basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V2

FEBRUARIE/MAART 2016

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 14 bladsye, 1 inligtingsblad en 'n 25 bladsy-antwoordeboek.

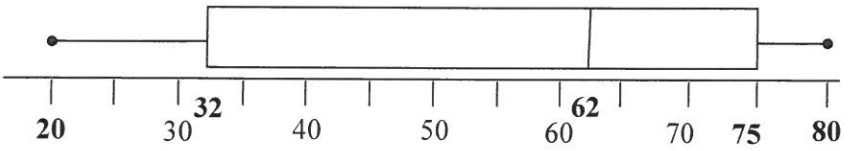
INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat jy die vraestel begin beantwoord.

1. Hierdie vraestel bestaan uit 10 vrae.
2. Beantwoord AL die vrae in die SPESIALE ANTWOORDEBOEK wat verskaf word.
3. Dui ALLE berekeninge, diagramme, grafieke ensovoorts wat jy gebruik het om die antwoorde te bepaal, duidelik aan.
4. Antwoorde alleenlik sal NIE noodwendig volpunte verdien NIE.
5. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.
6. Diagramme is NIE noodwendig volgens skaal geteken NIE.
7. Jy mag 'n goedgekeurde wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders vermeld.
8. 'n INLIGTINGSBLAD met formules is aan die einde van die vraestel ingesluit.
9. Skryf netjies en leesbaar.

VRAAG 1

Die mond-en-snordiagram hieronder toon die punte (uit 80) wat 'n klas van nege leerders in 'n Geskiedenistoets behaal het.



1.1 Lewer kommentaar op die skeefheid van die data. (1)

1.2 Skryf die omvang (variasiewydte) neer van die punte wat behaal is. (2)

1.3 Indien die leerders 32 punte moes behaal het om die toets te slaag, beraam watter persentasie van die klas die toets gedruip het. (2)

1.4 In stygende volgorde is die tweede punt 28, die derde punt 36 en die sesde punt 69. Die sewende en agste punte is dieselfde. Die gemiddelde punt vir hierdie toets is 54.

	28	36			69			

Vul die punte van die oorblywende leerders in stygende volgorde in. (6)
[11]

VRAAG 2

'n Maatskappy het die getal boodskappe aangeteken wat oor 'n tydperk van 60 werksdae per e-pos gestuur is. Die data word in die tabel hieronder getoon.

GETAL BOODSKAPPE	GETAL DAE
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2

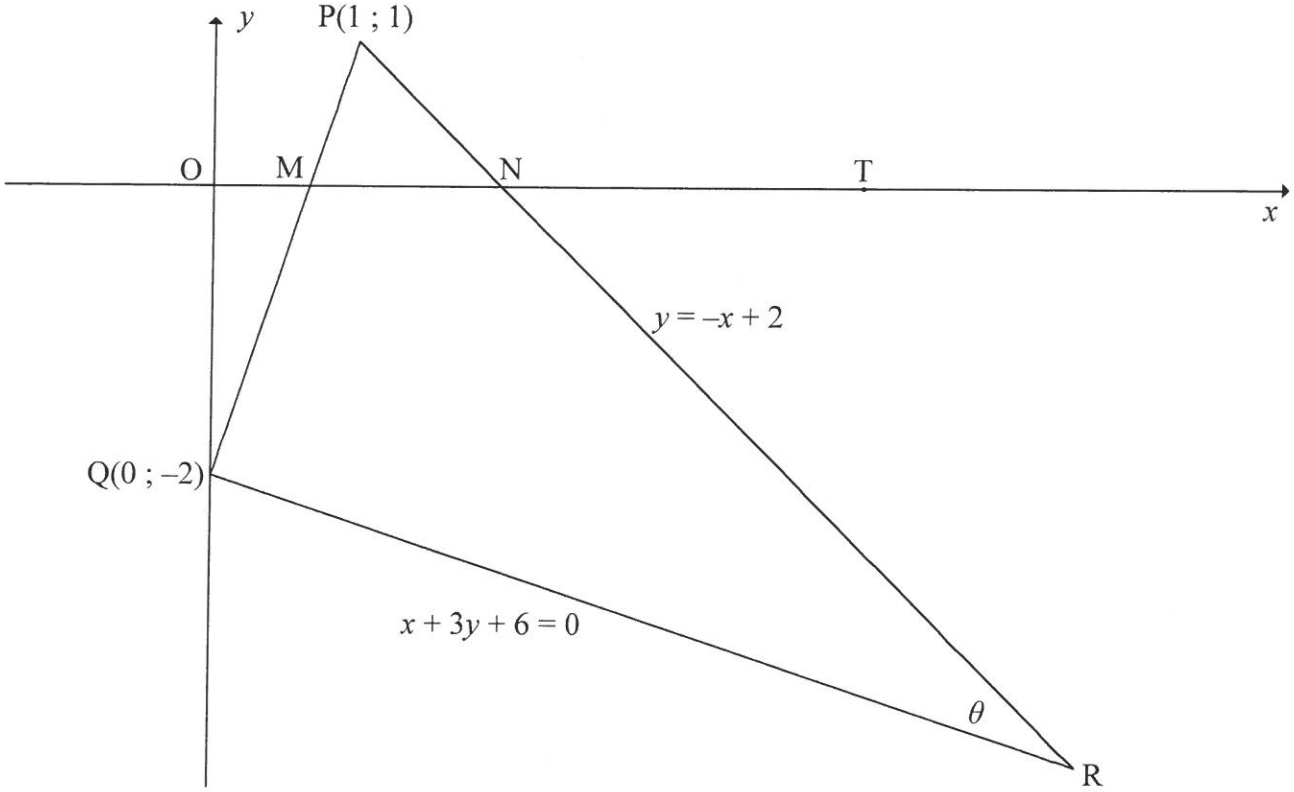
2.1 Benader die gemiddelde getal boodskappe wat per dag gestuur is, afgerond tot TWEE desimale plekke. (3)

