basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

FEBRUARIE/MAART 2017

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 14 bladsye, 1 inligtingsblad en 'n antwoordeboek van 28 bladsye.

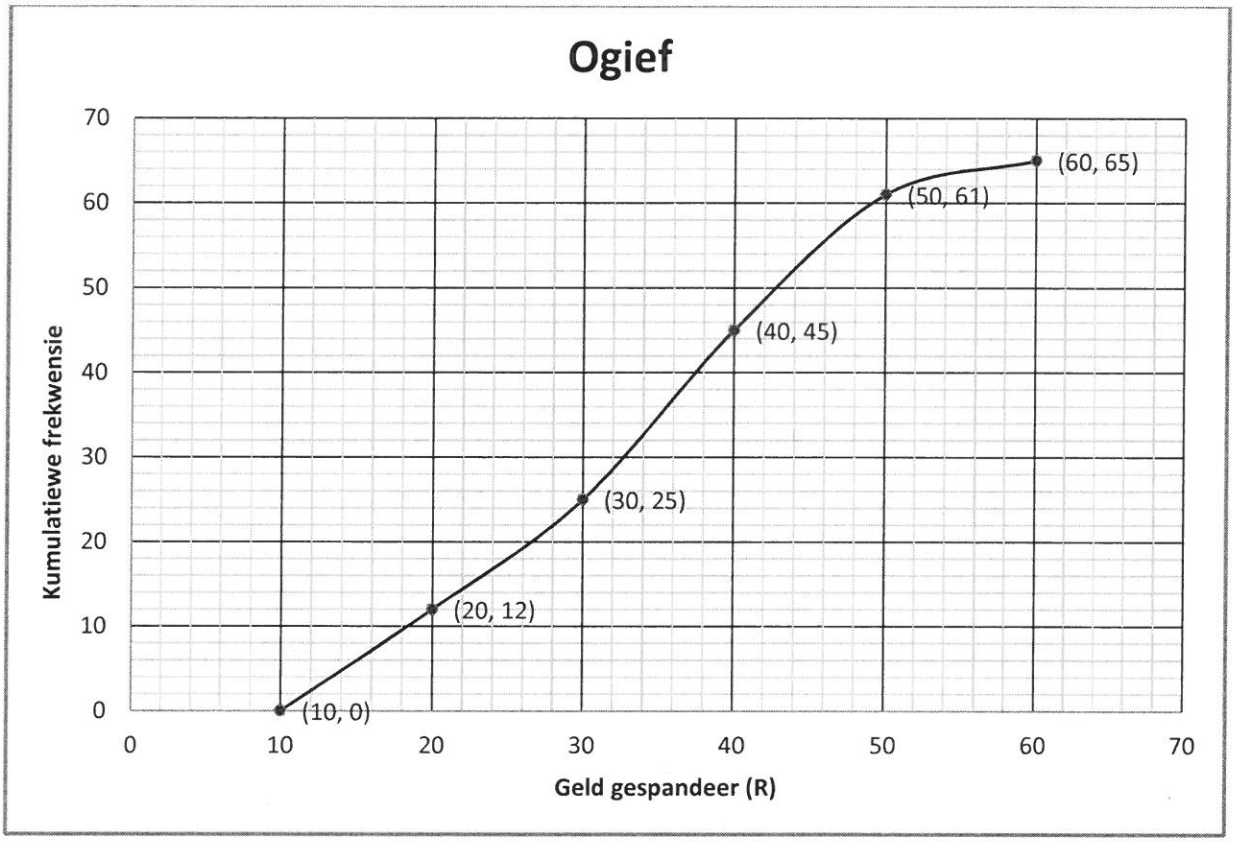
INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat jy die vraestel begin beantwoord.

1. Hierdie vraestel bestaan uit 11 vrae.
2. Beantwoord AL die vrae in die ANTWOORDEBOEK wat verskaf word.
3. Dui ALLE berekeninge, diagramme, grafieke ensovoorts wat jy in die beantwoording van die vrae gebruik duidelik aan.
4. Antwoorde alleenlik sal nie noodwendig volpunte verdien nie.
5. Jy mag 'n goedgekeurde wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders vermeld.
6. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.
7. Diagramme is NIE noodwendig volgens skaal geteken NIE.
8. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
9. Skryf netjies en leesbaar.

VRAAG 1

Die bedrag geld, in rand, wat leerders op 'n spesifieke dag by die skool se snoepwinkel spandeer het, is aangeteken. Die data word in die ogief hieronder voorgestel.



'n Onvoltooide frekwensietabel van die data word ook gegee.

Bedrag geld (in R)	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frekwensie	a	13	20	b	4

1.1 Hoeveel leerders het die snoepwinkel daardie dag besoek? (1)

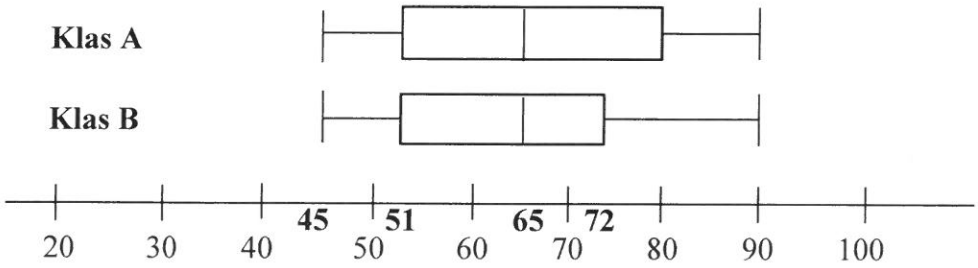
1.2 Skryf die modale klas van hierdie data neer. (1)

1.3 Bepaal die waardes van a en b in die frekwensietabel. (2)

1.4 Gebruik die ogief om die getal leerders te skat wat op die dag wat die data aangeteken is, ten minste R45 by die snoepwinkel spandeer het. (2)
[6]

VRAAG 2

2.1 Mev. Smith het twee klasse van 30 leerders elk. Hulle finale punte (uit 100) vir die jaar word in die mond-en-snordiagram hieronder aangetoon.



2.1.1 Bepaal die interkwartielomvang (interkwartielvariasiewydte) van Klas B. (2)

2.1.2 Verduidelik die betekenis van die verskil in lengte van die monde in die diagram. (2)

2.1.3 Mev. Smith het die resultate bestudeer en beweer dat daar geen wesenlike verskil in die prestasie van die twee klasse is nie. Gee TWEE redes wat jy dink mev. Smith sal gebruik om haar bewering te staaf. (2)

2.2 Agt pare het vir 'n danskompetisie ingeskryf. Twee beoordelaars het punte vir hulle vertonings gegee. Die punte (uit 20) word in die tabel hieronder gegee.

PAAR	1	2	3	4	5	6	7	8
BEOORDELAAR 1	18	4	6	8	5	12	10	14
BEOORDELAAR 2	15	6	3	5	5	14	8	15

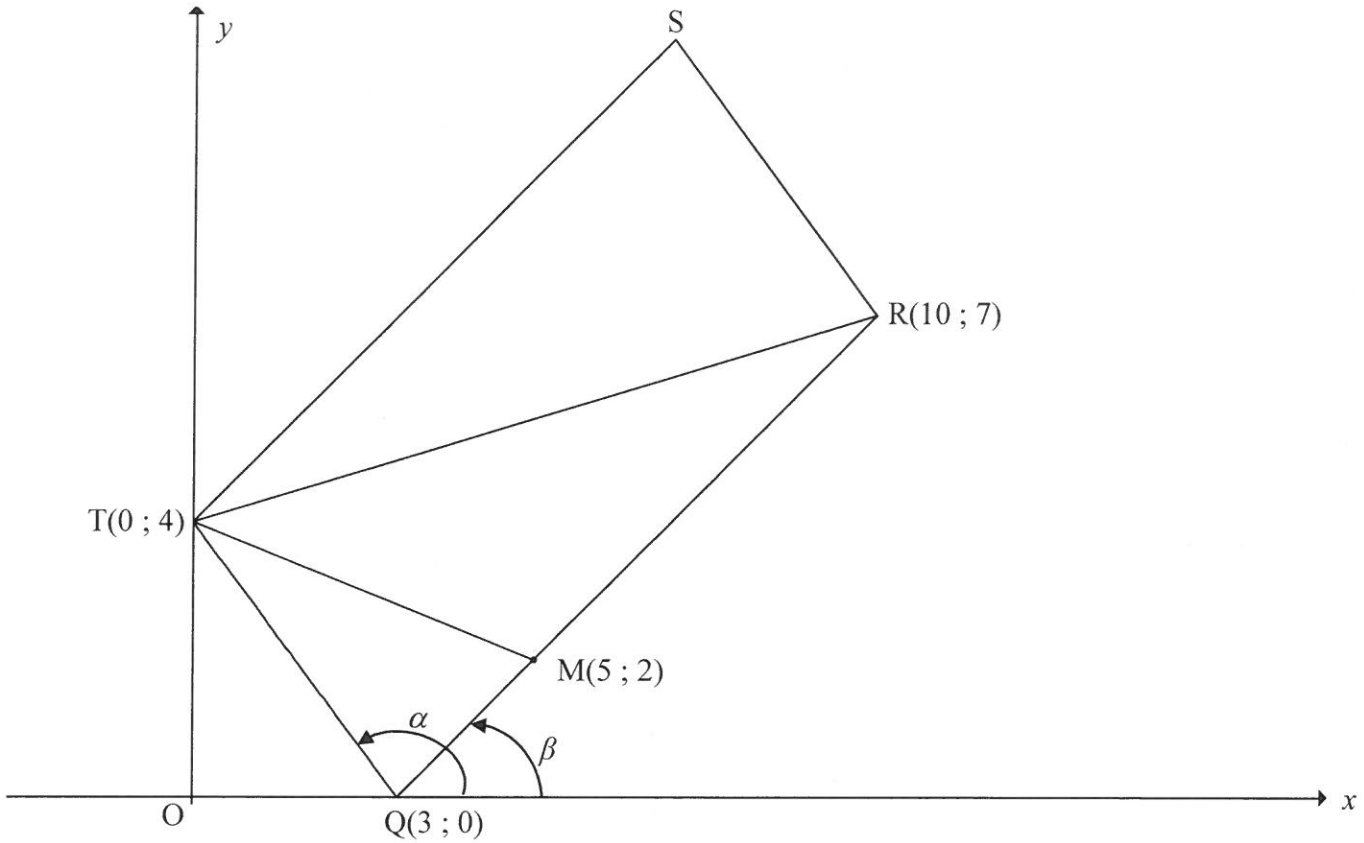
2.2.1 Bepaal die vergelyking van die kleinstekwadrate-regressielyn van die punte wat die twee beoordelaars gegee het. (3)

2.2.2 'n Negende paar het laat vir die kompetisie ingeskryf en ontvang 'n punt van 15 by BEOORDELAAR 1. Skat die punt, tot die naaste heeltallige waarde, wat hulle moontlik by BEOORDELAAR 2 ontvang het. (2)

2.2.3 Is die beoordelaars konsekwent in die toekenning van punte vir die vertonings van die pare? Bewys jou antwoord en ondersteun dit met toepaslike statistiek. (2)
[13]

VRAAG 3

In die diagram is Q(3 ; 0), R(10 ; 7), S en T(0 ; 4) hoekpunte van parallelogram QRST. Vanaf T word 'n reguitlyn getrek om QR by M(5 ; 2) te ontmoet. Die inklinasiehoeke van TQ en RQ is onderskeidelik α en β.



3.1 Bereken die gradiënt van TQ. (1)

3.2 Bereken die lengte van RQ. Laat jou antwoord in wortelvorm. (2)

3.3 F(k ; –8) is 'n punt op die Cartesiese vlak sodat T, Q en F saamlynig is. Bereken die waarde van k. (4)

3.4 Bereken die koördinate van S. (4)

3.5 Bereken die grootte van $T\hat{S}R$. (6)

3.6 Bereken, in die eenvoudigste vorm, die verhouding van:

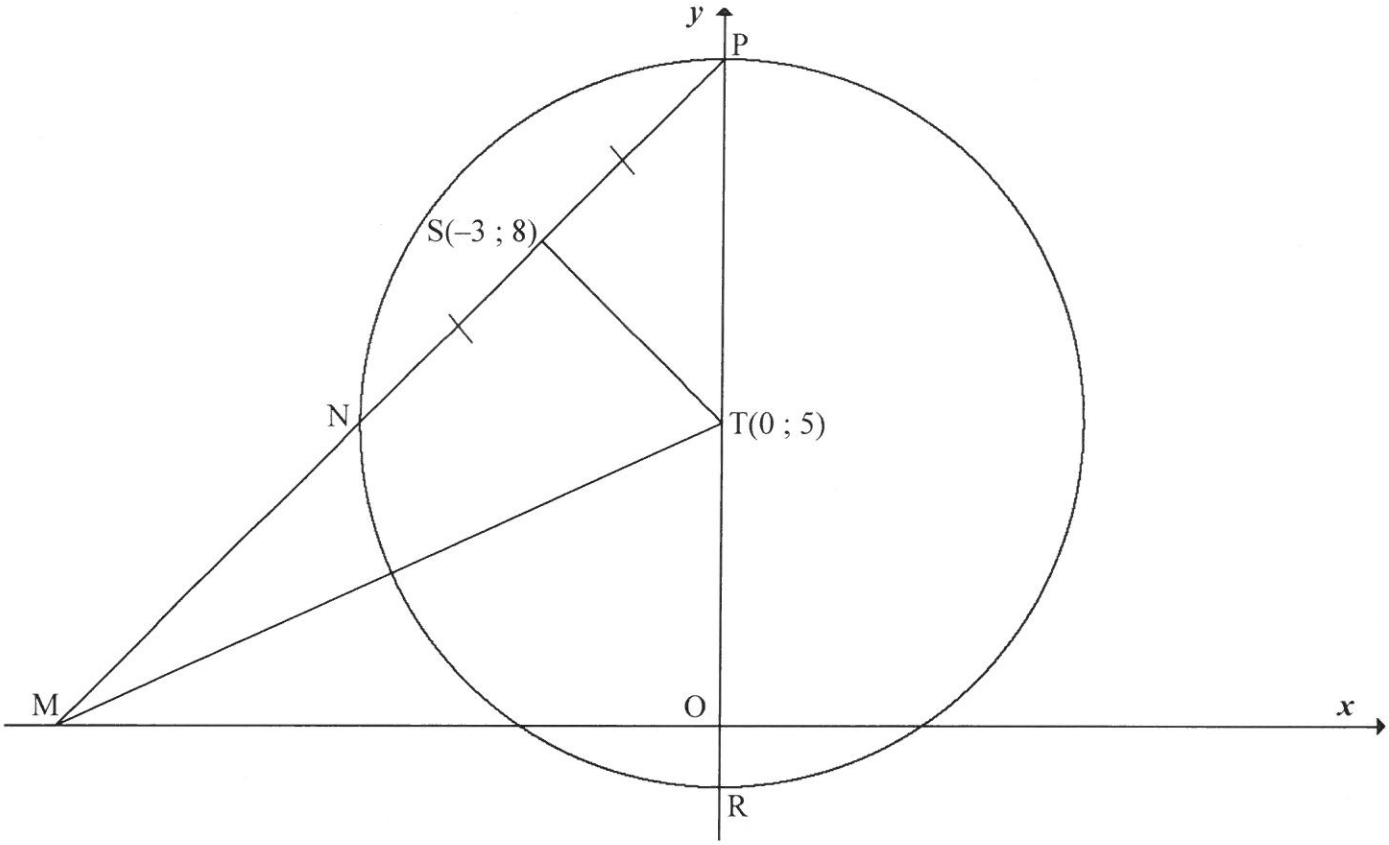
3.6.1 $\dfrac{MQ}{RQ}$ (3)

3.6.2 $\dfrac{\text{oppervlakte van } \Delta TQM}{\text{oppervlakte van parallelogram } RQTS}$ (3)

[23]

VRAAG 4

In die diagram sny die sirkel met middelpunt T(0 ; 5) die y-as by P en R. Die lyn deur P en S(–3 ; 8) sny die sirkel by N en die x-as by M. NS = PS. MT word getrek.



