+4}{2}=-2$ $x=-3 \qquad y=-8$ $\text{C}(-3\,;-8)$ **OR** by translation $\text{F}(0;-2)$ $\text{A}\to\text{F}\,(x;y)\to(x-3;y-6)$ $\text{F}\to\text{C}(0;-2)\to(0-3;-2-6)=(-3;-8)$	✓ $\text{F}(0\,;-2)$ ✓ $\frac{x+3}{2}=0\;;\;\frac{y+4}{2}=-2$ ✓ *x*-value ✓ *y*-value (4) ✓ $\text{F}(0;-2)$ ✓ $(x-3\,;y-6)$ ✓ *x*-value ✓ *y*-value (4)
3.4	$m_{\text{BC}}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{\text{BC}}=\frac{-8-0}{-3-(-9)}$ $m_{\text{BC}}=-\frac{4}{3}$ $y=-\frac{4}{3}x+c$ $(-2)=-\frac{4}{3}(-15)+c$ $\text{c}=-22$ $y=-\frac{4}{3}x-22$ **OR** $m_{\text{BC}}=\frac{0-(-8)}{-9-(-3)}$ OR $m_{\text{BC}}=\frac{-8-0}{-3-(-9)}$ $m_{\text{BC}}=-\frac{4}{3}$ $y-y_1=-\frac{4}{3}(x-x_1)$ $y-(-2)=-\frac{4}{3}(x-(-15))$ $y+2=-\frac{4}{3}x-20$ $y=-\frac{4}{3}x-22$	✓ substitution of B and C into the gradient formula ✓ m_{BC} ✓ $m_{\text{line}}=m_{\text{BC}}$ ✓ substitution of $\text{S}(-15;-2)$ ✓ equation (5) ✓ substitution into the gradient formula ✓ m_{BC} ✓ $m_{\text{line}}=m_{\text{BC}}$ ✓ substitution of $\text{S}(-15;-2)$ ✓ equation (5)



3.5	$\tan\alpha = m_{AC} = 2$ $\alpha = 63,43°$ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ $A\hat{B}D = 18,43°$ $B\hat{A}C = \alpha - A\hat{B}D$ $B\hat{A}C = 63,43° - 18,43°$ $B\hat{A}C = 45°$ **OR** $AB = \sqrt{(-9-3)^2 + (0-4)^2}$ $AB = 4\sqrt{10}$ $BD = 10$ $AD = \sqrt{(3-1)^2 + (4-0)^2}$ $AD = 2\sqrt{5}$ $BD^2 = AB^2 + AD^2 - 2AB.AD\cos B\hat{A}C$ $(10)^2 = (4\sqrt{10})^2 + (2\sqrt{5})^2 - 2(4\sqrt{10})(2\sqrt{5})\cos B\hat{A}C$ $\cos B\hat{A}C = \frac{\sqrt{2}}{2}$ $B\hat{A}C = 45°$	✓ $\tan\alpha = m_{AC} = 2$ ✓ $\alpha = 63,43°$ ✓ $\tan A\hat{B}D = m_{AS} = \frac{1}{3}$ ✓ $A\hat{B}D = 18,43°$ ✓ answer (5) ✓ length of AB ✓ calculation of remaining 2 lengths ✓ substitution into cosine-rule ✓ rewriting in terms of $\cos B\hat{A}C$ ✓ answer (5)

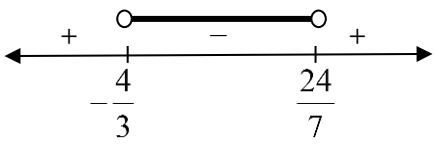
3.6	A(3 ; 4) and S(–15 ; – 2)	
	$AS = \sqrt{(x_A - x_S)^2 + (y_A - y_S)^2}$	
	$AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$	✓ $AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$
	$AS = \sqrt{360} = 6\sqrt{10} = 18,97$	✓ length of AS
	$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \frac{\frac{1}{2}(BD)(\perp h)}{\frac{1}{2}(AS)(AC)\sin B\hat{A}C}$	
	$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \frac{\frac{1}{2}(10)(4)}{\frac{1}{2}(6\sqrt{10})(6\sqrt{5})\sin 45°}$	✓ Area ΔABD ✓ Area ΔASC
	$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \frac{2}{9}$	✓ answer (5)
	OR	
	$AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$	✓ $AS = \sqrt{(3-(-15))^2 + (4-(-2))^2}$
	$AS = \sqrt{360} = 6\sqrt{10} = 18,97$	✓ length of AS
	$AB = \sqrt{(-9-3)^2 + (0-4)^2} = 4\sqrt{10}$	
	$AD = \sqrt{(3-1)^2 + (4-0)^2} = 2\sqrt{5}$	
	$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ASC} = \frac{\frac{1}{2}(AB)(AD)\sin \hat{A}}{\frac{1}{2}(AS)(AC)\sin \hat{A}}$	
	$= \frac{\frac{1}{2}(4\sqrt{10})(2\sqrt{5})\sin \hat{A}}{\frac{1}{2}(6\sqrt{10})(6\sqrt{5})\sin \hat{A}}$	✓ Area ΔABD ✓ Area ΔASC
	$= \frac{2}{9}$	✓ answer (5)
		[22]