2.2 Teken 'n kumulatiewefrekwensie-grafiek (ogief) van die data op die rooster wat in die ANTWOORDEBOEK verskaf word. (4)

2.3 Benader vervolgens die getal dae waarop 65 of meer boodskappe gestuur is. (2)
[9]

VRAAG 3

In die diagram hieronder is P(1 ; 1), Q(0 ; –2) en R die hoekpunte van 'n driehoek en $P\hat{R}Q = \theta$. Die x-afsnitte van PQ en PR is M en N onderskeidelik. Die vergelykings van die sye PR en QR is $y = -x + 2$ en $x + 3y + 6 = 0$ onderskeidelik. T is 'n punt op die x-as, soos getoon.



3.1 Bepaal die gradiënt van QP. (2)

3.2 Bewys dat $P\hat{Q}R = 90°$. (2)

3.3 Bepaal die koördinate van R. (3)

3.4 Bereken die lengte van PR. Laat jou antwoord in wortelvorm. (2)

3.5 Bepaal die vergelyking van 'n sirkel wat deur P, Q en R gaan in die vorm $(x-a)^2 + (y-b)^2 = r^2$. (6)

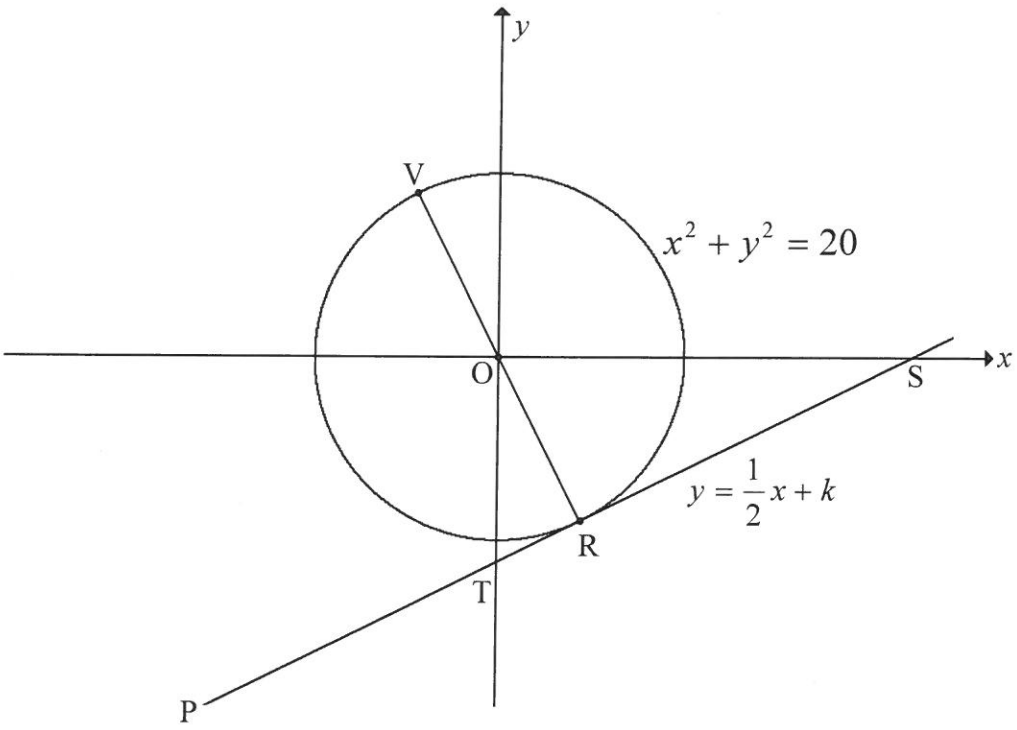
3.6 Bepaal die vergelyking van 'n raaklyn aan die sirkel wat deur P, Q en R by punt P gaan, in die vorm $y = mx + c$. (3)

3.7 Bereken die grootte van θ. (5)

[23]

VRAAG 4

In die diagram hieronder is $x^2 + y^2 = 20$ die vergelyking van die sirkel met middelpunt O. Die raaklyn PRS aan die sirkel by R het die vergelyking $y = \frac{1}{2}x + k$. PRS sny die y-as by T en die x-as by S.