4.1 Gee 'n rede waarom TS ⊥ NP. (1)

4.2 Bepaal die vergelyking van die lyn deur N en P in die vorm $y = mx + c$. (5)

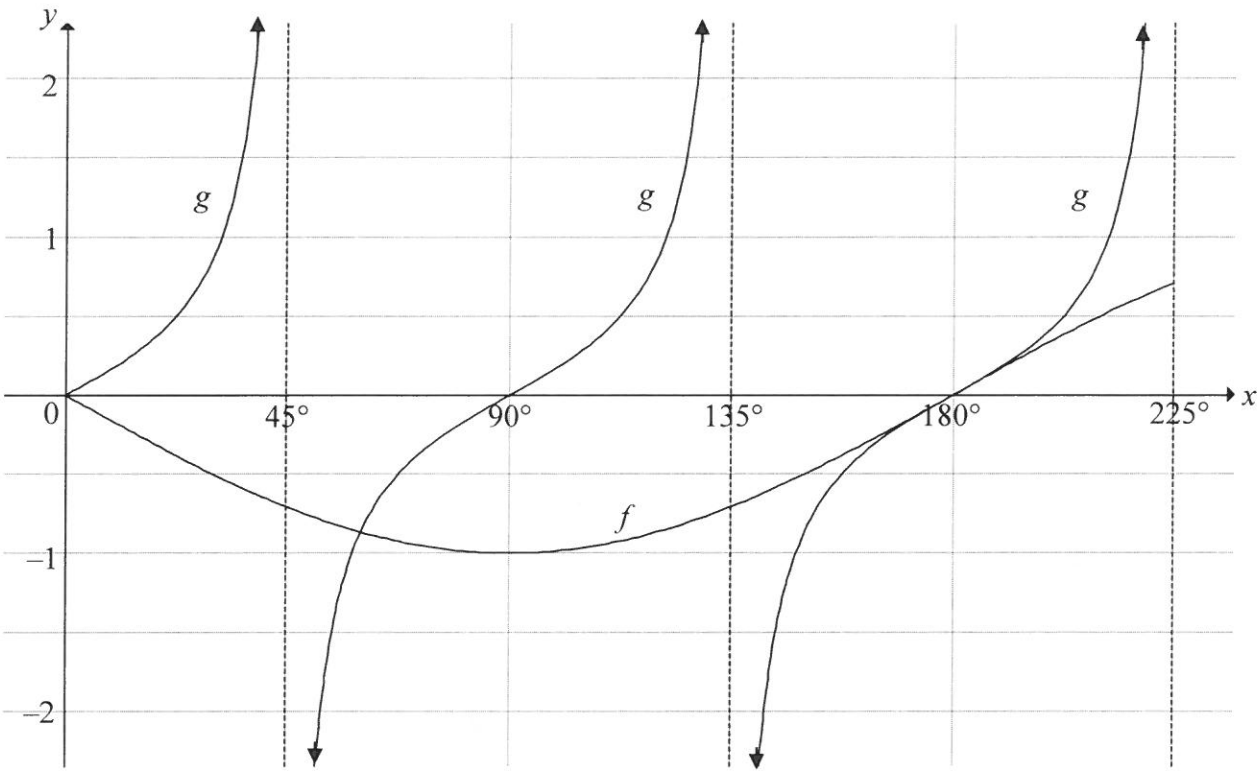
4.3 Bepaal die vergelykings van die raaklyne aan die sirkel, ewewydig aan die x-as. (4)

4.4 Bepaal die lengte van MT. (4)

4.5 'n Ander sirkel word deur die punte S, T en M getrek. Bepaal, met redes, die vergelyking van hierdie sirkel STM in die vorm $(x-a)^2 + (y-b)^2 = r^2$. (5)
[19]

VRAAG 5

In die diagram word die grafieke van die funksies $f(x) = a\sin x$ en $g(x) = \tan bx$ op dieselfde assestelsel geskets vir die interval $0° \leq x \leq 225°$.



5.1 Skryf die waardes van a en b neer. (2)

5.2 Skryf die periode van $f(3x)$ neer. (2)

5.3 Bepaal die waardes van x in die interval $90° \leq x \leq 225°$ waarvoor $f(x).g(x) \leq 0$. (3)
[7]

VRAAG 6

6.1 **Sonder die gebruik van 'n sakrekenaar**, bepaal die volgende in terme van $\sin 36°$:

6.1.1 $\sin 324°$ (1)

6.1.2 $\cos 72°$ (2)

6.2 Bewys die identiteit: $1-\dfrac{\tan^2\theta}{1+\tan^2\theta}=\cos^2\theta$ (4)

6.3 Gebruik VRAAG 6.2 om die algemene oplossing van die volgende te bepaal:

$$1-\frac{\tan^2\frac{1}{2}x}{1+\tan^2\frac{1}{2}x}=\frac{1}{4}$$ (6)

6.4 Gegee: $\cos(A-B)=\cos A\cos B+\sin A\sin B$

6.4.1 Gebruik die formule van $\cos(A-B)$ en lei 'n formule vir $\sin(A-B)$ af. (4)

6.4.2 **Sonder die gebruik van 'n sakrekenaar**, toon aan dat

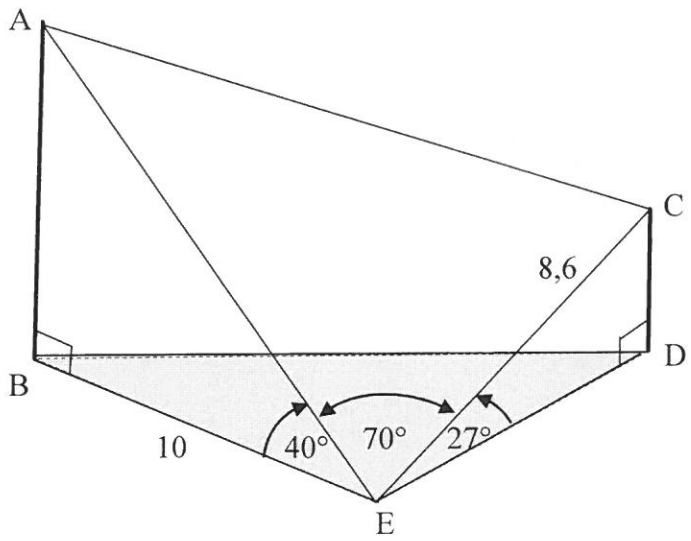
$$\sin(x+64°)\cos(x+379°)+\sin(x+19°)\cos(x+244°)=\frac{1}{\sqrt{2}}$$

vir alle waardes van x. (6)

[23]

VRAAG 7

In die diagram is B, E en D punte in dieselfde horisontale vlak. AB en CD is vertikale pale. Staalkabels AE en CE anker die pale by E. 'n Ander staalkabel verbind A en C. CE = 8,6 m, BE = 10 m, $A\hat{E}B = 40°$, $A\hat{E}C = 70°$ en $C\hat{E}D = 27°$.



Bereken die:

7.1 Hoogte van paal CD (2)

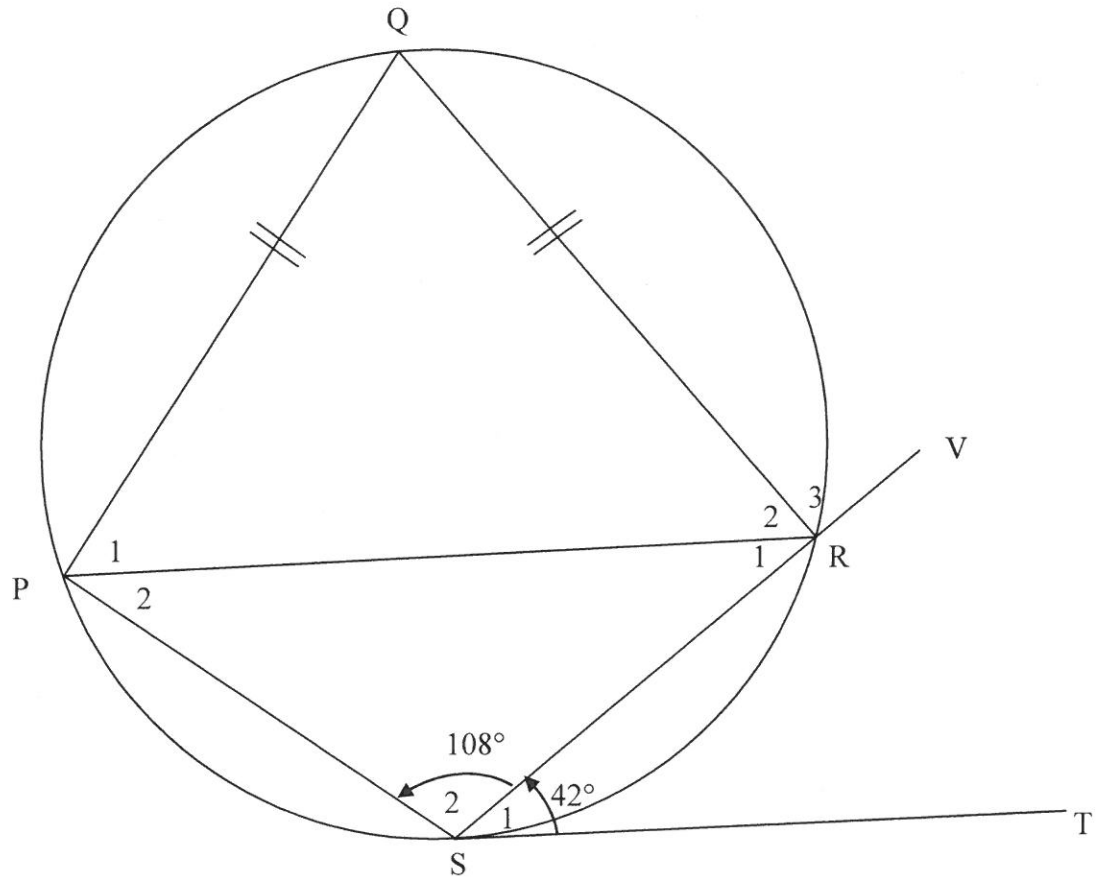
7.2 Lengte van kabel AE (2)

7.3 Lengte van kabel AC (4)
[8]

Gee redes vir ALLE bewerings en berekeninge in VRAAG 8, 9, 10 en 11.

VRAAG 8

In die diagram is PQRS 'n koordevierhoek. ST is 'n raaklyn aan die sirkel by S en koord SR is verleng na V. PQ = QR, $\hat{S}_1 = 42°$ en $\hat{S}_2 = 108°$.



Bepaal, met redes, die grootte van die volgende hoeke:

8.1 $\hat{Q}$ (2)

8.2 $\hat{R}_2$ (2)

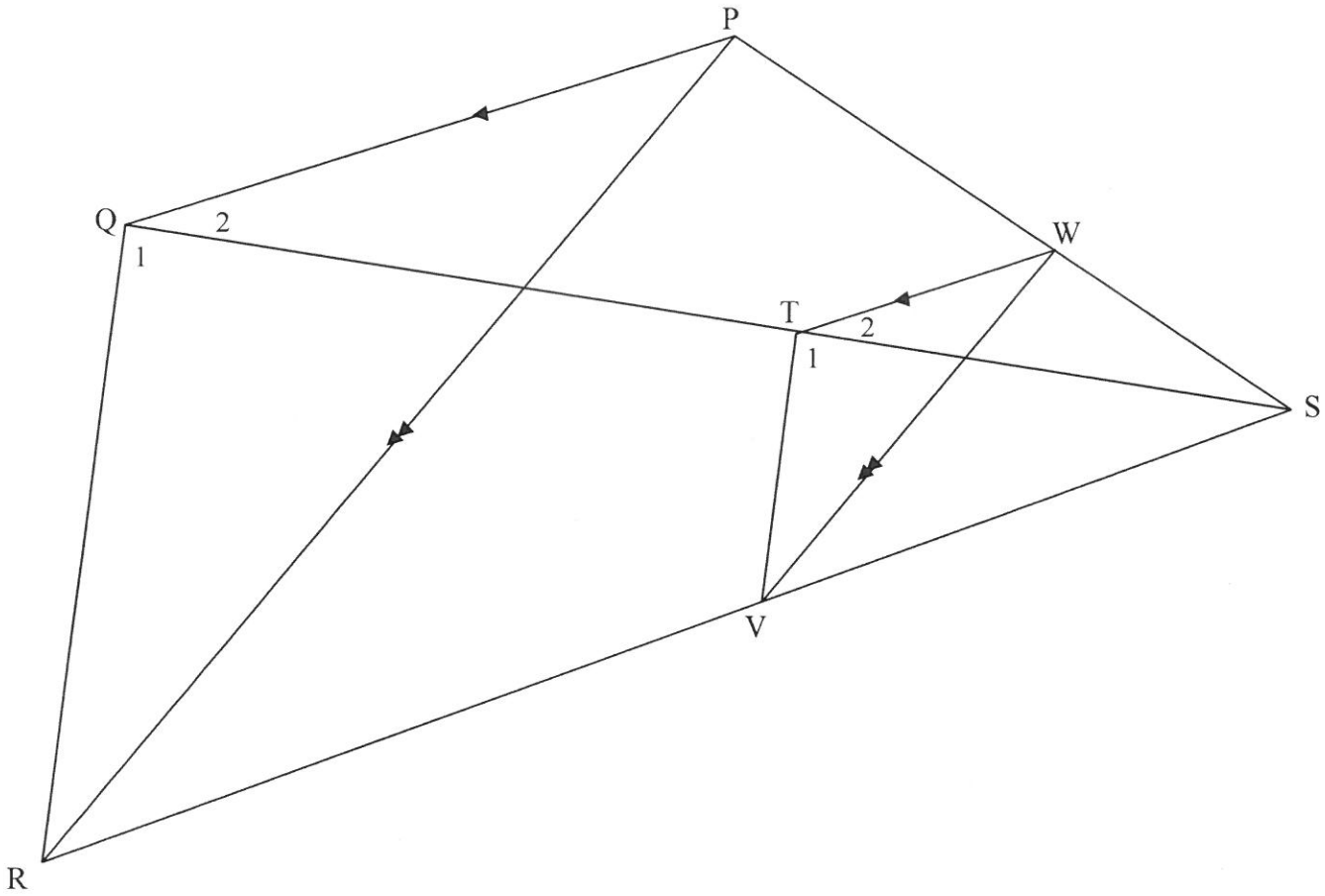
8.3 $\hat{P}_2$ (2)

8.4 $\hat{R}_3$ (2)

[8]

VRAAG 9

In die diagram is PQRS 'n vierhoek met hoeklyne PR en QS verbind. W is 'n punt op PS. WT is ewewydig aan PQ met T op QS. WV is ewewydig aan PR met V op RS. TV word getrek. PW : WS = 3 : 2.