QUESTION/*VRAAG* 4



4.1	$\text{E}\left(\frac{5+(-3)}{2};\frac{1+(-3)}{2}\right)$ $\therefore \text{E}(1;-1)$	✓ $x=1$ ✓ $y=-1$ (2)
4.2	$\text{AB}=\sqrt{(x_\text{A}-x_\text{B})^2+(y_\text{A}-y_\text{B})^2}$ $\text{AB}=\sqrt{(5-(-3))^2+(1-(-3))^2}$ $\text{AB}=\sqrt{80}=4\sqrt{5}=8,94\text{ units}$	✓ $\text{AB}=\sqrt{80}=4\sqrt{5}=8,94$ (1)
4.3	$m_\text{AB}=\frac{1-(-3)}{5-(-3)}$ $m_\text{AB}=\frac{1}{2}$ $\therefore m_\text{CE}=-2 \quad [\text{CE}\perp\text{AB}]$ $\text{E}(1;-1)$ $y=-2x+c$ **OR** $y-y_1=-2(x-x_1)$ $(-1)=-2(1)+c$ $y-(-1)=-2(x-1)$ $c=1$ $y=-2x+1$ $y=-2x+1$	✓ $m_\text{AB}=\frac{1}{2}$ ✓ m_CE ✓ substitution of E ✓ equation (4)

4.4	$y=-2x+1$ $p=-2(2)+1$ $p=-3$ **OR** $m_{CE}=-2$ $\frac{p-(-1)}{2-1}=-2$ $p+1=-2$ $p=-3$	✓ substitution of C(2 ; *p*) into ⊥ bisector of AB (1) ✓ substitution of C and E into the gradient formula (1)
4.5	BC = *r* = 5 units $\therefore (x-2)^2+(y+3)^2=25$ $x^2-4x+4+y^2+6y+9=25$ $x^2+y^2-4x+6y-12=0$	✓ BC = *r* = 5 units ✓ $(x-2)^2+(y+3)^2$ ✓ r^2 ✓ $x^2-4x+4+y^2+6y+9=25$ (4)

4.6	$(x-2)^2+(y+3)^2=25$ **OR** $x^2+y^2-4x+6y-12=0$ $y=tx+8$ $(x-2)^2+(tx+8+3)^2=25$ **OR** $x^2+(tx+8)^2-4x+6(tx+8)-12=0$ $x^2-4x+4+t^2x^2+22tx+121-25=0$ **OR** $x^2+t^2x^2+16tx+64-4x+6tx+48-12=0$ $x^2(t^2+1)+x(22t-4)+100=0$ $\Delta<0$ $(22t-4)^2-4(t^2+1)(100)<0$ $484t^2-176t+16-400t^2-400<0$ $84t^2-176t-384<0$ $21t^2-44t-96<0$ $(7t-24)(3t+4)<0$ CV: $\frac{24}{7}$; $-\frac{4}{3}$  $\therefore t\in\left(-\frac{4}{3};\frac{24}{7}\right)$ **OR** $-\frac{4}{3}<t<\frac{24}{7}$	✓ substitution of $y=tx+8$ ✓ standard form ✓ $\Delta<0$ ✓ standard form of Δ ✓ critical values ✓ answer (6)
		[18]

QUESTION/*VRAAG* 5

5.1.1	$\sin 220°$ $= -\sin 40°$ $= -p$	✓ $-\sin 40°$ ✓ answer (2)
5.1.2	$\cos^2 50°$ $= \sin^2 40°$ $= p^2$	✓ $\sin^2 40$ ✓ answer (2)
5.1.3	$\cos(-80°)$ $= \cos 80°$ $= 1 - 2\sin^2 40°$ $= 1 - 2p^2$ **OR** $\cos(-80°)$ $= \cos 80°$ $= \cos(30° + 50°)$ $= \cos 30° \cos 50° - \sin 30° \sin 50°$ $= \frac{\sqrt{3}p}{2} - \frac{\sqrt{1-p^2}}{2}$	✓ $\cos 80°$ ✓ double angle ✓ answer (3) ✓ $\cos 80°$ ✓ expansion ✓ answer (3)
5.2.1	LHS $= \tan x(1 - \cos^2 x) + \cos^2 x$ $= \frac{\sin x}{\cos x}(\sin^2 x) + \cos^2 x$ $= \frac{\sin^3 x + \cos^3 x}{\cos x}$ $= \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\cos x}$ $= \frac{(\sin x + \cos x)(1 - \sin x \cos x)}{\cos x}$ $=$ RHS **OR**	✓ $\frac{\sin x}{\cos x}$ ✓ $\sin^2 x$ ✓ simplification ✓ factorisation of cubes ✓ $\sin^2 x + \cos^2 x = 1$ (5)