4.1 Bepaal, met redes, die vergelyking van OR in die vorm $y = mx + c$. (3)

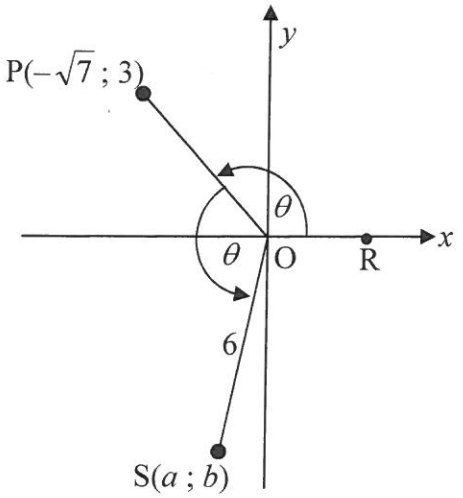
4.2 Bepaal die koördinate van R. (4)

4.3 Bepaal die oppervlakte van ΔOTS, gegee dat R(2 ; –4). (6)

4.4 Bereken die lengte van VT. (4)
[17]

VRAAG 5

5.1 $P(-\sqrt{7}\,;\,3)$ en $S(a\,;\,b)$ is punte in die Cartesiese vlak soos in die diagram hieronder getoon. $P\hat{O}R = P\hat{O}S = \theta$ en $OS = 6$.



Bepaal, SONDER die gebruik van 'n sakrekenaar, die waarde van:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 a (4)

5.2 5.2.1 Vereenvoudig $\dfrac{4\sin x\cos x}{2\sin^2 x - 1}$ tot 'n enkele trigonometriese verhouding. (3)

5.2.2 Bereken vervolgens die waarde van $\dfrac{4\sin 15°\cos 15°}{2\sin^2 15° - 1}$ SONDER om 'n sakrekenaar te gebruik. (Laat jou antwoord in die eenvoudigste wortelvorm.) (2)

[13]