9.1 Skryf die waarde van die volgende verhoudings neer:

9.1.1 $\frac{ST}{TQ}$ (2)

9.1.2 $\frac{SV}{VR}$ (1)

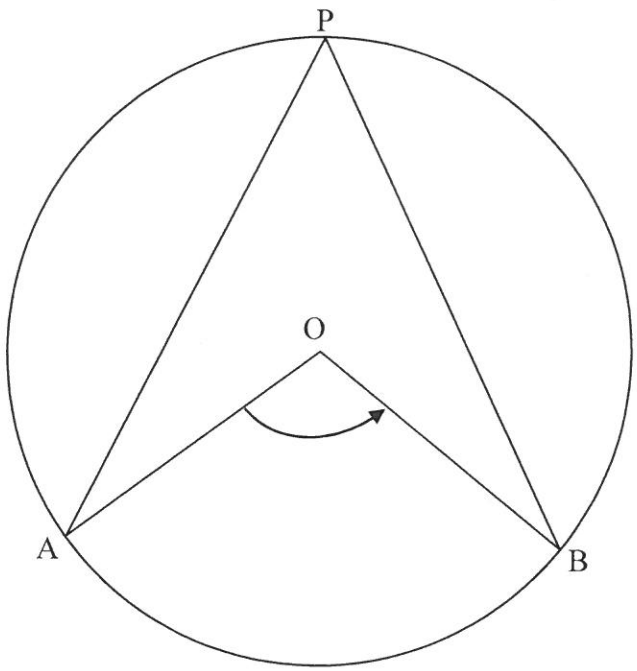
9.2 Bewys dat $\hat{T}_1 = \hat{Q}_1$. (4)

9.3 Voltooi die volgende bewering: $\Delta VWS \;|||\; \Delta\ldots$ (1)

9.4 Bepaal WV : PR. (2)
[10]

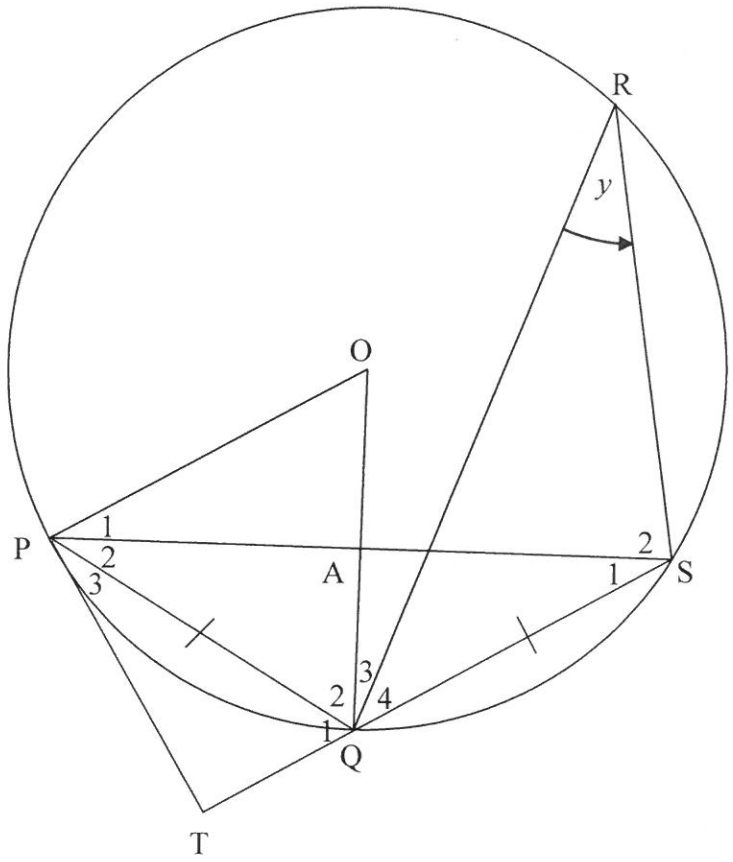
VRAAG 10

10.1 In die diagram is O die middelpunt van die sirkel en P is 'n punt op die omtrek van die sirkel. Boog AB onderspan $A\hat{O}B$ by die middelpunt van die sirkel en $A\hat{P}B$ by die omtrek van die sirkel.



Gebruik die diagram om die stelling te bewys wat beweer dat $A\hat{O}B = 2A\hat{P}B$. (5)

10.2 In die diagram is O die middelpunt van die sirkel en P, Q, S en R punte op die sirkel. PQ = QS en $Q\hat{R}S = y$. Die raaklyn by P ontmoet SQ verleng by T. OQ sny PS by A.



10.2.1 Gee 'n rede waarom $\hat{P}_2 = y$. (1)

10.2.2 Bewys dat PQ vir $T\hat{P}S$ halveer. (4)

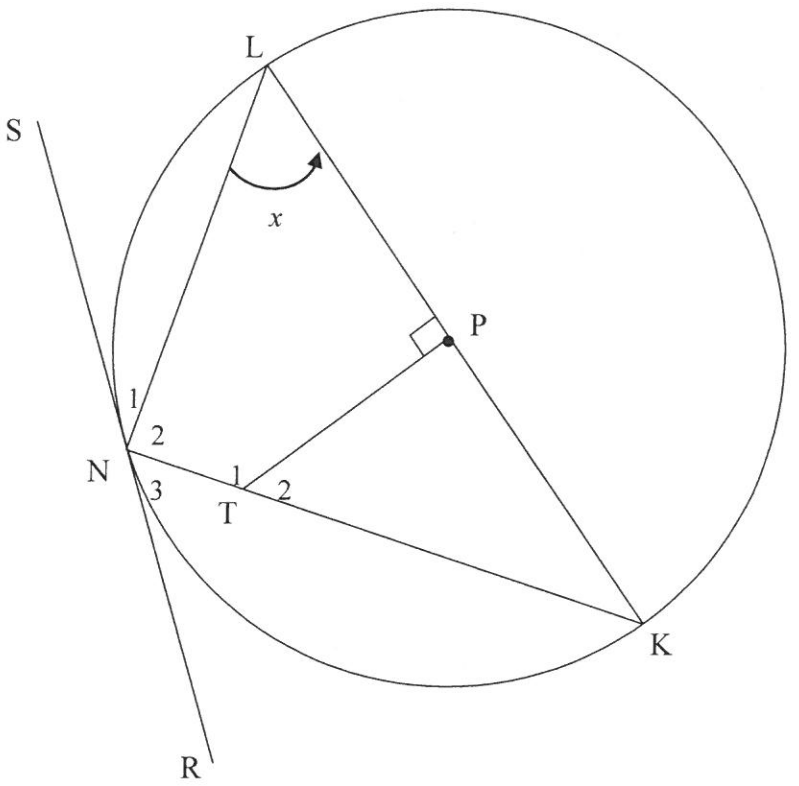
10.2.3 Bepaal $P\hat{O}Q$ in terme van y. (2)

10.2.4 Bewys dat PT 'n raaklyn is aan die sirkel wat deur die punte P, O en A gaan. (2)

10.2.5 Bewys dat $O\hat{A}P = 90°$. (5)
[19]

VRAAG 11

In die diagram is LK die middellyn van die sirkel met middelpunt P. RNS is 'n raaklyn aan die sirkel by N. T is 'n punt op NK en TP ⊥ KL. $P\hat{L}N = x$.



11.1 Bewys dat TPLN 'n koordevierhoek is. (3)

11.2 Bepaal, met redes, die grootte van $\hat{N}_1$ in terme van x. (3)

11.3 Bewys dat:

11.3.1 $\Delta KTP \,|||\, \Delta KLN$ (3)

11.3.2 $KT \,.\, KN = 2KT^2 - 2TP^2$ (5)

[14]

TOTAAL: 150

INLIGTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

InΔABC: $\dfrac{a}{\sin \mathrm{A}} = \dfrac{b}{\sin \mathrm{B}} = \dfrac{c}{\sin \mathrm{C}}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\textit{oppervlakte}\ \Delta\mathrm{ABC} = \frac{1}{2}ab.\sin \mathrm{C}$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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Candidate Barcode Label/*Kandidaat se Stafieskodeplakker*

NSC Answer Book
NSS-antwoordeboek

National Senior Certificate/*Nasionale Senior Sertifikaat* (Grade 12/*Graad 12*)

Sequence No. on mark sheet
Volgnommer op puntestaat

CENTRE NUMBER *SENTRUMNOMMER*	
EXAMINATION NUMBER *EKSAMENNOMMER*	
DATE *DATUM*	
BOOK NUMBER *BOEKNOMMER*	OF *VAN* BOOKS *BOEKE*
SUBJECT CODE *VAKKODE*	
PAPER NUMBER *VRAESTELNOMMER*	2
SUBJECT NAME *VAKNAAM*	MATHEMATICS/*WISKUNDE*

This answer book consists of 28 pages./*Hierdie antwoordeboek bestaan uit 28 bladsye.*

MARKER/*NASIENER*			MODERATOR'S INITIALS IN RELEVANT BLOCK *MODERATORPARAAF IN BETROKKE BLOKKIE*					
Question *Vraag*	Marks *Punte*	Marker's Code & Initials	Marks *Punte*	SM	DCM	CM	IM	EM
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RE-MARK/RE-CHECK *HERNASIEN/HERKONTROLEER*		
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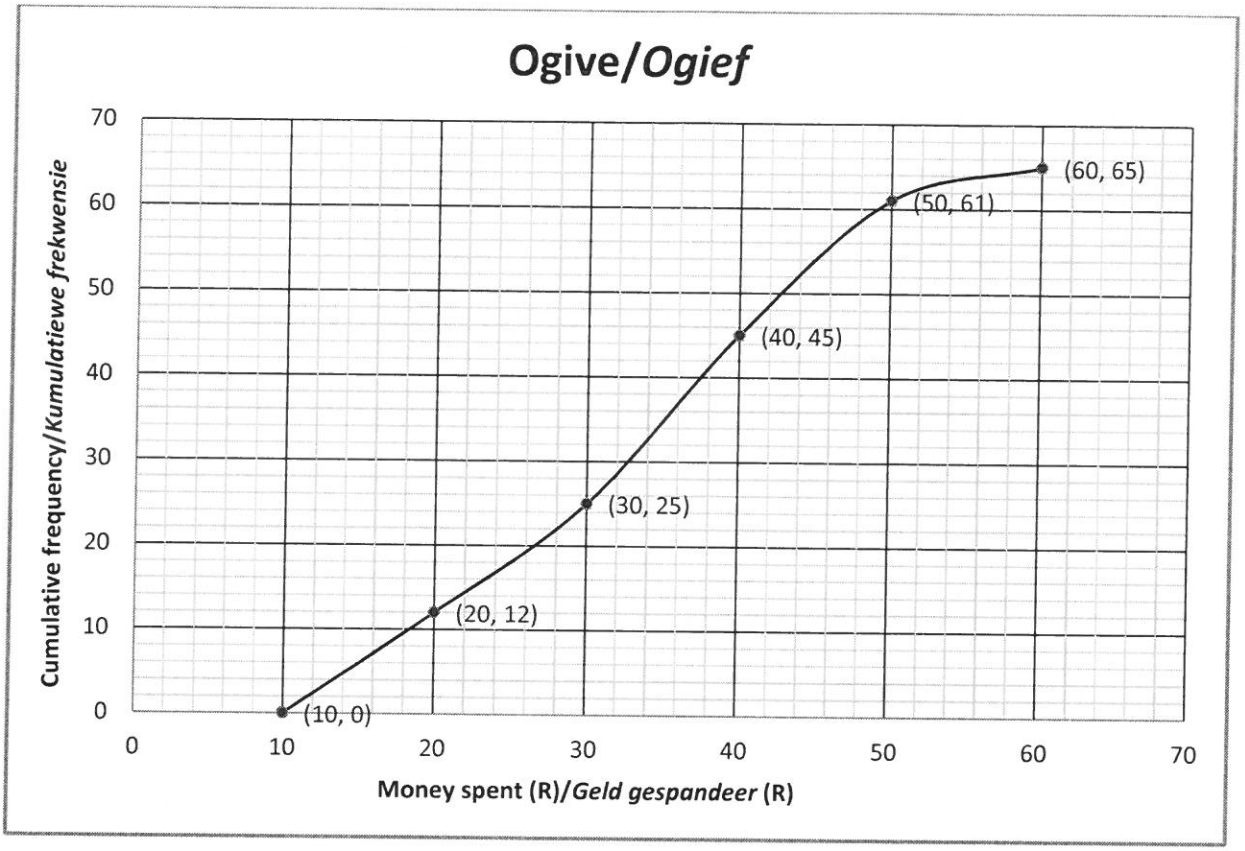
TOTAL *TOTAAL*		
HASH TOTAL *KAFTOTAAL*		

CONTROLLED AND CERTIFIED CORRECT SURNAME AND INITIALS OF EA	

PLEASE READ INSTRUCTIONS ON THE NEXT PAGE
LEES ASSEBLIEF INSTRUKSIES OP VOLGENDE BLADSY

PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG
1. Write your examination number and centre number clearly in the spaces provided and attach your examination number sticker in the space provided.	1. Skryf jou eksamennommer en sentrumnommer duidelik in die ruimtes soos verskaf en plak jou eksamennommeretiket in die ruimte soos verskaf.
2. Remember that your own name (or the name of your school) may not appear anywhere on or in this answer book.	2. Onthou dat jou eie naam (of die naam van jou skool) nie op of in hierdie antwoordeboek mag voorkom nie.
3. Answer ALL questions in the spaces provided.	3. Beantwoord ALLE vrae in die ruimtes wat voorsien is.
4. No pages may be torn from this answer book.	4. Geen bladsye mag uit hierdie antwoordeboek geskeur word nie.
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	5. Lees die instruksies wat op jou eksamenrooster gedruk is sorgvuldig deur, asook enige ander instruksies wat in elke eksamenvraestel gegee word.
6. Candidates may not retain an answer book or remove it from the examination room. Answer books must be returned to the Invigilator at the end of the examination session.	6. Geen antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word nie. Antwoordeboeke moet aan die Toesighouer terugbesorg word aan die einde van die eksamensessie.
7. Answers must be written in black/blue ink as distinctly as possible. Do not write in the margins.	7. Skryf die antwoorde so duidelik moontlik met swart/blou ink. Moenie in die kantlyn skryf nie.
8. Indicate the questions you have answered by drawing a circle around the relevant numbers on the front cover of the answer book where marks are to be recorded.	8. Dui die vrae wat jy beantwoord het aan op die voorblad van die antwoordeboek waar die punte aangebring word, deur 'n kringetjie te trek om die nommers van die vrae wat jy beantwoord het.
9. Rule off through any work/rough work that must not be marked.	9. Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.
10 In the event that you use the additional space provided: 10.1 Write down the number of the question. 10.2 Leave a line and rule off after your answer.	10 In geval jy die addisionele ruimte wat voorsien word, gebruik: 10.1 Skryf die nommer van die vraag neer. 10.2 Laat 'n lyn oop en trek 'n lyn na jou antwoord.