	$\text{RHS} = \dfrac{(\sin x+\cos x)(1-\sin x\cos x)}{\cos x}$ $= \dfrac{\sin x-\sin^2 x\cos x+\cos x-\sin x\cos^2 x}{\cos x}$ $= \tan x-\sin^2 x+1-\sin x\cos x$ $= \tan x+\cos^2 x-\sin x\cos x$ $= \tan x\left(1-\dfrac{\sin x\cos x}{\tan x}\right)+\cos^2 x$ $= \tan x\left(1-\dfrac{\sin x\cos x}{\frac{\sin x}{\cos x}}\right)+\cos^2 x$ $= \tan x\left(1-\cos^2 x\right)+\cos^2 x$ $= \text{LHS}$	✓ multiplication ✓ ÷ by $\cos x$ ✓ $-\sin^2 x+1=\cos^2 x$ ✓ factorisation ✓ $\tan x=\dfrac{\sin x}{\cos x}$ (5)
5.2.2	$\cos x=0$ or where tan x is undefined $x=90°+k.360°$ or $x=270°+k.360°$ $x=90°$ or $x=-90°$	✓ $\cos x=0$ or tan x undefined ✓ $x=90°$ ✓ $x=-90°$ (3)
5.3.1	$\dfrac{\sin 150°+\cos^2 x-1}{2}$ $= \dfrac{\sin 30°+\cos^2 x-1}{2}$ $= \dfrac{\frac{1}{2}-\left(1-\cos^2 x\right)}{2}$ $= \left(\dfrac{1}{2}-\sin^2 x\right)\times\dfrac{1}{2}$ $= \dfrac{1-2\sin^2 x}{4}$ $= \dfrac{\cos 2x}{4}$	✓ $\sin 30°$ ✓ $\sin 30^\circ=\dfrac{1}{2}$ ✓ factor ✓ $1-\cos^2 x=\sin^2 x$ ✓ simplification ✓ answer in terms of cos $2x$ (6)
5.3.2	$\dfrac{\sin 150°+\cos^2 x-1}{2}=\dfrac{1}{25}$ $\dfrac{\cos 2x}{4}=\dfrac{1}{25}$ $\cos 2x=\dfrac{4}{25}$ $\text{ref}\angle = 80,79...°$ $2x=80,79...°+k.360°$ or $2x=279,20...°+k.360°$ $x=40,40°+k.180°$ or $x=139,60°+k.180°$; $k\in$ Z	✓ answer $5.3.1=\dfrac{1}{25}$ ✓ $2x=80,79°$ ✓ $2x=279,20...°$ ✓ $x=40,40°$ and $x=139,60°$ ✓ $+k.180°$; $k\in$ Z (5)

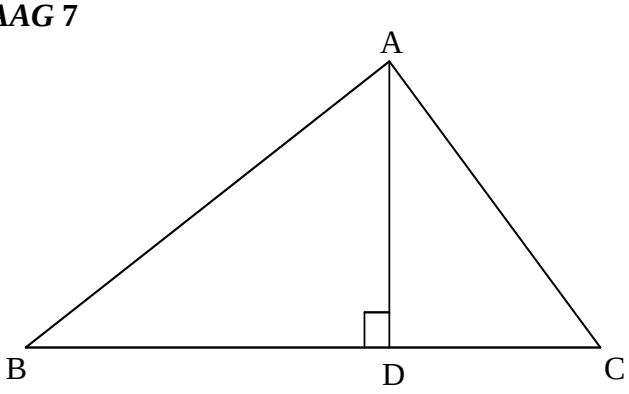
	OR $\frac{\sin 150° + \cos^2 x - 1}{2} = \frac{1}{25}$ $\sin 150° + \cos^2 x - 1 = \frac{2}{25}$ $\sin 30° + \cos^2 x - 1 = \frac{2}{25}$ $\cos^2 x = \frac{29}{50}$ $\cos x = \pm\sqrt{\frac{29}{50}}$ $x = 40, 40° + k.360°$ or $x = 319, 60° + k.360°$; $k \in$ Z or $x = 139, 60° + k.360°$ or $x = 220, 40° + k.360°$; $k \in$ Z	✓ $\cos^2 x = \frac{29}{50}$ ✓ $x = 40, 40°$ ✓ $x = 139, 60°$ ✓ x = 220,40° and $x = 319, 60°$ ✓ + k.360° ; $k \in$ Z (5)
		[26]