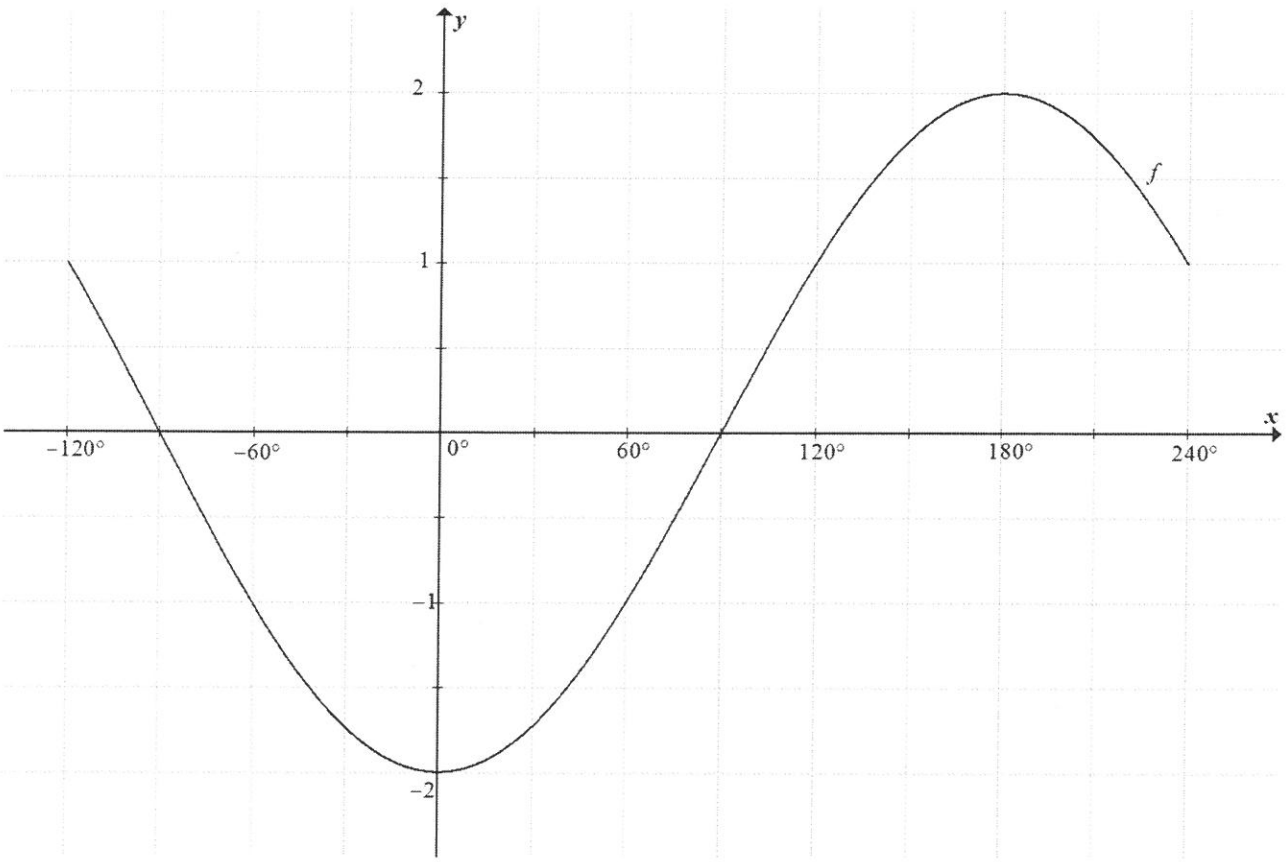
VRAAG 6

Gegee die vergelyking: $\sin(x + 60°) + 2\cos x = 0$

6.1 Toon dat die vergelyking ook as $\tan x = -4 - \sqrt{3}$ geskryf kan word. (4)

6.2 Bepaal die oplossings van die vergelyking $\sin(x + 60°) + 2\cos x = 0$ in die interval $-180° \leq x \leq 180°$. (3)

6.3 In die diagram hieronder is die grafiek van $f(x) = -2\cos x$ vir $-120° \leq x \leq 240°$ geskets.

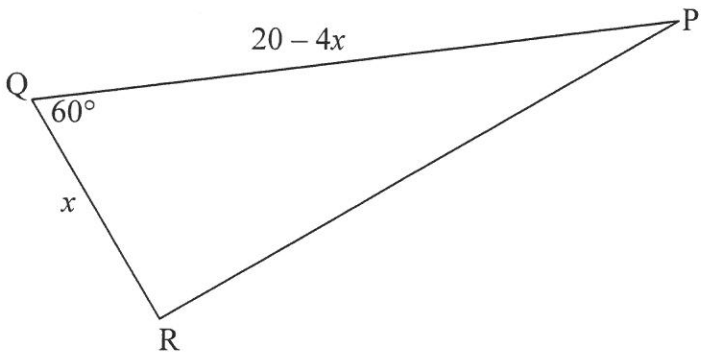


6.3.1 Skets die grafiek van $g(x) = \sin(x + 60°)$ vir $-120° \leq x \leq 240°$ op die rooster wat in die ANTWOORDEBOEK verskaf word. (3)

6.3.2 Bepaal die waardes van x in die interval $-120° \leq x \leq 240°$ waarvoor $\sin(x + 60°) + 2\cos x > 0$. (3)
[13]

VRAAG 7

7.1 In die diagram hieronder is ΔPQR geskets met $PQ = 20 - 4x$, $RQ = x$ en $\hat{Q} = 60°$.

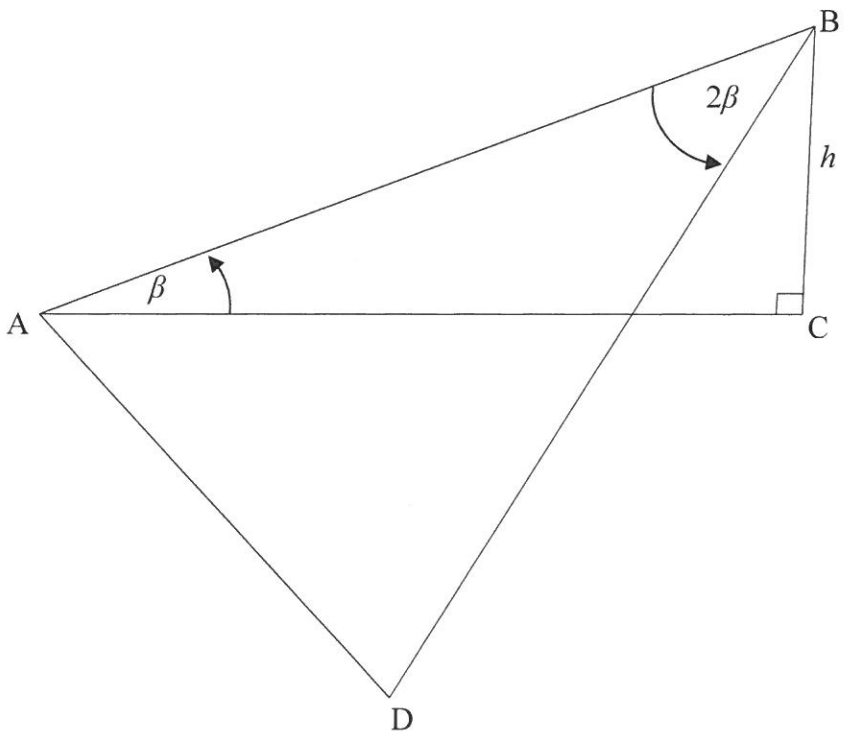


7.1.1 Toon dat die oppervlakte van $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

7.1.2 Bepaal die waarde van x waarvoor die oppervlakte van ΔPQR 'n maksimum sal wees. (3)

7.1.3 Bereken die lengte van PR indien die oppervlakte van ΔPQR 'n maksimum is. (3)

7.2 In die diagram hieronder is BC 'n mas wat deur twee kabels by A en D geanker is. A, D en C is in dieselfde horisontale vlak. Die hoogte van die mas is h en die hoogtehoek vanaf A na die bopunt van die mas, B, is β. $A\hat{B}D = 2\beta$ en BA = BD.