QUESTION/*VRAAG* 1



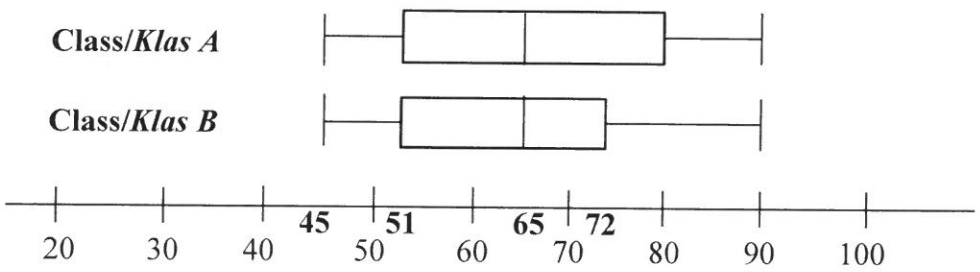
Amount of money (in R) *Bedrag geld (in R)*	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frequency *Frekwensie*	a	13	20	b	4

	Solution/*Oplossing*	**Marks** *Punte*
1.1		(1)
1.2		(1)

	Solution/*Oplossing*	**Marks *Punte***
1.3		(2)
1.4		(2)
		[6]

QUESTION/*VRAAG* 2

2.1



	Solution/*Oplossing*	**Marks *Punte***
2.1.1		(2)
2.1.2		(2)

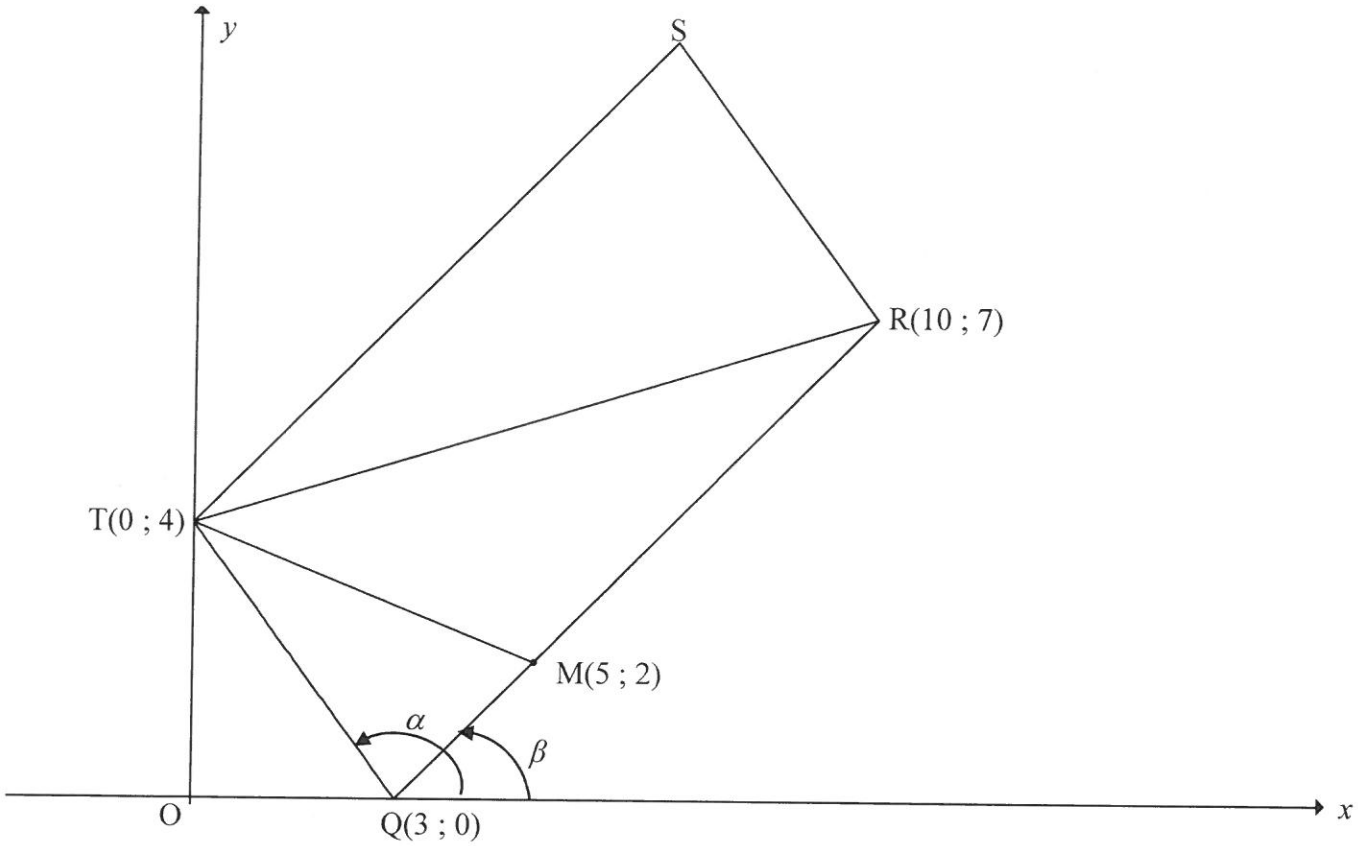
	Solution/*Oplossing*	**Marks *Punte***
2.1.3		(2)

2.2

COUPLE/PAAR	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
JUDGE 1/ BEOORDELAAR 1	18	4	6	8	5	12	10	14
JUDGE 2/ BEOORDELAAR 2	15	6	3	5	5	14	8	15

	Solution/*Oplossing*	**Marks *Punte***
2.2.1		(3)
2.2.2		(2)
2.2.3		(2)
		[13]

QUESTION/*VRAAG* 3

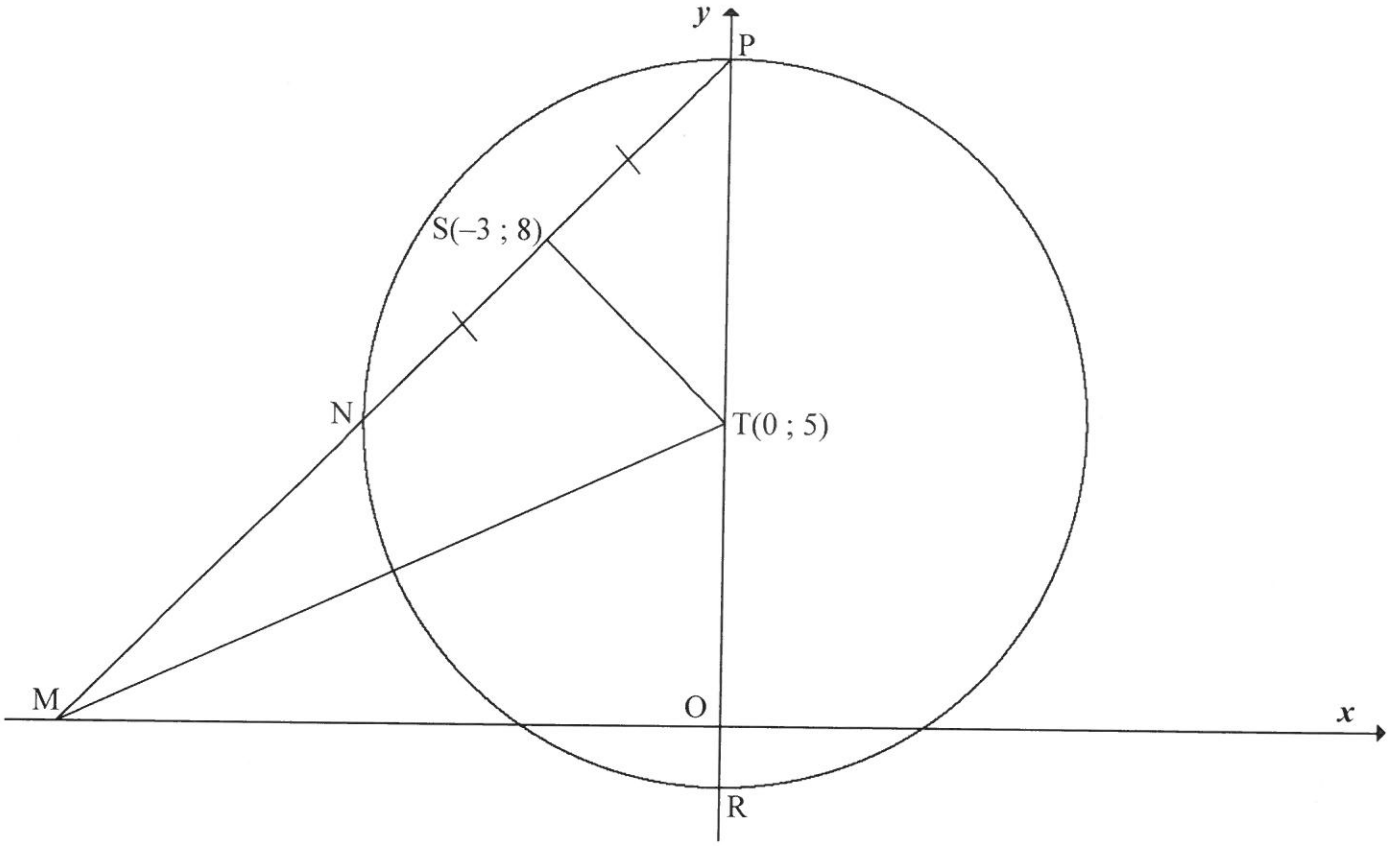


	Solution/*Oplossing*	**Marks** ***Punte***
3.1		(1)
3.2		(2)

	Solution/*Oplossing*	**Marks *Punte***
3.3		(4)
3.4		(4)

	Solution/*Oplossing*	**Marks** ***Punte***
3.5		(6)
3.6.1		(3)
3.6.2		(3)
		[23]

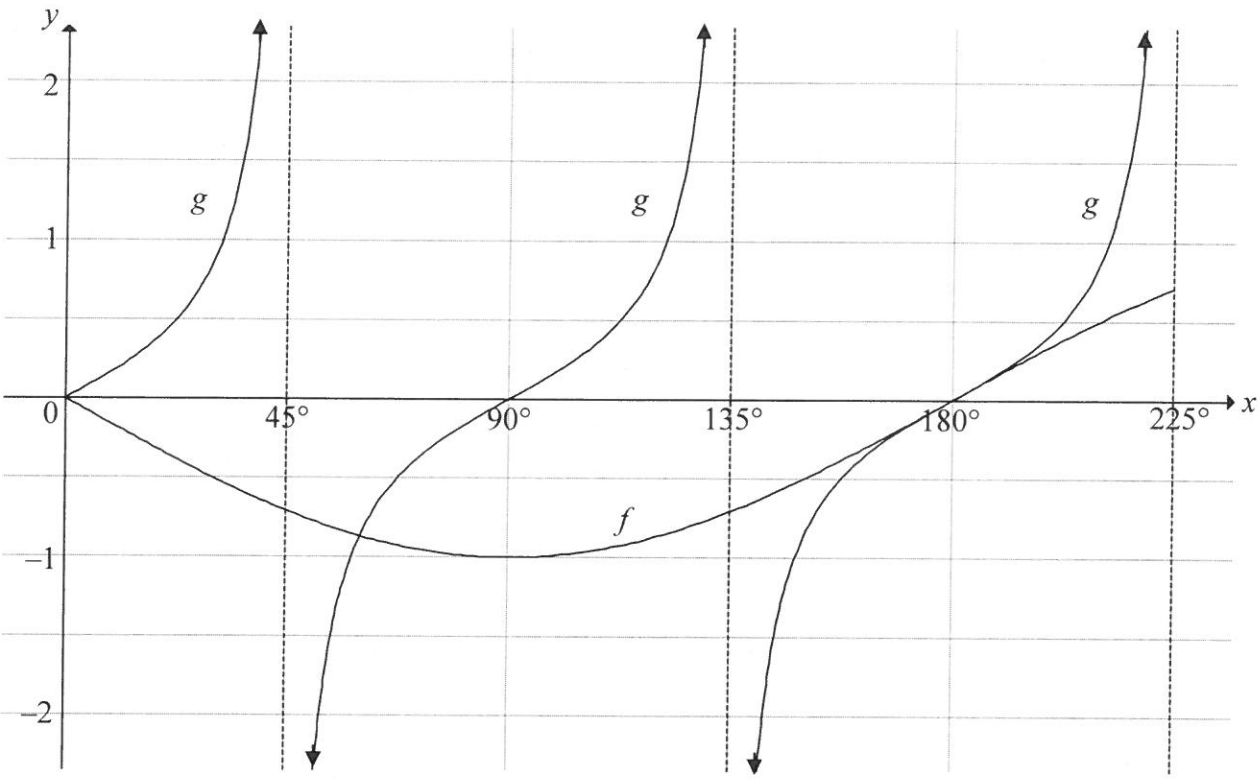
QUESTION/*VRAAG* 4



	Solution/*Oplossing*	**Marks** ***Punte***
4.1		(1)
4.2		(5)

	Solution/*Oplossing*	**Marks** ***Punte***
4.3		(4)
4.4		(4)
4.5		(5)
		[19]

QUESTION/*VRAAG* 5



	Solution/*Oplossing*	**Marks** ***Punte***
5.1		(2)
5.2		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
5.3		(3)
		[7]

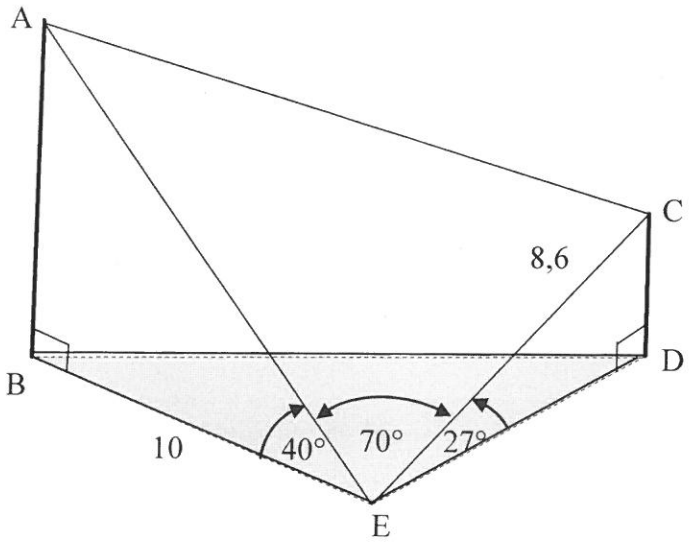
QUESTION/*VRAAG* 6

	Solution/*Oplossing*	**Marks** ***Punte***
6.1.1		(1)
6.1.2		(2)
6.2		

	Solution/*Oplossing*	**Marks** ***Punte***
6.2 (contd/ *vervolg*)		(4)
6.3		(6)
6.4.1		(4)

	Solution/*Oplossing*	**Marks** ***Punte***
6.4.2		(6)
		[23]

QUESTION/*VRAAG* 7

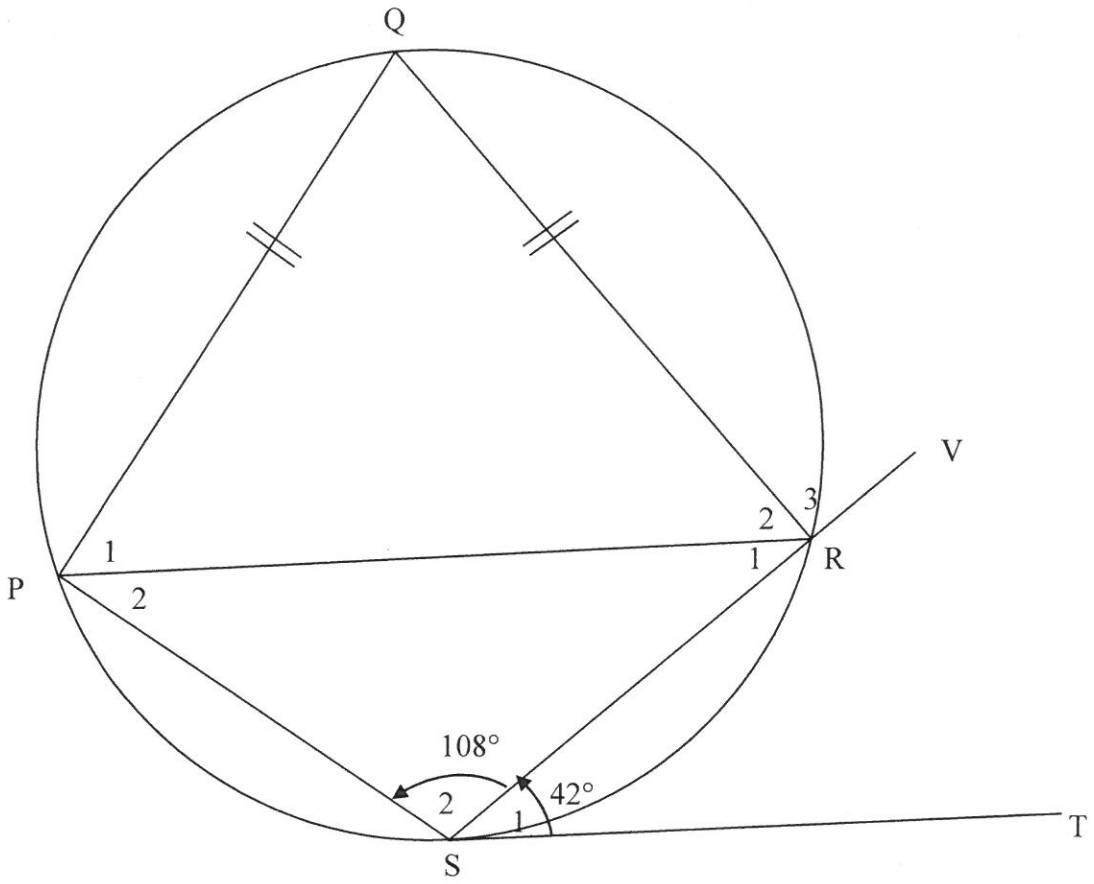


	Solution/*Oplossing*	**Marks** ***Punte***
7.1		(2)
7.2		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
7.3		(4)
		[8]

Give reasons for your statements and calculations in QUESTIONS 8, 9, 10 and 11.
Gee redes vir jou bewerings en berekeninge in VRAAG 8, 9, 10 en 11.