QUESTION/*VRAAG* 6

6.1	Period = $360°$	✓ $360°$ (1)
6.2	Amplitude $= 1$	✓ 1 (1)
6.3	$a = -45°$	✓ $a = -45°$ (1)
6.4	$\sin 2x = k$ $k = \sin(2 \times 165°)$ **OR** $k = \sin\left(2 \times (-75°)\right)$ $k = \sin 330°$ $k = \sin(-150°)$ $k = -\sin 30°$ $k = -\frac{1}{2}$ **OR** $k = \cos(165° - 45°)$ **OR** $k = \cos(-75° - 45°)$ $k = \cos 120°$ $k = \cos(-120°)$ $k = -\cos 60°$ $k = -\frac{1}{2}$	✓ $-\sin 30°$ ✓ $-\frac{1}{2}$ (2) ✓ $-\cos 60°$ ✓ $-\frac{1}{2}$ (2)
6.5	Points of intersection are translated $60°$ to the left $x = -15°$	✓ $x = -15°$ (1)
6.6	$\sqrt{2}\sin 2x = \sin x + \cos x$ $\sin 2x = \frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x$ $\sin 2x = \sin 45° \sin x + \cos 45° \cos x$ $\sin 2x = \cos(45° - x)$ **OR** $\sin 2x = \cos(x - 45°)$ $\therefore$ 2 roots in the interval $x \in [-90°; 90°]$	✓ division by $\sqrt{2}$ ✓ special angles ✓ $\cos(45° - x)$ or $\cos(x - 45°)$ ✓ answer (4)
		[10]

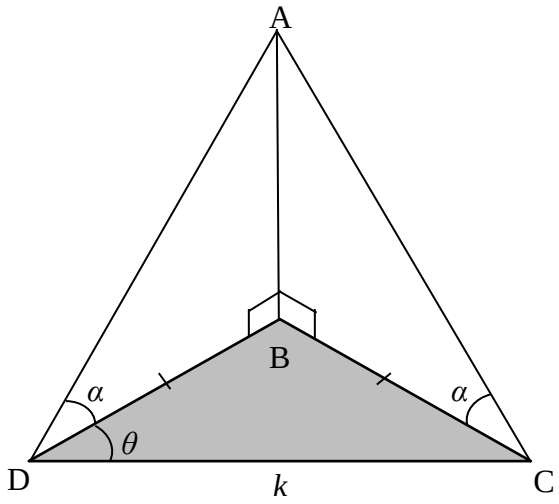
QUESTION/*VRAAG* 7

7.1



7.1.1	$\sin \hat{B} = \frac{AD}{AB}$ $AD = AB\sin \hat{B}$	✓ $\sin \hat{B} = \frac{AD}{AB}$ ✓ answer (2)
7.1.2	Area of $\Delta ABC = \frac{1}{2}(BC)(AD)$ $\therefore$ Area of $\Delta ABC = \frac{1}{2}(BC)(AB)\sin \hat{B}$	✓ $\frac{1}{2}(BC)(AD)$ (1)

7.2



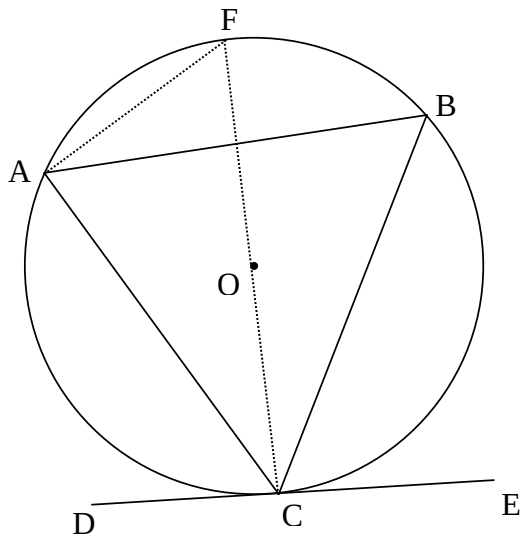
7.2.1	In ΔADB $\sin\alpha = \frac{AB}{AD}$ $AD = \frac{AB}{\sin\alpha}$ In ΔABC $\sin\alpha = \frac{AB}{AC}$ $AC = \frac{AB}{\sin\alpha}$ $AD = AC$ **OR** In ΔADB and ΔACB	✓ $\sin\alpha = \frac{AB}{AD}$ ✓ $\sin\alpha = \frac{AB}{AC}$ (2)