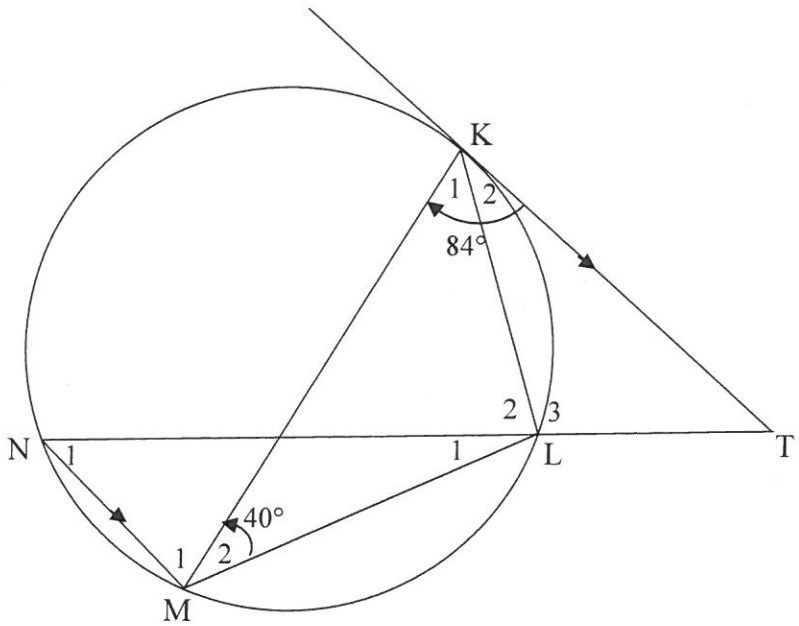


Bepaal die afstand AD tussen die twee ankerpunte in terme van h. (7)
[15]

Gee redes vir ALLE bewerings in VRAAG 8, 9 en 10.

VRAAG 8

8.1 In die diagram hieronder is raaklyn KT aan die sirkel by K ewewydig aan die koord NM. NT sny die sirkel by L. ΔKML is getrek. $\hat{M}_2 = 40°$ en $M\hat{K}T = 84°$.



Bepaal, met redes, die grootte van:

8.1.1 $\hat{K}_2$ (2)

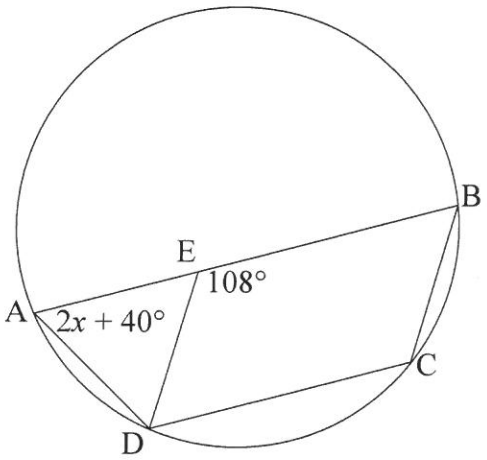
8.1.2 $\hat{N}_1$ (3)

8.1.3 $\hat{T}$ (2)

8.1.4 $\hat{L}_2$ (2)

8.1.5 $\hat{L}_1$ (1)

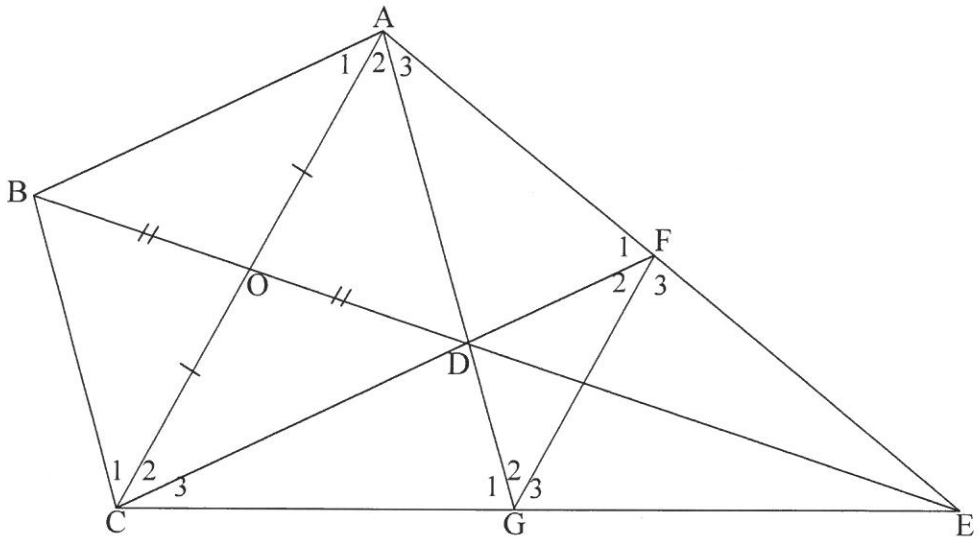
8.2 In die diagram hieronder is AB en DC koorde van 'n sirkel. E is 'n punt op AB sodat BCDE 'n parallelogram vorm. $D\hat{E}B = 108°$ en $D\hat{A}E = 2x + 40°$.