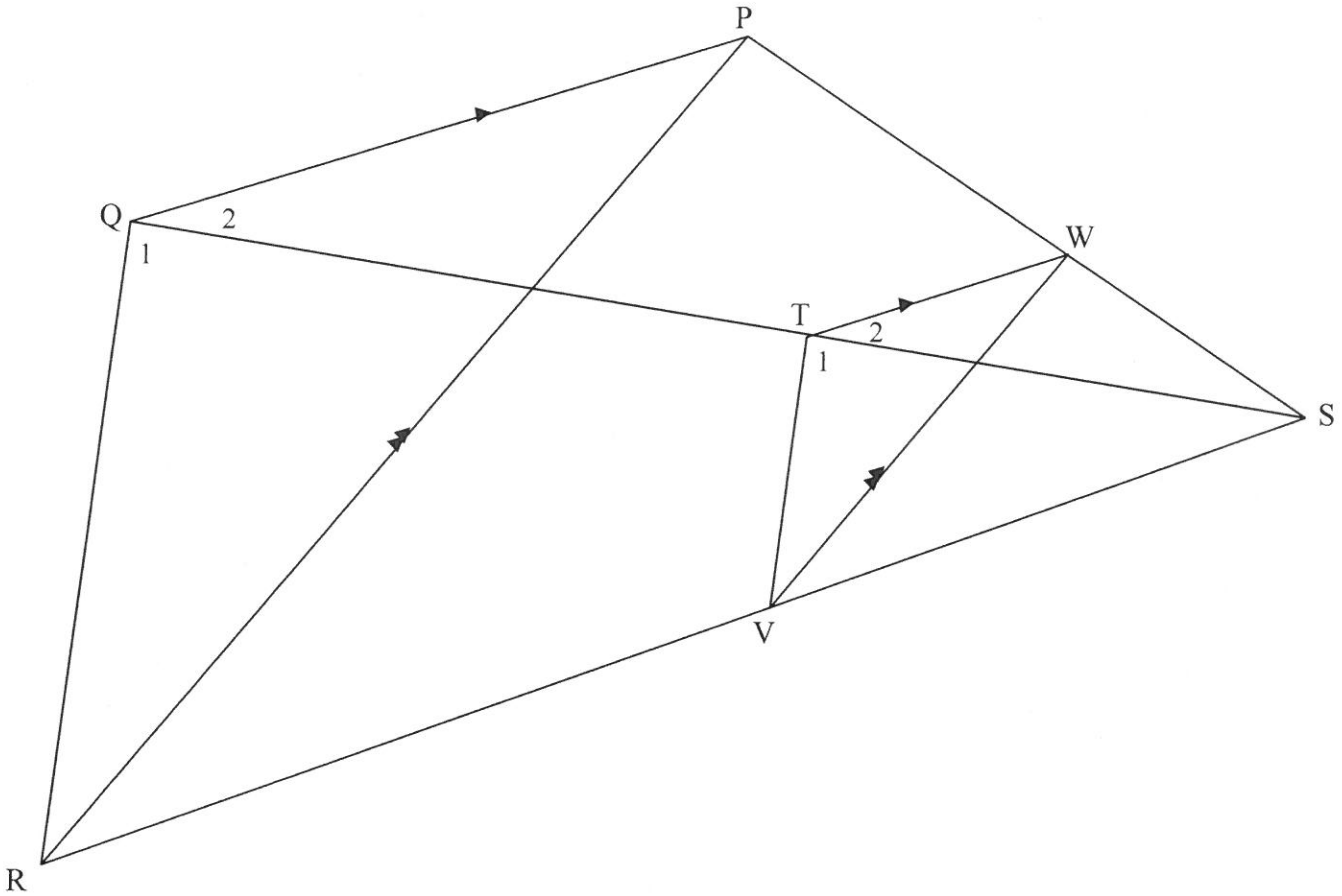
QUESTION/*VRAAG* 8



	Solution/*Oplossing*	**Marks** ***Punte***
8.1		(2)
8.2		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
8.3		(2)
8.4		(2)
		[8]

QUESTION/*VRAAG* 9

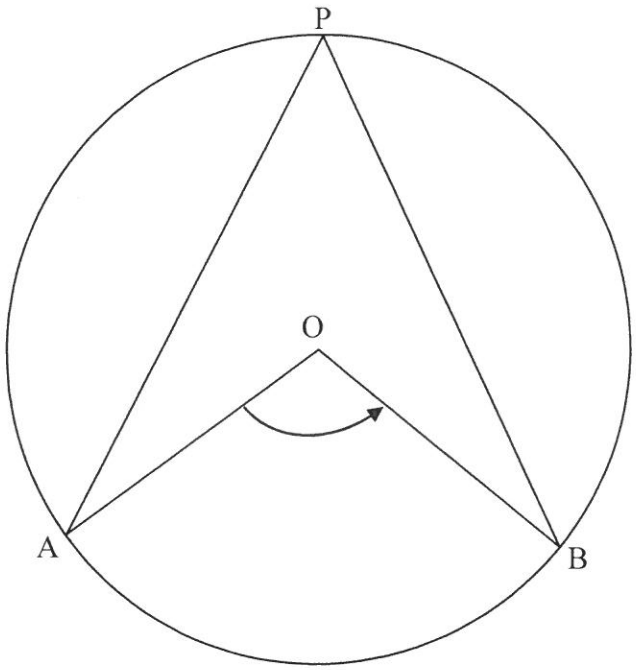


	Solution/*Oplossing*	**Marks** ***Punte***
9.1.1		(2)
9.1.2		(1)

	Solution/*Oplossing*	**Marks** ***Punte***
9.2		(4)
9.3		(1)
9.4		(2)
		[10]

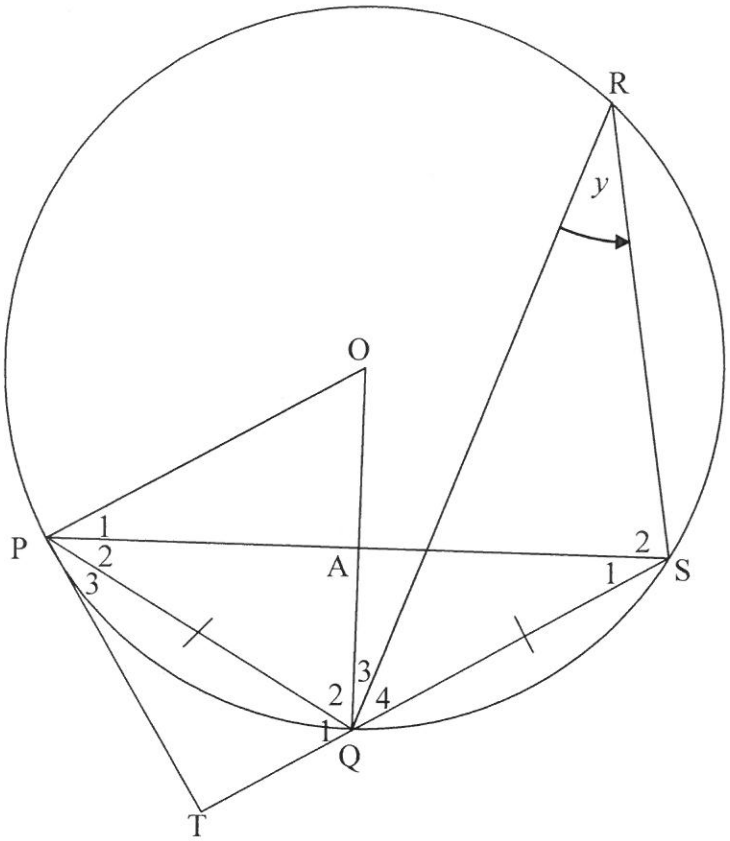
QUESTION/*VRAAG* 10

10.1



	Solution/*Oplossing*	**Marks** ***Punte***
		(5)

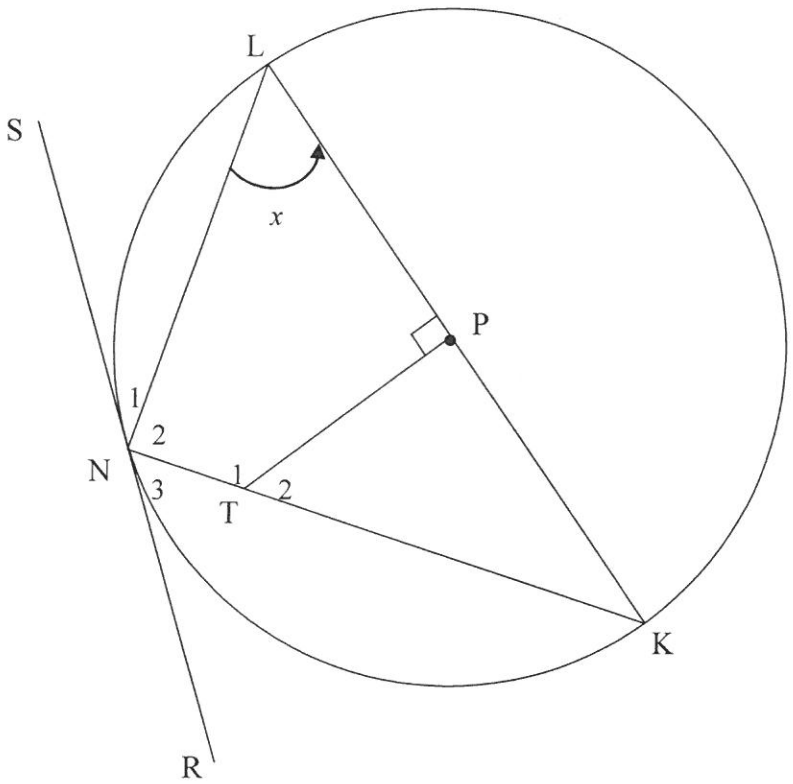
10.2



	Solution/*Oplossing*	**Marks** ***Punte***
10.2.1		(1)
10.2.2		(4)

	Solution/*Oplossing*	**Marks** ***Punte***
10.2.3		(2)
10.2.4		(2)
10.2.5		(5)
		[19]

QUESTION/*VRAAG* 11



	Solution/*Oplossing*	**Marks** ***Punte***
11.1		(3)
11.2		(3)

	Solution/*Oplossing*	**Marks** ***Punte***
11.3.1		(3)
11.3.2		(5)
		[14]

	Additional space/*Addisionele ruimte*	**Marks** ***Punte***

	Additional space/*Addisionele ruimte*	**Marks** ***Punte***

	Additional space/*Addisionele ruimte*	**Marks** ***Punte***

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL/*NASIONALE*
SENIOR
CERTIFICATE/*SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2017

MEMORANDUM

MARKS / PUNTE: 150

This memorandum consists of 21 pages.
Hierdie memorandum bestaan uit 21 bladsye.

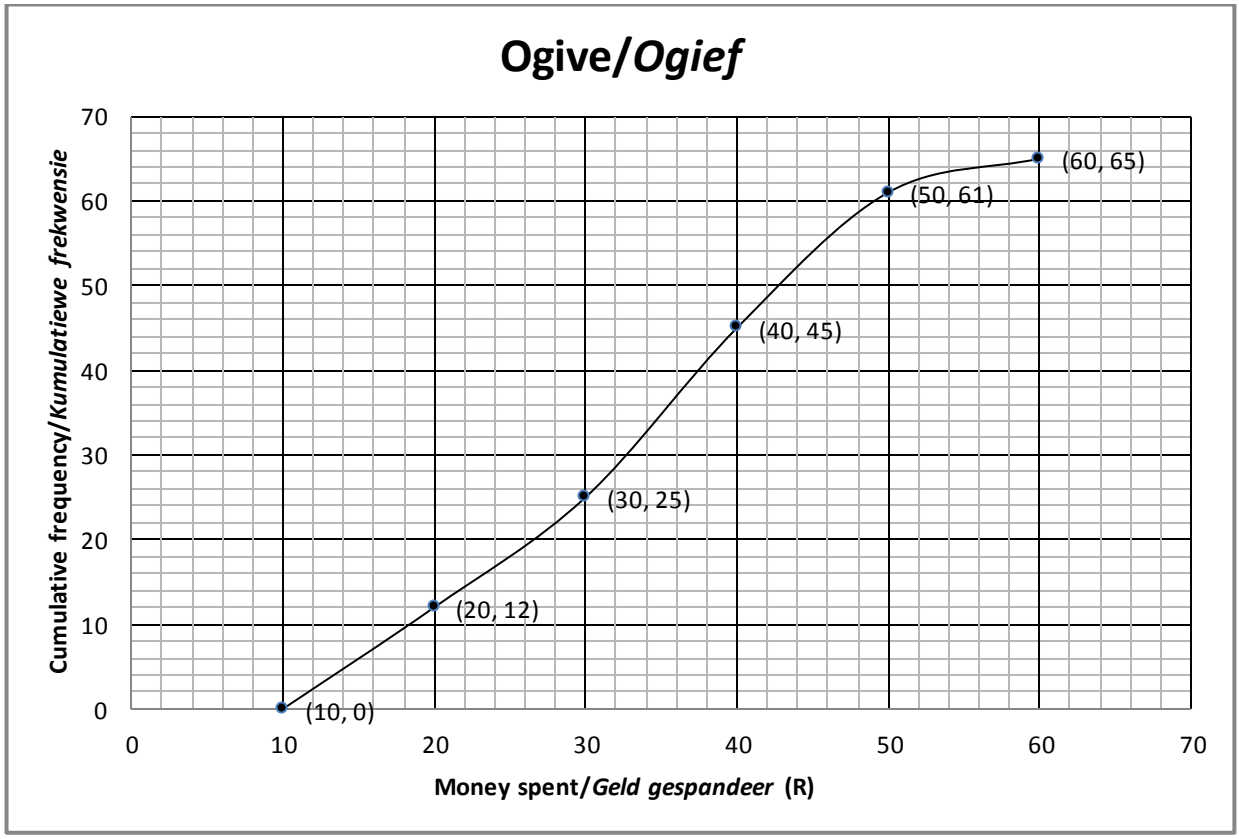
NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate has crossed out an attempt to answer a question and did not redo it, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord, doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing. Staak nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.*