	$AB = AB$ [common side] $A\hat{B}D = A\hat{B}C = 90°$ [given] $BD = BC$ [given] $\Delta ADB \equiv \Delta ACB$ [S∠S] $\therefore AD = AC$	✓ $\Delta ADB \equiv \Delta ACB$ ✓ R (2)
	OR In ΔADB and ΔACB $A\hat{D}B = A\hat{C}B = \alpha$ [given] $A\hat{B}D = A\hat{B}C = 90°$ [given] $AB = AB$ **OR** $BD = BC$ [common side **OR** given] $\therefore \Delta ADB \equiv \Delta ACB$ [∠∠S] $\therefore AD = AC$	✓ $\Delta ADB \equiv \Delta ACB$ ✓ R (2)
	OR $AD^2 = AB^2 + DB^2$ [Pythagoras] $AC^2 = AB^2 + BC^2$ [Pythagoras] But $DB = BC$ [given] $\therefore AD^2 = AC^2$ $\therefore AD = AC$	✓ both Pythagoras statements ✓ DB = BC (2)
7.2.2	$\frac{BD}{\sin\theta} = \frac{k}{\sin(180° - 2\theta)}$ $BD = \frac{k\sin\theta}{\sin 2\theta}$ $BD = \frac{k\sin\theta}{2\sin\theta\cos\theta}$ $BD = \frac{k}{2\cos\theta}$ **OR** $BC^2 = k^2 + BD^2 - 2k(BD)\cos\theta$ $BD^2 = k^2 + BD^2 - 2k(BD)\cos\theta$ $k^2 - 2k(BD)\cos\theta = 0$ $2k(BD)\cos\theta = k^2$ $\therefore BD = \frac{k}{2\cos\theta}$	✓ substitution of $(180° - 2\theta)$ into sine rule ✓ reduction ✓ double angle (3) ✓ substitution into cosine-rule ✓ substitution BC with BD into cosine-rule ✓ simplification in terms of BD (3)

7.2.3	Area of $\Delta BCD = \frac{1}{2}(DC)(BD)\left(\sin C\hat{D}B\right)$ $= \frac{1}{2}k\left(\frac{k}{2\cos\theta}\right)\sin\theta$ $= \frac{1}{4}k^2\tan\theta$ **OR** Area of $\Delta BCD = \frac{1}{2}(BD)(BC)\left(\sin(180° - 2\theta)\right)$ $= \frac{1}{2}\left(\frac{k}{2\cos\theta}\right)\left(\frac{k}{2\cos\theta}\right)(\sin 2\theta)$ $= \frac{2k^2\sin\theta\cos\theta}{8\cos\theta\cos\theta}$ $= \frac{1}{4}k^2\tan\theta$	✓ substitution into area rule ✓ $\frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓ $\frac{1}{4}k^2\tan\theta$ (3) ✓ substitution into area rule ✓ $\frac{\sin\theta}{\cos\theta} = \tan\theta$ ✓ $\frac{1}{4}k^2\tan\theta$ (3)
		[11]

QUESTION/*VRAAG* 8

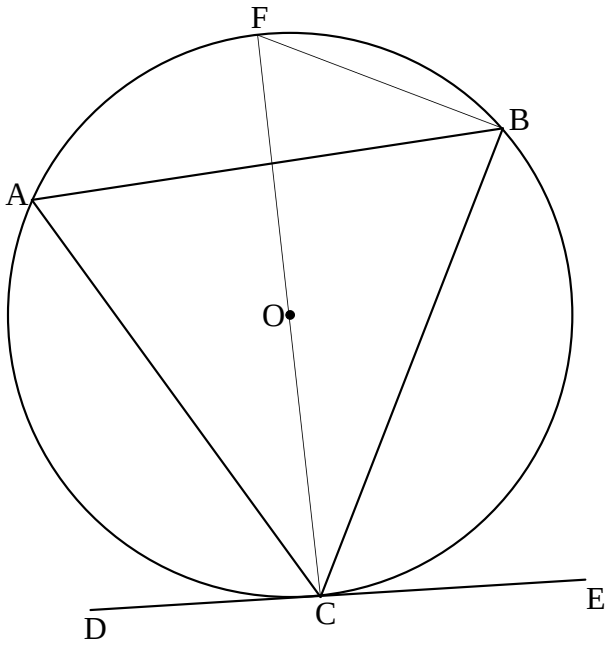
8.1



	Construction: Draw diameter CF and draw AF *Konstruksie: Trek middellyn* CF *en verbind* AF	✓Constr
	$F\hat{C}E = 90°$ [tan ⊥ radius/*raaklyn ⊥ radius*]	✓ S ✓ R
	$F\hat{A}C = 90°$ [∠ in semi circle/*∠ in halwe sirkel*]	✓ S/R
	$F\hat{A}B = F\hat{C}B$ [∠s same segment/*∠e dieselfde segm*]	✓ S/R
	$\therefore B\hat{A}C = B\hat{C}E$ $\therefore B\hat{C}E = \hat{A}$	(5)