Bereken, met redes, die waarde van x. (5)
[15]

VRAAG 9

In die diagram hieronder halveer EO die sy AC van ΔACE. EDO is verleng na B sodat BO = OD. AD en CD verleng, ontmoet EC en EA by G en F onderskeidelik.



9.1 Gee 'n rede waarom ABCD 'n parallelogram is. (1)

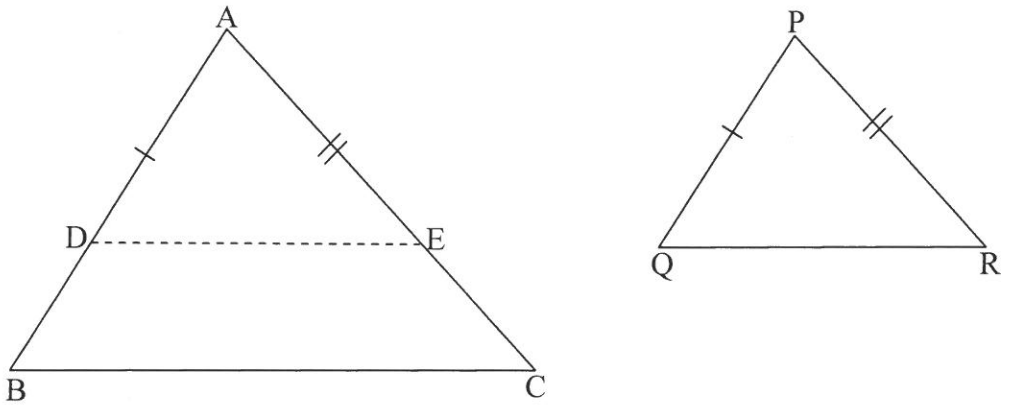
9.2 Skryf neer, met redes, TWEE verhoudings wat elk aan $\frac{ED}{DB}$ gelyk is. (4)

9.3 Bewys dat $\hat{A}_1 = \hat{F}_2$. (5)

9.4 Dit word verder gegee dat ABCD 'n ruit is. Bewys dat ACGF 'n koordevierhoek is. (3)
[13]

VRAAG 10

10.1 In die diagram hieronder word ΔABC en ΔPQR gegee met $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ en $\hat{C} = \hat{R}$.



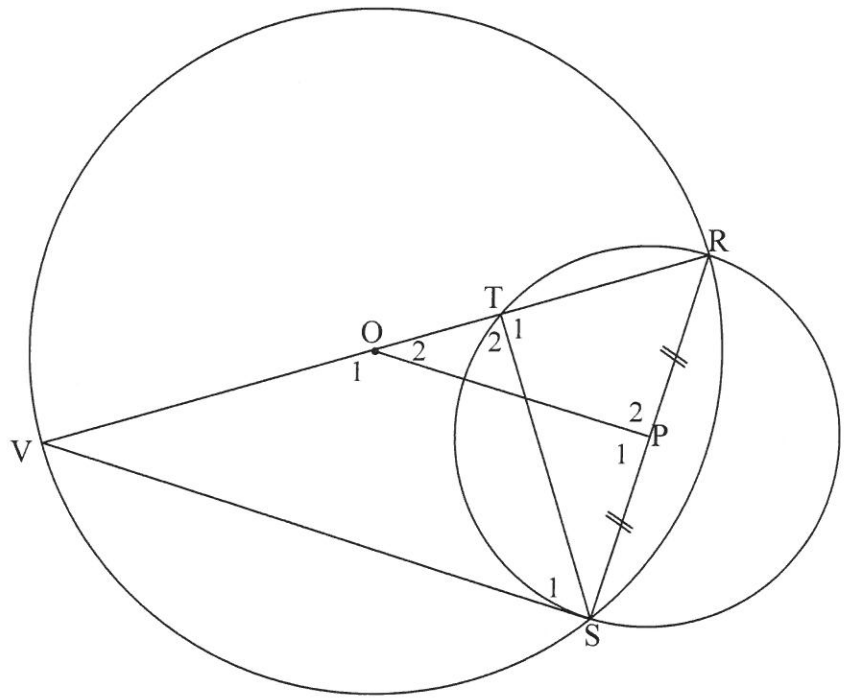
DE word getrek sodat $AD = PQ$ en $AE = PR$.

10.1.1 Bewys dat $\Delta ADE \equiv \Delta PQR$. (2)

10.1.2 Bewys dat $DE \mid\mid BC$. (3)

10.1.3 Bewys vervolgens dat $\frac{AB}{PQ} = \frac{AC}{PR}$. (2)

10.2 In die diagram hieronder is VR 'n middellyn van 'n sirkel met middelpunt O. S is enige punt op die omtrek. P is die middelpunt van RS. Die sirkel met RS as middellyn sny VR by T. ST, OP en SV is geteken.