QUESTION/*VRAAG* 1

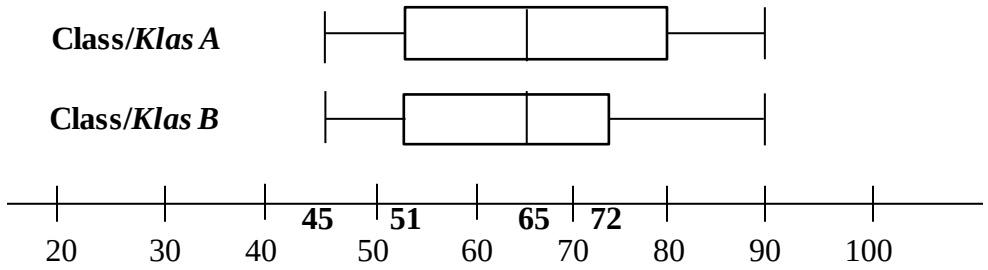


Amount of money/ *Bedrag geld* (in R)	$10 \le x < 20$	$20 \le x < 30$	$30 \le x < 40$	$40 \le x < 50$	$50 \le x < 60$
Frequency *Frekwensie*	*a*	13	20	*b*	4

1.1	65 learners/*leerders*	✓ answer (1)
1.2	Modal class/*Modale klas*: $30 \le x < 40$	✓ answer (1)
1.3	$a = 12$ $b = 61 - 45$ $= 16$	✓ answer ✓ answer (2)
1.4	No. of learners/*Aantal leerders* = 65 – 54 **OR/*OF*** 65 – 55 = 11 = 10 Answer only: full marks	✓ 54 or 55 ✓ 11 or 10 (2) **[6]**

QUESTION/*VRAAG* 2

2.1



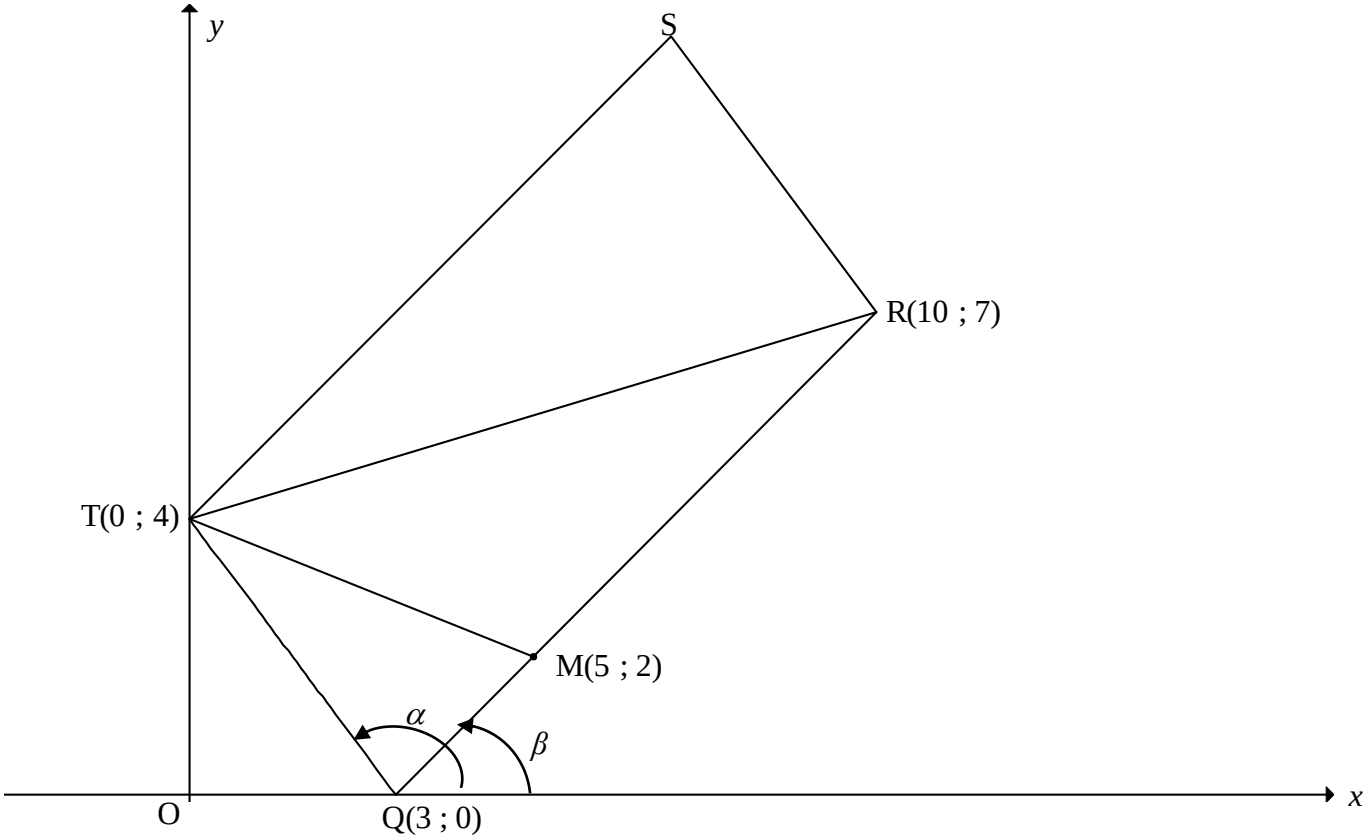
2.1.1	IQR of Class B/*IKV van Klas B* = $Q_3 - Q_1$ = 72 – 51 = 21 marks/*punte*	✓72 and 51 ✓ 21 only (2)
2.1.2	Although the boxes contain the same number of data points, the marks for Class A are more widely spread./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas A meer verspreid.* **OR/OF** Although the boxes contain the same number of data points, the marks for Class B are more clustered./*Alhoewel die monde dieselfde aantal datapunte bevat, is die punte van Klas B nader aan mekaar.*	✓ ✓ Class A is more widely spread (2) ✓ ✓ Class B is more clustered (2)
2.1.3	Medians are the same/*Mediane is dieselfde* Ranges are the same **OR** Maximum and minimum values are the same/*Variasiewydtes is dieselfde **OF** die maksimum en minimum waarde is dieselfde* 75% of both classes obtained 51 and above/*75% van albei klasse behaal 51 en meer.*	✓ ✓ any TWO of the 3 reasons mentioned (2)

2.2

COUPLE/*PAAR*	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
JUDGE 1/ *BEOORDELAAR 1*	18	4	6	8	5	12	10	14
JUDGE 2/ *BEOORDELAAR 2*	15	6	3	5	5	14	8	15

2.2.1	$a = -0,03$ $b = 0,93$ $\hat{y} = -0,03 + 0,93x$	✓ value a ✓ value b ✓ equation (3)
2.2.2	$\hat{y} = -0,03 + 0,93(15)$ = 13,92 **OR/*OF*** 13,85 ≈ 14	✓ substitution ✓ answer (2)
2.2.3	Yes **OR** they are consistent, because $r = 0,9$. ($r = 0,89567\ldots$)/*Ja OF hulle is konsekwent, want $r = 0,9$. ($r = 0,89567\ldots$)*	✓ statement ✓ $r = 0,9$ (2) **[13]**

QUESTION/*VRAAG* 3



3.1	$m_{TQ} = \frac{4-0}{0-3}$ $= -\frac{4}{3}$	✓ answer (1)
3.2	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $RQ = \sqrt{(10-3)^2 + (7-0)^2}$ $RQ = \sqrt{98} = 7\sqrt{2}$	✓substitution/*substitusie* ✓ answer in surd form (2)
3.3	$m_{FQ} = m_{TQ}$ $\frac{-8}{k-3} = -\frac{4}{3}$ $4k - 12 = 24$ $k = 9$ **OR/*OF*** $m_{FT} = m_{QT}$ $\frac{-8-4}{k-0} = -\frac{4}{3}$ $-36 = -4k$ $k = 9$ **OR/*OF*** Equation of TQ: $y = -\frac{4}{3}x + 4$ $-8 = -\frac{4}{3}k + 4$ $k = 9$	✓ equating gradients/*stel gradient gelyk* ✓ $m_{FQ} = \frac{-8}{k-3}$ ✓ simplification/*vereenvoudig* ✓ answer (4) ✓ gradient ✓ equation of TQ/*vgl van TQ* ✓ substitution of $(k ; -8)$ /*substitusie van* $(k ; -8)$ ✓ answer (4)

3.4	Using transformation/*Gebruik transformasie*: $\therefore$ S(7 ; 11) **OR/*OF*** Midpoint of TR = midpoint of SQ [diag ||m/*hkle*||*m*] Midpoint of TR = $(5 ; \frac{11}{2})$ $\frac{x_S + 3}{2} = 5$ and $\frac{y_S + 0}{2} = \frac{11}{2}$ $\therefore x_S = 7$ and $y_S = 11$ $\therefore$ S(7 ; 11) **OR/*OF*** Equation of TS: $y = \left(\frac{7-2}{10-5}\right)x + 4 = x + 4$ Equation of RS: $y - 7 = -\frac{4}{3}(x - 10)$ $y = -\frac{4}{3}x + \frac{61}{3}$ $x + 4 = -\frac{4}{3}x + \frac{61}{3}$ $7x = 49$ $x = 7$ $\therefore \quad y = 11$ $\therefore$ S(7 ; 11)	✓ ✓ *x*–value/*waarde* ✓ ✓ *y*–value/*waarde* (4) ✓ *x*–value/*waarde* of/*van* T ✓ *y*–value/*waarde* of/*van* T ✓ *x*–value/*waarde* of/*van* S ✓ *y*–value/*waarde* of/*van* S (4) ✓ equations of TS and RS/*vgls van TS en RS* ✓ equating / *gelykstel* ✓ *x*–value/*waarde* ✓ *y*–value/*waarde* (4)
3.5	$T\hat{S}R = T\hat{Q}R$ [opp ∠s of ||m/*teenoorst ∠e* ||*m*] $T\hat{Q}R = \alpha - \beta$ $tan\,\alpha = m_{TQ} = -\frac{4}{3}$ $\therefore \alpha = 180° - 53{,}13° = 126{,}87°$ $tan\,\beta = m_{RQ} = \frac{7}{7} = 1$ $\therefore \beta = 45°$ $T\hat{Q}R = 126{,}87° - 45°$ $= 81{,}87°$ $T\hat{S}R = 81{,}87°$ **OR/*OF***	✓ $T\hat{Q}R = \alpha - \beta$ ✓ $\tan\alpha = m_{TQ}$ ✓ α ✓ $\tan\beta = m_{RQ}$ ✓ β ✓ answer (6)

	TQ = SR = 5 TR = $\sqrt{100+9}=\sqrt{109}$ RQ = TS = $\sqrt{49+49}=\sqrt{98}$ $\cos R\hat{Q}T = \cos T\hat{S}R = \frac{TQ^2+RQ^2-TR^2}{2.TQ.RQ}$ $= \frac{25+98-109}{2(5)(\sqrt{98})}$ $= 0{,}141...$ $R\hat{Q}T = T\hat{S}R = 81{,}87°$	✓ length of TQ **OR** SR ✓ length of TR ✓ length of RQ **OR** TS ✓ correct subst into cosine rule ✓ simplification ✓ answer (6)
3.6.1	$MQ=\sqrt{(5-3)^2+(2-0)^2}$ $MQ=\sqrt{8}$ $\frac{MQ}{RQ}=\frac{\sqrt{8}}{\sqrt{98}}$ Answer only: full marks $=\frac{2}{7}$ or 0,29	✓substitution/*substitusie* ✓ $MQ=\sqrt{8}=2\sqrt{2}$ ✓ answer (3)
3.6.2	$\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR}=\frac{\frac{1}{2}.QM.\perp h}{\frac{1}{2}.QR.\perp h}$ [⊥h same/*dieselfde*] $=\frac{QM}{QR}=\frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS}=\frac{\text{area of } \Delta TQM}{2\times\text{area of } \Delta TQR}$ $=\frac{1}{2}\left(\frac{2}{7}\right)=\frac{1}{7}$ **OR/*OF*** $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR}=\frac{QM}{QR}$ $=\frac{2}{7}$ $\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS}=\frac{\text{area of } \Delta TQM}{2\text{area of } \Delta TQR}$ $=\frac{1}{2}\left(\frac{2}{7}\right)=\frac{1}{7}$ **OR/*OF***	✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR}=\frac{2}{7}$ ✓ area parm RQTS = 2area ΔTQR ✓ answer (3) ✓ $\frac{\text{area of } \Delta TQM}{\text{area of } \Delta TQR}=\frac{2}{7}$ ✓ area parm RQTS = 2area ΔTQR ✓ answer (3)

$$\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\frac{1}{2}QM.\perp h}{RQ.\perp h}$$

✓ $\frac{\frac{1}{2}QM.\perp h}{RQ.\perp h}$

$$= \frac{1}{2}\left(\frac{2}{7}\right)$$

✓ $\frac{1}{2}\left(\frac{2}{7}\right)$

$$= \frac{1}{7}$$

✓ answer

(3)

OR/*OF*

$$\frac{\text{area of } \Delta TQM}{\text{area of parm } RQTS} = \frac{\frac{1}{2}QT.QM.\sin(\alpha-\beta)}{2\text{area of } \Delta QTR}$$

✓ area parm $RQTS = 2\text{area } \Delta TQR$

$$= \frac{\frac{1}{2}QT.QM.\sin(\alpha-\beta)}{2[\frac{1}{2}.QT.QR.\sin(\alpha-\beta)]}$$

✓ $\frac{\frac{1}{2}QT.QM.\sin(\alpha-\beta)}{2[\frac{1}{2}.QT.QR.\sin(\alpha-\beta)]}$

$$= \frac{1}{2}\left(\frac{2}{7}\right)$$

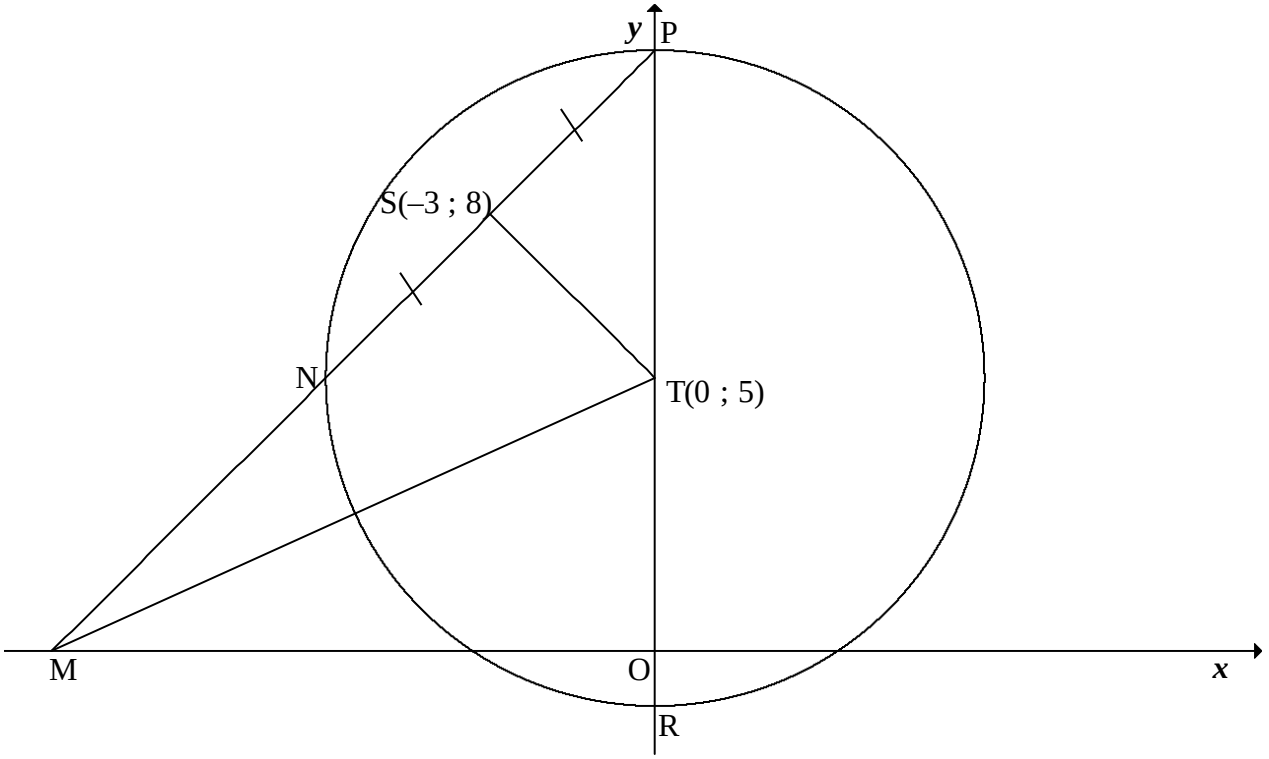
$$= \frac{1}{7}$$

✓ answer

(3)