OR

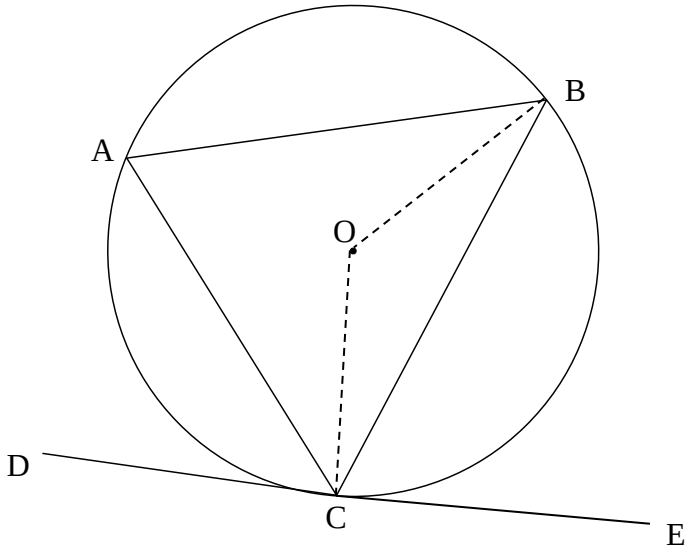
8.1



	Construction: Draw diameter CF and draw FB *Konstruksie: Trek middellyn* CF *en verbind* FB	✓ construction
	$F\hat{B}C = 90°$ [∠ in semi circle/∠*in halwe sirkel*]	✓ S / R
	$B\hat{F}C + F\hat{C}B = 90°$ [sum of ∠s in Δ/*binne ∠e v Δ*]	
	$O\hat{C}E = 90°$ [tan ⊥ radius/ *raaklyn ⊥ radius*]	✓ S ✓ R
	$\therefore B\hat{C}E = \hat{F}$	
	but $\hat{A} = \hat{F}$ [∠s in same seg/∠*in dies. segment*]	✓ S / R
	$\therefore B\hat{C}E = \hat{A}$	(5)

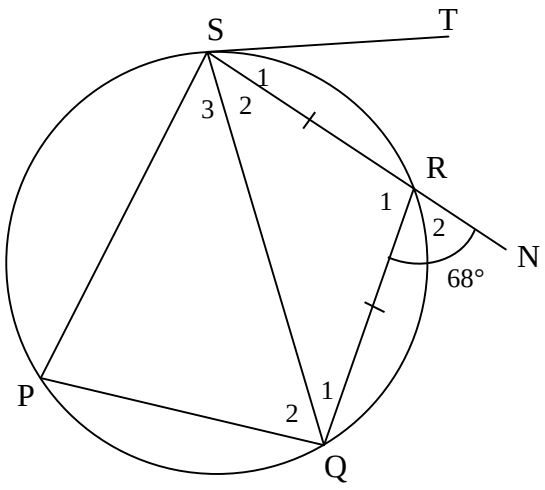
OR

8.1



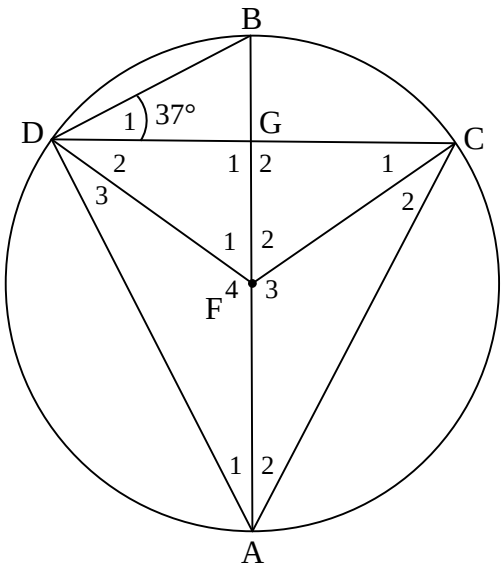
	Construction: Draw radii BO and OC *Konstruksie: Trek radiusse BO en OC*	✓ construction
	$O\hat{C}E = 90°$ or $B\hat{C}E = 90° - O\hat{C}B$ [tan ⊥ radius / *raaklyn ⊥ radius*]	✓ S ✓R
	$O\hat{C}B = O\hat{B}C$ [∠s opp equal sides/ *∠e teenoor gelyke sye*] $\therefore C\hat{O}B = 180° - 2O\hat{C}B$ [∠s of Δ/*∠e van Δ*]	✓ S
	$C\hat{A}B = 90° - O\hat{C}B$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore B\hat{C}E = C\hat{A}B$	✓ S/R (5)

8.2



8.2.1	$\hat{P} = \hat{R}_2 = 68°$	[ext ∠ of cyclic quad /*buite ∠ van kvh*]	✓ S ✓ R (2)
8.2.2	$\hat{Q}_1 = \hat{S}_2$ $\hat{Q}_1 + \hat{S}_2 = 68°$ $\therefore \hat{Q}_1 = 34°$	[∠s opp equal sides /*∠e teenoor gelyke sye*] [ext ∠ of Δ/ *buite ∠ van* Δ]	✓ S ✓ S (2)
8.2.3	$\hat{S}_1 = \hat{Q}_1 = 34°$	[tan-chord theorem/*∠ tussen rkl en koord*]	✓ S ✓ R (2)
			[11]