10.2.1 Waarom is OP ⊥ PS? (1)

10.2.2 Bewys dat ΔROP ||| ΔRVS. (4)

10.2.3 Bewys dat ΔRVS ||| ΔRST. (3)

10.2.4 Bewys dat $ST^2 = VT \cdot TR$. (6)
[21]

TOTAAL: 150

INLIGTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin \text{A}} = \frac{b}{\sin \text{B}} = \frac{c}{\sin \text{C}}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta\text{ABC} = \frac{1}{2}ab.\sin \text{C}$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$\text{P(A)} = \frac{n(\text{A})}{n(\text{S})} \qquad \text{P(A of B)} = \text{P(A)} + \text{P(B)} - \text{P(A en B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

EXAMINATION NUMBER:/ ***EKSAMENNOMMER:***													
CENTRE NUMBER:/ ***SENTRUMNOMMER:***													

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

MATHEMATICS P2/*WISKUNDE V2*

GRADE/*GRAAD* 12

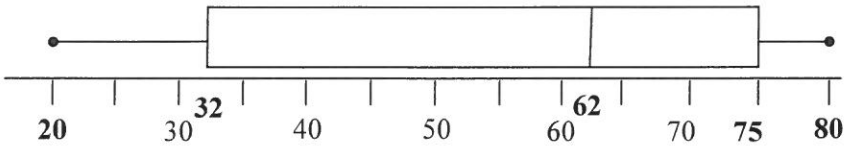
FEBRUARY/MARCH/*FEBRUARIE/MAART* 2016

SPECIAL ANSWER BOOK/ *SPESIALE ANTWOORDEBOEK*

QUESTION/ *VRAAG*	MARK/ *PUNT*			INITIAL/ *PARAAF*	MOD.	REMARK/ *PUNT*			INITIAL/ *PARAAF*
1									
2									
3									
4									
5									
6									
7									
8									
9									
10									
TOTAL/ *TOTAAL* (150)									

This answer book consists of 25 pages./
Hierdie antwoordeboek bestaan uit 25 bladsye.

QUESTION/*VRAAG* 1



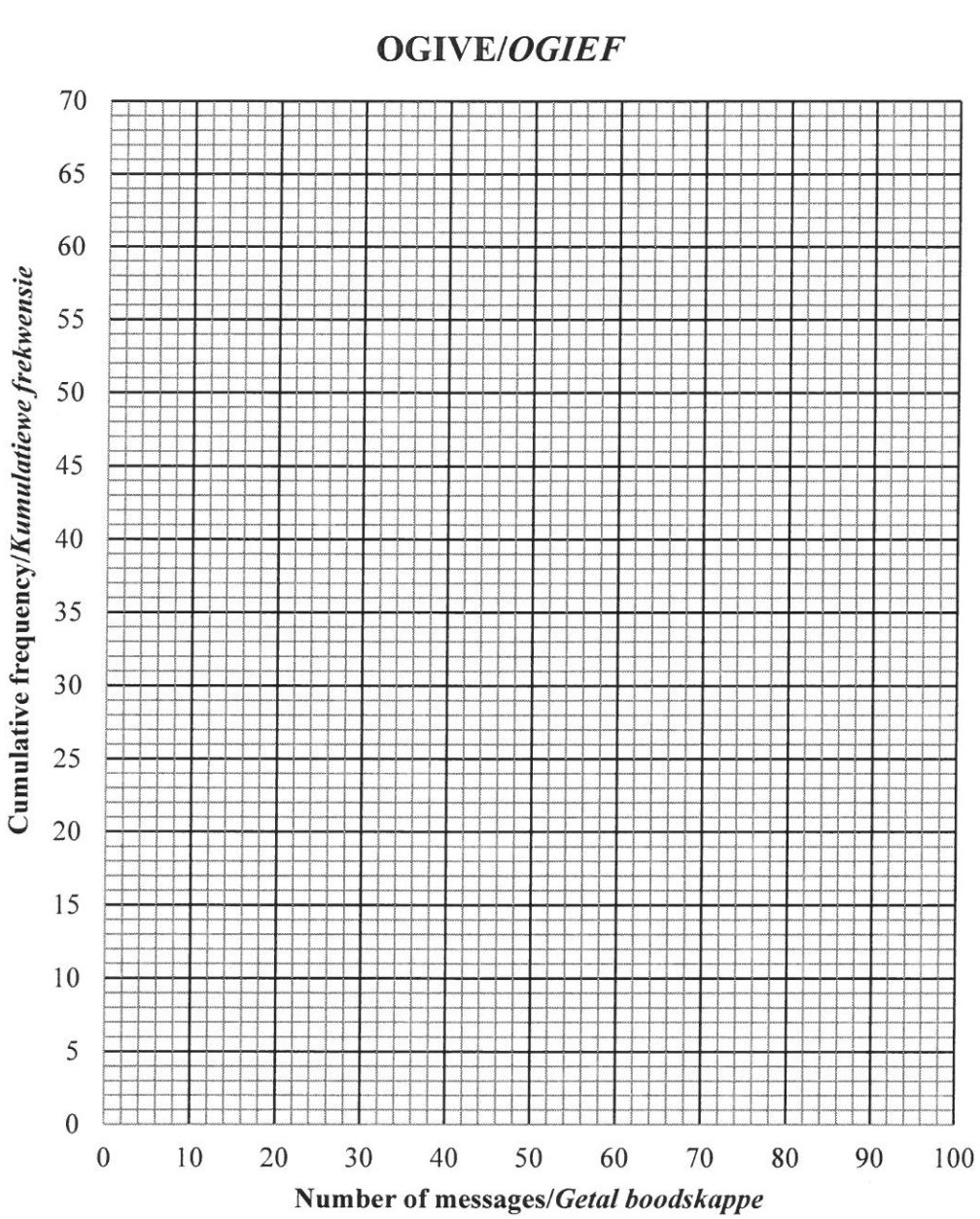
	Solution/*Oplossing*	Marks/ *Punte*
1.1		(1)
1.2		(2)
1.3		(2)
1.4		(6)
		[11]