[23]

QUESTION/*VRAAG* 4



4.1	line from centre to **midpt of chord** / *lyn vanaf midpt na midpt van koord*	✓ answer (1)
4.2	$m_{ST} = \frac{8-5}{-3-0}$ $= -1$ $m_{ST} \times m_{NP} = -1$ [TS ⊥ NP] $\therefore\ m_{NP} = 1$ $\therefore y = x + c$ $8 = -3 + c$ $c = 11$ $\therefore y = x + 11$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 8 = 1(x + 3)$ $y = x + 11$	✓ subst (–3 ; 8) and (0 ; 5) into gradient formula ✓ m_{ST} ✓ m_{NP} ✓ subst (–3 ; 8) into equation of a line ✓ equation (5)
4.3	P(0 ; 11) [*y*-intercept of chord NP] ∴ radius is 6 units R(0 ; –1) Equations of the tangents to the circle parallel to the *x*-axis/ *Vgls van die raaklyne aan die sirkel || aan die x-as*: $y = 11$ and $y = -1$	✓ coordinates of P/ koördinate v P ✓ coordinates of R koördinate van R ✓✓ answers (4)
4.4	M(–11 ; 0) [*x*–intercept of/*x-afsnit van* NP] $MT = \sqrt{(0-11)^2 + (5-0)^2}$ $MT = \sqrt{146} = 12{,}08$	✓✓ coordinates of M ✓ substitution ✓ answer (4)

4.5	MT = diameter/*middellyn* [conv∠ in $\frac{1}{2}$ circle/*omgek ∠ in $\frac{1}{2}$ sirkel*] radius = $\frac{\sqrt{146}}{2}$ units Centre of circle/*Middelpunt v sirkel* = Midpoint MT /*Middelpunt MT* = $\left(\frac{-11}{2};\frac{5}{2}\right)$ Equation of circle through S, T and M: $\left(x+\frac{11}{2}\right)^2+\left(y-\frac{5}{2}\right)^2=\frac{146}{4}$ **OR/*OF*** $\left(x+5\frac{1}{2}\right)^2+\left(y-2\frac{1}{2}\right)^2=\frac{73}{2}=6{,}04$	✓ radius of circle ✓ *x* value of M ✓ *y* value of M ✓ LHS of equation ✓ RHS of equation (5) **[19]**

QUESTION/*VRAAG* 5

5.1	$a=-1$ $b=2$	✓ answer ✓ answer (2)
5.2	$f(3x)=-\sin 3x$ Period of $f(3x)=\frac{360°}{3}$ $=120°$ Answer only: Full marks	✓ $\frac{360°}{3}$ ✓ answer (2)
5.3	$x \in [90° ; 135°) \cup \{180°\}$ **OR/*OF*** $90° \leq x < 135°$ or $x = 180°$	✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct brackets (3) ✓ 90° and 135° in interval form ✓ 180° as single value ✓ correct inequalities (3) **[7]**

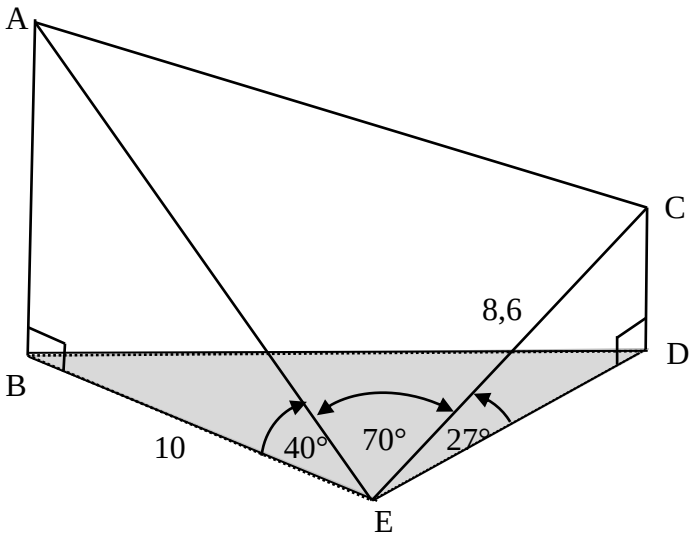
QUESTION/*VRAAG* 6

6.1.1	$\sin(360° - 36°) = -\sin 36°$	✓ answer (1)
6.1.2	$\cos 72° = \cos(2 \times 36°)$ $= 1 - 2\sin^2 36°$ Answer only: Full marks	✓ double angle/*dubbelhoek* ✓answer (2)
6.2	R.T.P.: $1 - \frac{\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta$ $\text{LHS} = \frac{1+\tan^2\theta-\tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{\frac{1}{\cos^2\theta}}$ $= \cos^2\theta$ = RHS **OR/*OF*** $\text{LHS} = \frac{1+\tan^2\theta-\tan^2\theta}{1+\tan^2\theta}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ $= \frac{1}{1+\frac{\sin^2\theta}{\cos^2\theta}} \times \frac{\cos^2\theta}{\cos^2\theta}$ $= \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}$ $= \frac{\cos^2\theta}{1}$ $= \cos^2\theta$ = RHS **OR/*OF***	✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity/ *kwosiëntidentiteit* ✓ denominator as a single fraction / *Noemer as enkelbreuk* ✓ square identity/*vierkantidentiteit* (4) ✓ writing as a single fraction/*skryf as enkelbreuk* ✓ quotient identity / *kwosiëntidentiteit* ✓ $\times \frac{\cos^2\theta}{\cos^2\theta}$ ✓ square identity/*vierkantidentiteit* (4) ✓ quotient identity/

	$\text{LHS} = 1-\left(\frac{\sin^2\theta}{\cos^2\theta} \div \left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\right)$ $= 1-\left(\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}\right)$ $= 1-\left(\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{1}\right)$ $= 1-\sin^2\theta$ $= \cos^2\theta$ $= \text{RHS}$	*kwosiëntidentiteit* ✓ writing as a single fraction/ *skryf as enkelbreuk* ✓ square identity/*vierkantidentiteit* ✓ simplification/*vereenvoudiging* (4)
6.3	$\cos^2\frac{1}{2}x = \frac{1}{4}$ $\cos\frac{1}{2}x = \frac{1}{2}$ *or* $-\frac{1}{2}$ $\frac{1}{2}x = 60°+k.360°$ or $\frac{1}{2}x = 300°+k.360°$ or $\frac{1}{2}x = 120°+k.360°$ or $\frac{1}{2}x = 240°+k.360°$ $x = 120°+k.720°$ or $x = 600°+k.720°$ or $x = 240°+k.720°$ or $x = 480°+k.720°;\ k \in Z$ **OR/*OF*** $\cos^2\frac{1}{2}x = \frac{1}{4}$ $\cos\frac{1}{2}x = \frac{1}{2}$ *or* $-\frac{1}{2}$ $\frac{1}{2}x = \pm 60°+k.360°$ or $\frac{1}{2}x = \pm 120°+k.360°$ $x = \pm 120°+k.720°$ or $x = \pm 240°+k.720°;\ k \in Z$	✓✓ $\cos^2\frac{1}{2}x = \frac{1}{4}$ ✓ 60° and 300° ✓ 120° and 240° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°;\ k \in Z$ (6) ✓✓ $\cos^2\frac{1}{2}x = \frac{1}{4}$ ✓ ±60° ✓±120° ✓ write at least one general solution as $\frac{1}{2}x = \angle + k.360°$ ✓ write at least one general solution as $x = \angle + k.720°\ k \in Z$ (6)

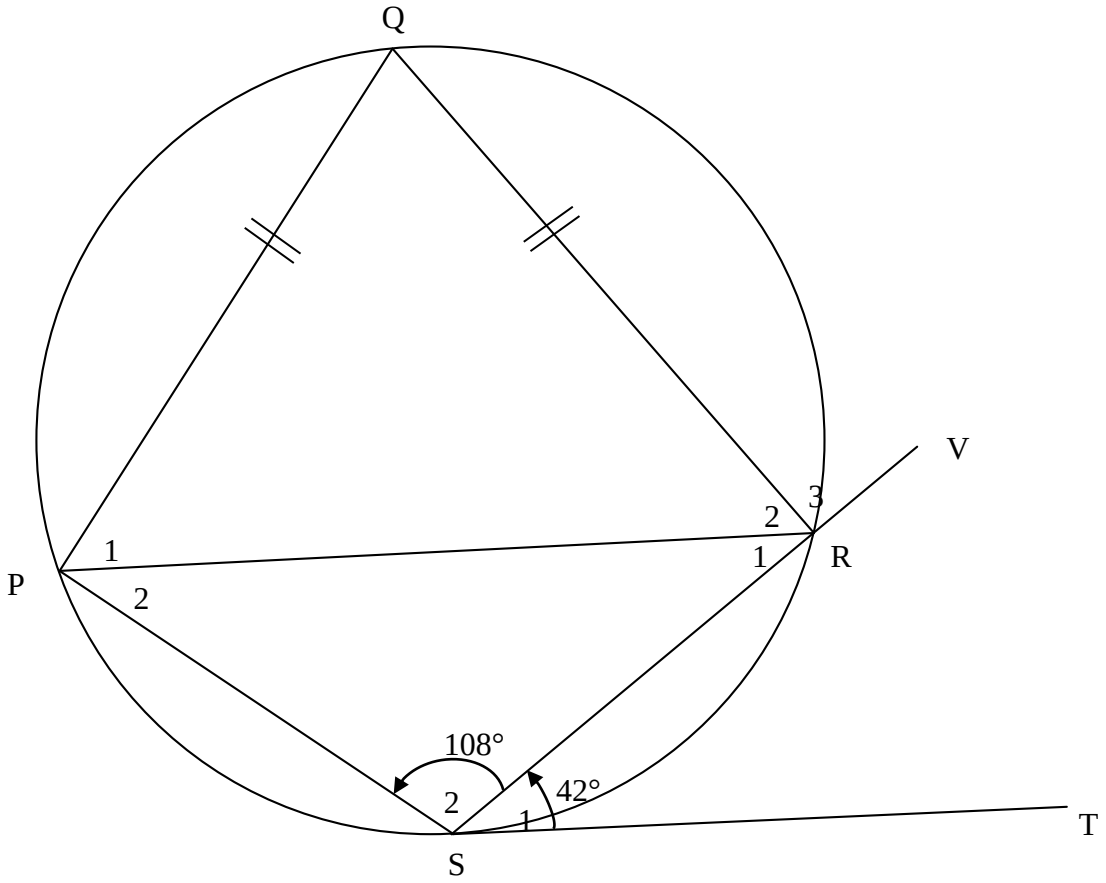
6.4.1	$\sin(A-B) = \cos[90°-(A-B)]$ $= \cos[(90°-A)-(-B)]$ $= \cos(90°-A)\cos(-B)+\sin(90°-A)\sin(-B)$ $= \sin A\cos B+\cos A(-\sin B)$ $= \sin A\cos B-\cos A\sin B$ **OR/*OF*** $\sin(A-B) = \cos[90°-(A-B)]$ $= \cos[(90°+B)-A]$ $= \cos(90°+B)\cos A+\sin(90°+B)\sin A$ $= -\sin B\cos A+\cos B\sin A$ $= \sin A\cos B-\cos A\sin B$	✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4) ✓ co–ratio/*ko-verhouding* ✓ writing as a difference of A & B/ *skryf as verskil van A & B* ✓ expansion/*uitbreiding* ✓ all reductions/*alle reduksies* (4)
6.4.2	$\sin(x+64°)\cos(x+379°)+\sin(x+19°)\cos(x+244°)$ $= \sin(x+64°)\cos(x+19°)+\sin(x+19°)[-\cos(x+64°)]$ $= \sin(x+64°)\cos(x+19°)-\cos(x+64°)\sin(x+19°)$ $= \sin[x+64°-(x+19°)]$ $= \sin 45°$ $= \frac{1}{\sqrt{2}}$	✓ $\cos(x+379°) = \cos(x+19°)$ ✓✓ $\cos(x+244°) = -\cos(x+64°)$ ✓✓compound formula identity/ *saamgestelde identiteit* ✓ $\sin 45°$ (6) **[23]**

QUESTION/*VRAAG* 7



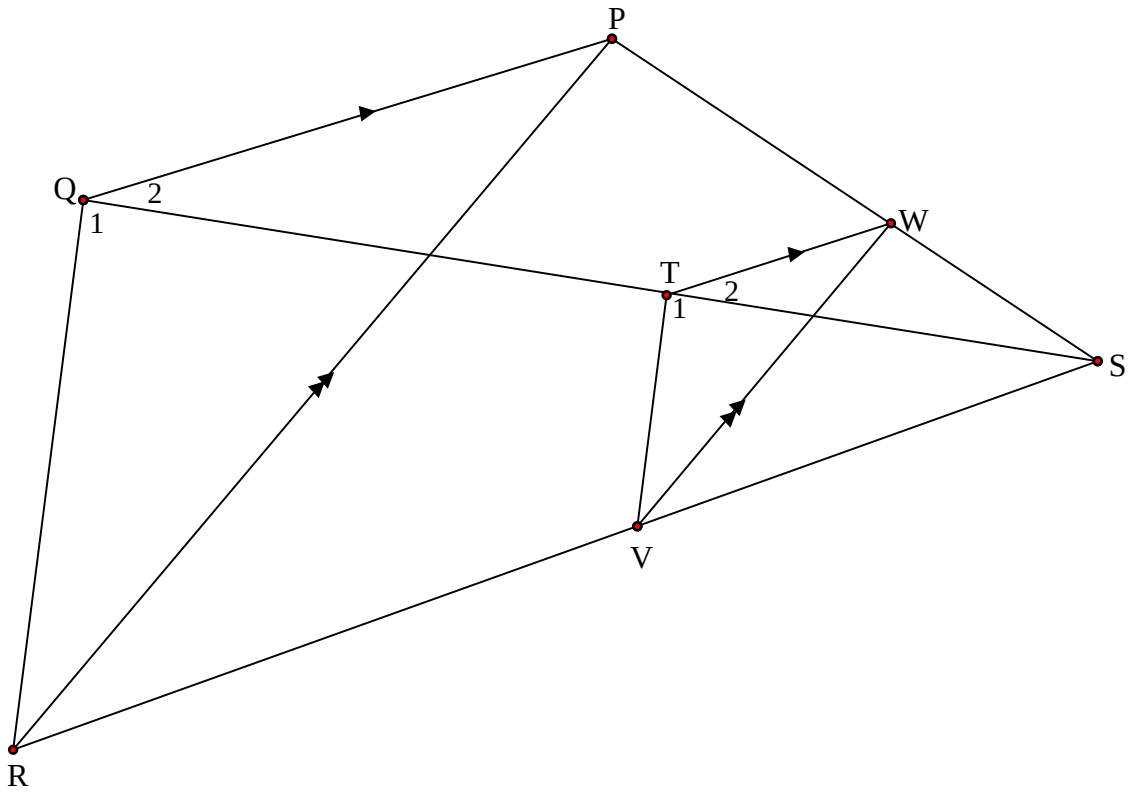
7.1	$\sin 27° = \frac{CD}{8,6}$ $CD = 8,6\sin 27°$ $CD = 3,90\text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.2	$\cos 40° = \frac{10}{AE}$ $AE = \frac{10}{\cos 40°}$ $AE = 13,05\text{ m}$	✓ substitution in correct trig ratio / *substitusie in korrekte trig verh* ✓ answer (2)
7.3	$AC^2 = CE^2 + AE^2 - 2\,CE.AE(\cos A\hat{E}C)$ $= (8,6)^2 + (13,05)^2 - 2(8,6)(13,05)(\cos 70°)$ $= 167,49$ $AC = 12,94\text{ m}$	✓ correct use of cosine rule in ΔACE/ *korrekte gebruik van reel in* ΔACE ✓ correct subst into cosine rule ✓ AC^2 ✓ answer (4) **[8]**