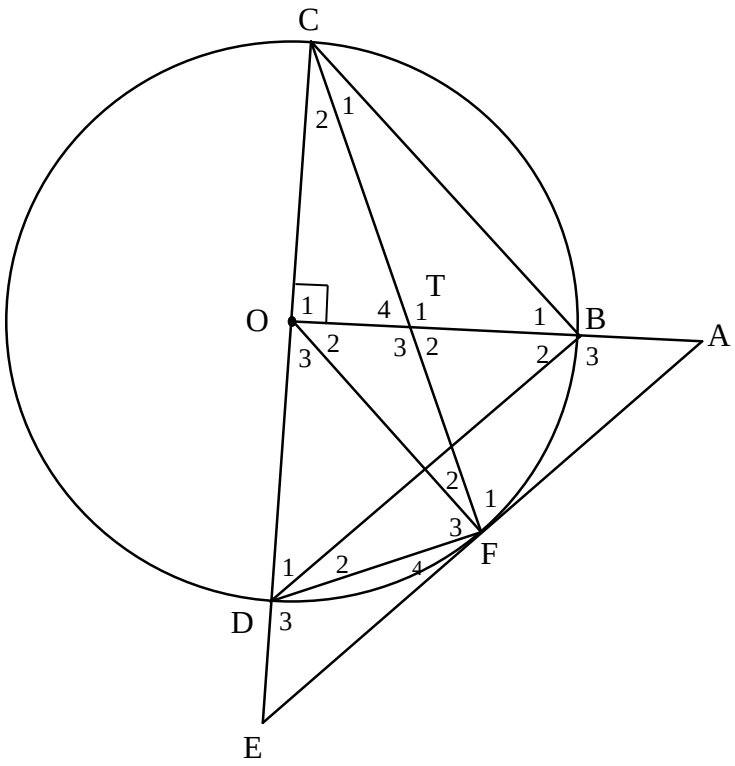
QUESTION/*VRAAG* 9



9.1	$\hat{A}_2 = \hat{D}_1 = 37°$	[∠s in the same seg/*∠e in dies segment*]	✓ S ✓ R
	$\hat{A}_1 = \hat{A}_2 = 37°$	[BA bisects CÂD/BA *halveer* CÂD]	
	$\hat{D}_3 = \hat{A}_1 = 37°$	[∠s opp equal sides/*∠e teenoor gelyke sye*]	✓✓ any other two statements
	$\hat{C}_2 = \hat{A}_2 = 37°$	[∠s opp equal sides/*∠e teenoor gelyke sye*]	(4)
9.2	$A\hat{D}G = 53°$	[∠ in semi circle/*∠ in halwe sirkel*]	✓ S ✓ R
	$\hat{A}_1 = 37°$	[proved in 9.1/*reeds bewys in 9.1*]	
	$\therefore \hat{G}_1 = 90°$	[sum of ∠s in Δ/*binne ∠e van* Δ]	✓ S
	$\therefore CG = DG$	[line from centre ⊥ to chord/ *lyn uit midpt.* ⊥ *op koord*]	✓ R (4)
	OR		
	$\hat{F}_2 = 2\hat{D}_1 = 74°$	[∠ at centre = 2 ×∠ at circumference/ *midpt. ∠s = 2×omtreks∠*]	✓ S ✓ R
	$\hat{D}_3 = 37°$	[proved in 9.1/*reeds bewys in 9.1*]	
	$\therefore \hat{D}_2 = 16°$	[∠ in semi circle/*∠ in halwe sirkel*]	
	$\hat{C}_1 = \hat{D}_2 = 16°$	[∠s opp equal sides/*∠e teenoor gelyke sye*]	
	$\therefore \hat{G}_2 = 90°$	[sum of ∠s in Δ/*binne ∠e van* Δ]	✓ S
	$\therefore CG = DG$	[line from centre ⊥ to chord/ *lyn uit midpt.* ⊥ op *koord*]	✓ R (4)

9.3	$\hat{F}_2 = 2\hat{D}_1 = 74°$ **OR** $\hat{F}_2 = 2\hat{A}_2 = 74°$ [$\angle$ at centre = 2 $\times\angle$ at circum./ *midpt. $\angle$s = 2×omtreks$\angle$*]	✓ S
	$\frac{FG}{20} = \cos 74°$	✓ trig ratio
	$FG = 5,51$	✓ FG
	$\therefore BG = 14,49$ units	✓ answer (4)
	OR	
	$\hat{F}_2 = 2\hat{D}_1 = 74°$ [$\angle$ at centre = 2 $\times\angle$ at circumference *midpt. $\angle$ = 2×omtreks$\angle$*]	✓ S
	$\frac{FG}{20} = \sin 16°$	✓ trig ratio
	$FG = 5,51$	✓ FG
	$\therefore BG = 14,49$ units	✓ answer (4)
	OR	
	$\frac{DG}{20} = \cos 16°$	✓ trig ratio
	$DG = 19,23$	✓ length of DG
	$\frac{BG}{19,23} = \tan 37°$	✓ trig ratio
	$BG = 14,49$ units	✓ answer (4)
	OR	
	$\frac{DG}{20} = \cos 16°$	✓ trig ratio
	$DG = 19,23$	✓ length of DG
	$FG^2 = FD^2 - DG^2$ [Pythagoras]	
	$FG^2 = 20^2 - (19,23)^2$	✓ correct use of Pythagoras
	$FG = 5,51$	
	$BG = 20 - 5,51$	
	$= 14,49$ units	✓ answer (4)
		[12]