QUESTION/*VRAAG* 8



8.1	$\hat{Q} = 72°$ [opp ∠s of cyclic quad/*teenoorst ∠e koordevh*]	✓ S ✓ R (2)
8.2	$\hat{R}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*] $\hat{R}_2 = \frac{180° - 72°}{2}$ [sum of ∠s in Δ/*som v ∠e in Δ*] $= 54°$	✓ S/R ✓ answer (2)
8.3	$\hat{P}_2 = 42°$ [tan chord theorem/*raakl-koordst*]	✓ S ✓ R (2)
8.4	$\hat{R}_3 = \hat{P}_1 + \hat{P}_2$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $= 54° + 42°$ $= 96°$ **OR/*OF*** $\hat{R}_1 = 180° - 108° - 42° = 30°$ [sum of/*som van*∠s/e in Δ] $\hat{R}_3 = 180° - \hat{R}_1 - \hat{R}_2$ [∠s on str line/*∠e op reguitlyn*] $= 180° - 30° - 54°$ [sum of/*som van*∠s/e in Δ] $= 96°$	✓ R ✓ S (2) ✓ $\hat{R}_1 = 30°$ ✓ S (2) **[8]**

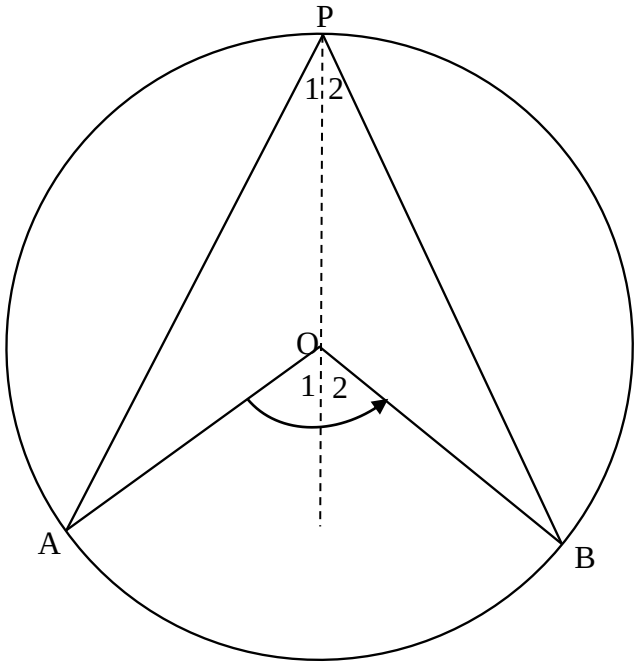
QUESTION/*VRAAG* 9



9.1.1	$\frac{ST}{TQ} = \frac{SW}{WP}$ [prop theorem/*eweredighst*; TW ‖ QP] $= \frac{2}{3}$	✓ S ✓ S (2)
9.1.2	$\frac{SV}{VR} = \frac{SW}{WP}$ [prop theorem/*eweredighst*; VW ‖ RP] $= \frac{2}{3}$	✓ answer (1)
9.2	$\frac{ST}{TQ} = \frac{SV}{VR}$ [both equal/*beide gelyk* $\frac{WS}{PW}$] $\therefore$ TV ‖ QR [line divides 2 sides of Δ in prop/*lyn verdeel 2 sye van Δ in dies verh*] $\therefore \hat{T}_1 = \hat{Q}_1$ [corresp/*ooreenkomst* ∠s/e; TV ‖ QR]	✓ S ✓ S ✓ R ✓ R (4)
9.3	ΔVWS ||| ΔRPS	✓ ΔRPS (any order) (1)
9.4	$\frac{WV}{PR} = \frac{SW}{SP}$ [ΔVWS ||| ΔRPS] $= \frac{2}{5}$ **OR/*OF*** $\frac{WV}{PR} = \frac{SV}{SR}$ [ΔVWS ||| ΔRPS] $= \frac{2}{5}$	✓ ratio ✓ answer (2) **[10]**

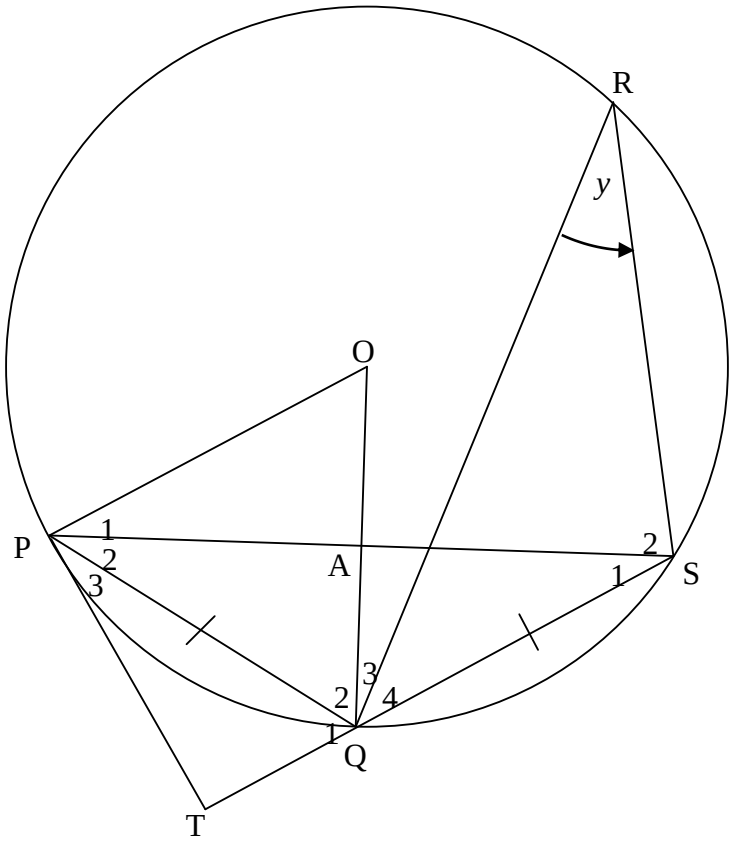
QUESTION/*VRAAG* 10

10.1



	Constr/*Konst* : Draw line PO and extend /*Trek lyn PO en verleng* Proof/*Bewys* : $OP = OA$ [radii] $\therefore \hat{P}_1 = \hat{A}$ [$\angle$s opp/*teenoor* = sides/*sye*] but $\hat{O}_1 = \hat{P}_1 + \hat{A}$ [ext $\angle$ of Δ] $\therefore \hat{O}_1 = 2\hat{P}_1$ Similarly/*Netso*, $\hat{O}_2 = 2\hat{P}_2$ $\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{P}_1 + \hat{P}_2)$ i.e. $A\hat{O}B = 2A\hat{P}B$	✓ construction ✓ S/R ✓ S/R ✓ S ✓ S (5)

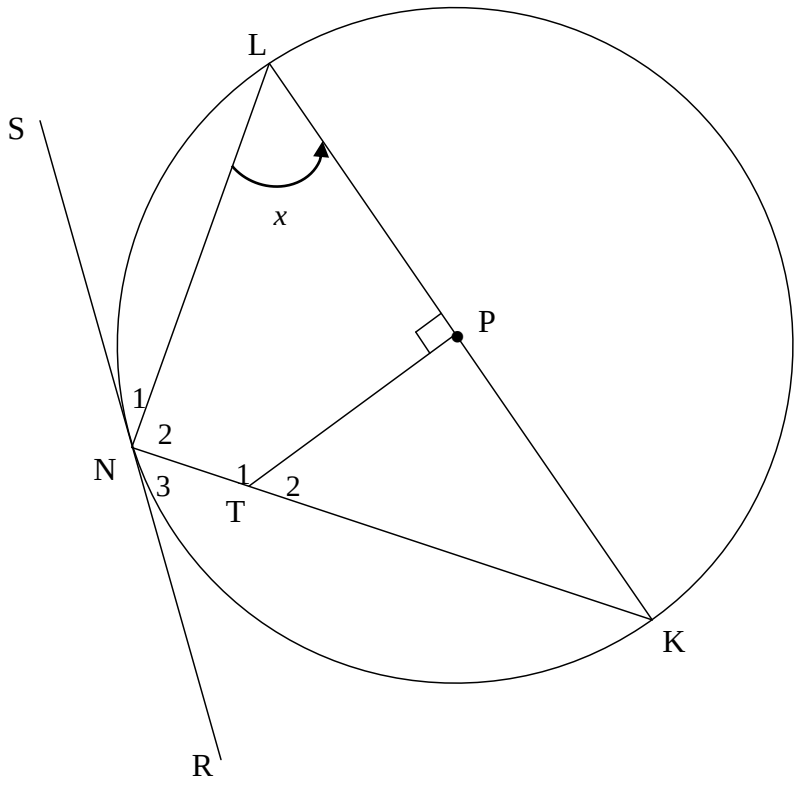
10.2



10.2.1	∠s in the same segment/ *∠e in dieselfde sirkelsegment*	✓ R (1)
10.2.2	$\hat{P}_2 = \hat{S}_1 = y$ [∠s opp equal sides/*∠e teenoor = sye*] $\hat{S}_1 = \hat{P}_3 = y$ [tan chord theorem/*raakl-koordst*] $\therefore\ \hat{P}_2 = \hat{P}_3$ $\therefore$ PQ bisects $T\hat{P}S$	✓ S ✓ R ✓ S ✓ R (4)
10.2.3	$P\hat{O}Q = 2\hat{S}_1 = 2y$ [∠at centre =2×∠at circ/*midpts∠= 2×omtreks∠]*	✓ S ✓ R (2)
10.2.4	$T\hat{P}A = \hat{P}_2 + \hat{P}_3 = 2y$ [proved/*bewys* in 11.2.2] $\therefore\ T\hat{P}A = P\hat{O}Q$ [proved/*bewys* in 11.2.3] $\therefore$ PT = tangent [converse tan chord theorem/*omgek raakl-koordst*]	✓ $T\hat{P}A = P\hat{O}Q$ ✓ R (2)

10.2.5	$O\hat{P}Q + O\hat{Q}P = 180° - 2y$ [sum of/*sum v ∠s/e* in Δ]	✓ S
	$\therefore O\hat{Q}P = 90° - y$ [∠s opp equal sides/*∠e to = sye;* OP = OQ]	✓ S ✓ R
	In ΔPAQ:	
	$O\hat{Q}P + \hat{P}_2 + Q\hat{A}P = 180°$	
	$90° - y + y + Q\hat{A}P = 180°$ [sum of/*sum v ∠s/e* in Δ]	✓ S
	$Q\hat{A}P = 90°$	✓ S
	$\therefore O\hat{A}P = 90°$ [∠s/*e* on straight line/*op reguitlyn*]	(5)
	OR/*OF*	
	$O\hat{P}T = 90°$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$\therefore \hat{P}_1 = 90° - 2y$	✓ S
	$\hat{P}_1 + \hat{O} + O\hat{A}P = 180°$ [sum of/*sum v ∠s/e* in Δ]	
	$(90° - 2y) + 2y + O\hat{A}P = 180°$	✓ S
	$\therefore O\hat{A}P = 90°$	✓ S (5)
	OR/*OF*	
	POSQ is a kite/'*n vlieër*	
	$\therefore OQ \perp PS$ [diag of a kite/*hoeklyne v vlieër*]	✓✓✓ S
	$\therefore O\hat{A}P = 90°$	✓✓ R (5)
	OR/*OF*	
	In ΔOAP and ΔOAS	
	OP = OS (radii)	✓ S
	OA is common	✓ S
	$P\hat{O}A = 2y$	
	$= 2\hat{P}_2$	
	$= Q\hat{O}S$	✓ S
	$\Delta OAP \equiv \Delta OAS$ (SAS)	✓ R
	$O\hat{A}P = O\hat{A}S$ (≡Δs)	
	$O\hat{A}P = O\hat{A}S = 90°$ (∠s on str line)	✓ S (5) **[19]**

QUESTION/*VRAAG* 11



11.1	$\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*] ∴ TPLN is a cyclic quad/ 'n *koordevh* [opp ∠s of quad is suppl/ **t***eenoor∠e v vh is suppl*] **OR** $\hat{N}_2 = 90°$ [∠ in semi-circle/*halfsirkel*] ∴ TPLN is a cyclic quad [ext ∠ = int opp ∠/*buite∠ = to binne ∠*]	✓ S ✓ R ✓ R (3) ✓ S ✓ R ✓ R (3)
11.2	$\hat{T}_2 = P\hat{L}N = x$ [ext ∠ of cyclic quad/*buite∠ van koordevh*] $\hat{K} = 90° - x$ [sum of/*som v ∠*s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{K} = 90° - x$ [sum of/*som v ∠*s/e in Δ] $\hat{N}_1 = \hat{K} = 90° - x$ [tan chord theorem/*raakl-koordst*] **OR/*OF*** $\hat{N}_3 = x$ [tan chord theorem/*raakl-koordst*] $\hat{N}_2 = 90°$ [∠in semi circle/ *halfsirkel*] $\hat{N}_1 = 90° - x$ [straight line/*reguitlyn*]	✓ R ✓ S✓ R (3) ✓ R ✓ S✓ R (3) ✓ R ✓ S ✓ S (3)

11.3.1	In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\therefore \Delta KTP \mid\mid\mid \Delta KLN$ [$\angle\angle\angle$] **OR/*OF*** In ΔKTP and ΔKLN: $P\hat{K}T = L\hat{K}N$ [common/*gemeen*] $K\hat{P}T = K\hat{N}L = 90°$ [given/*gegee*] $\hat{T}_2 = P\hat{L}N = x$ [proved in 11.2 **OR** sum of $\angle$s in Δ] $\therefore \Delta KTP \mid\mid\mid \Delta KLN$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
11.3.2	$\frac{KT}{KL} = \frac{KP}{KN}$ [$\mid\mid\mid$ Δs] $\therefore KT \cdot KN = KP \cdot KL$ But $KL = 2KP$ [radii: PK = LP] $\therefore KT \cdot KN = KP \cdot 2KP$ $= 2KP^2$ $= 2(KT^2 - TP^2)$ [Theorem of Pythagoras] $= 2KT^2 - 2TP^2$	✓ S/R ✓ S ✓ S ✓ S ✓ S (5) **[14]**

TOTAL/*TOTAAL*: 150