QUESTION/*VRAAG* 10



10.1	$\hat{O}_1 = 90°$	[given/*gegee*]	
	$\hat{F}_2 + \hat{F}_3 = 90°$	[∠ in semi circle/*∠ in halwe sirkel*]	✓ S ✓ R
	$\hat{O}_1 = \hat{F}_2 + \hat{F}_3 = 90°$		✓ S
	∴ TODF is a cyclic quad	[ext ∠ = int opp ∠/ *buite ∠ = teenoorst. binne ∠*] **OR** [converse ext ∠ of cyclic quad/ *omgekeerde buite ∠ v kvh*]	✓ R (4)
10.2	$\hat{T}_1 = \hat{T}_3$	[vert opp ∠s =/ *regoorstaande ∠e*]	✓ S / R
	But $\hat{D}_3 = \hat{T}_3$	[ext ∠ of cyclic quad/ *buite ∠ v kvh*]	✓ S ✓ R
	$\therefore \hat{T}_1 = \hat{D}_3$		(3)
10.3	In ΔDFE and ΔTFO		
	1) $\hat{D}_3 = \hat{T}_3$	[ext ∠ of cyclic quad/ *buite ∠ v kvh*]	✓ S
	2) $\hat{F}_4 = \hat{C}_2$	[tan-chord theorem/ *∠ tussen rkl en koord*]	✓ S / R
	but $\hat{C}_2 = \hat{F}_2$	[∠s opp equal sides/ *∠e teenoor gelyke sye*]	✓ S
	$\therefore \hat{F}_4 = \hat{F}_2$		✓ S
	3) $\hat{E} = \hat{O}_2$	[3rd ∠ of Δ/*∠e van* Δ]	✓ S **OR** R
	ΔTFO ||| ΔDFE	[∠∠∠]	(5)

	OR		
	In ΔDFE and ΔTFO		
	1) $\hat{D}_3 = \hat{T}_3$	[ext ∠ of cyclic quad/*buite ∠ van Δ*]	✓ S
	2) $\hat{F}_4 = \hat{C}_2$	[tan-chord theorem/*∠ tussen rkl en koord*]	✓ S / R
	$\hat{F}_2 + \hat{F}_3 = 90°$	[∠ in semi circle/*∠ in halwe sirkel*]	
	$\hat{D}_1 + \hat{D}_2 = 90° - \hat{C}_2$	[sum of ∠s in Δ/ *binne ∠e van Δ*]	
	$\hat{E} = 90° - 2\hat{F}_4$	[ext ∠ of Δ/ *buite ∠ van Δ*]	✓ S
	$\hat{O}_3 = 2\hat{C}_2$	[∠ at centre = 2 ×∠ at circumference/ *midpt. ∠s = 2×omtreks∠*]	
	$\hat{O}_2 = 90° - 2\hat{F}_4$	[∠s on a str line/*∠e op 'n reguitlyn*]	✓ S
	$\hat{O}_2 = \hat{E}$		
	3) $\therefore \hat{F}_4 = \hat{F}_2$	[3rd ∠ of Δ/*∠e van Δ*]	✓ S **OR** R
	ΔTFO ||| ΔDFE	[∠∠∠]	(5)
10.4	$\hat{B}_2 = \hat{D}_1$	[∠s opp equal sides/*∠e teenoor gelyke sye*]	✓ S / R
	$\hat{B}_2 = \hat{E}$	[given/*gegee*]	
	$\therefore \hat{D}_1 = \hat{E}$		
	$\therefore$ DB || EA	[corresp ∠s =/*ooreenkomstige ∠e gelyk*]	✓ R
			(2)
10.5	In ΔOEA		
	DB || EA	[proven/*reeds bewys*]	
	$\frac{OD}{DE} = \frac{OB}{BA}$	[line || one side of Δ/*lyn || een sy van Δ*]	✓ R
		OR [prop theorem; DB || EA/ *eweredigheid stelling; DB || EA*]	
	$\therefore DE = \frac{DO.AB}{OB}$		✓ S
	$\frac{FO}{FE} = \frac{TO}{DE}$	[ΔTFO ||| ΔDFE]	✓ S / R
	$DE = \frac{TO.FE}{FO}$		✓ S
	$\therefore \frac{DO.AB}{OB} = \frac{TO.FE}{FO}$		✓ S
	$\therefore \frac{DO.AB}{DO} = \frac{TO.FE}{DO}$	[DO = OB = FO]	
	$\therefore DO = \frac{TO.FE}{AB}$		(5)
			[19]

TOTAL/*TOTAAL*: 150