	28	**36**			**69**			

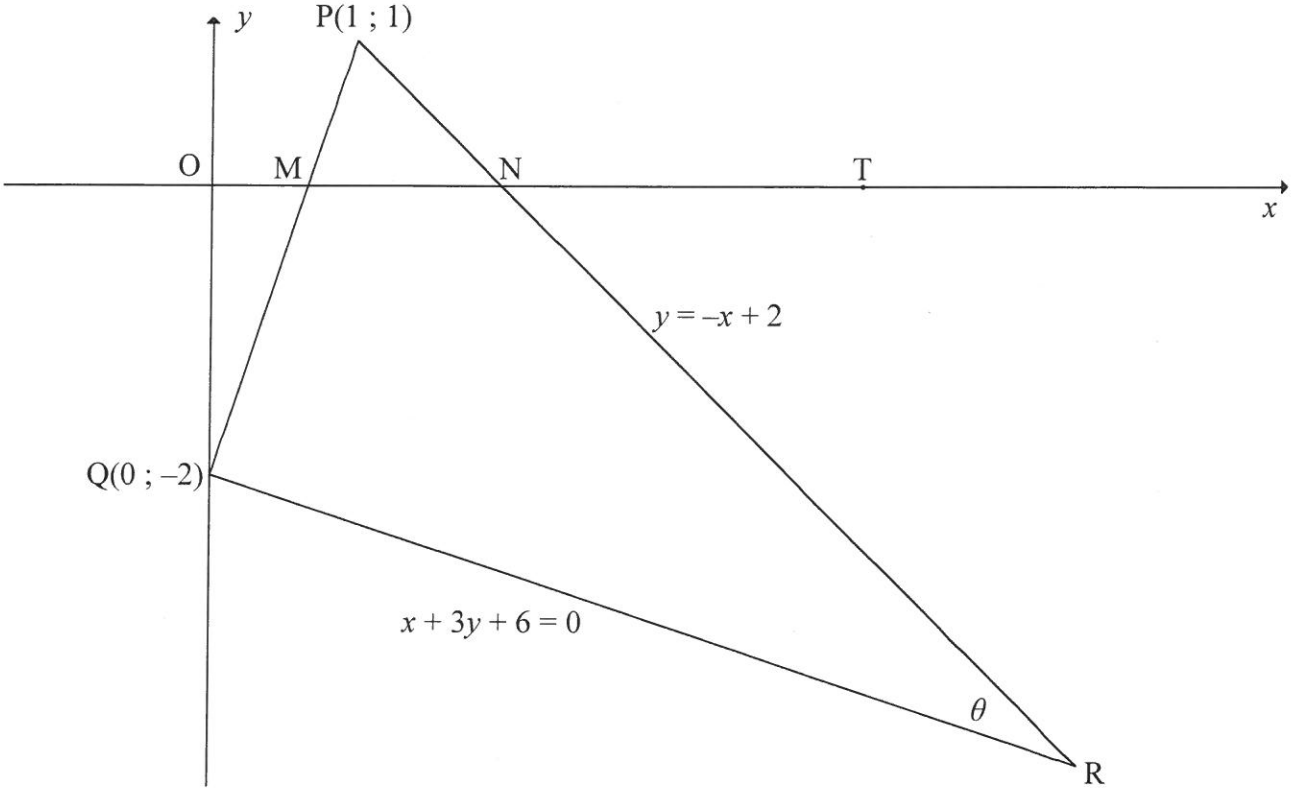
QUESTION/*VRAAG* 2

NUMBER OF MESSAGES/ ***GETAL BOODSKAPPE***	**NUMBER OF DAYS/** ***GETAL DAE***
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2

	Solution/*Oplossing*	**Marks/** ***Punte***
2.1		(3)

	Solution/*Oplossing*	**Marks/ *Punte***
2.2		(4)
2.3		(2)
		[9]

QUESTION/*VRAAG* 3



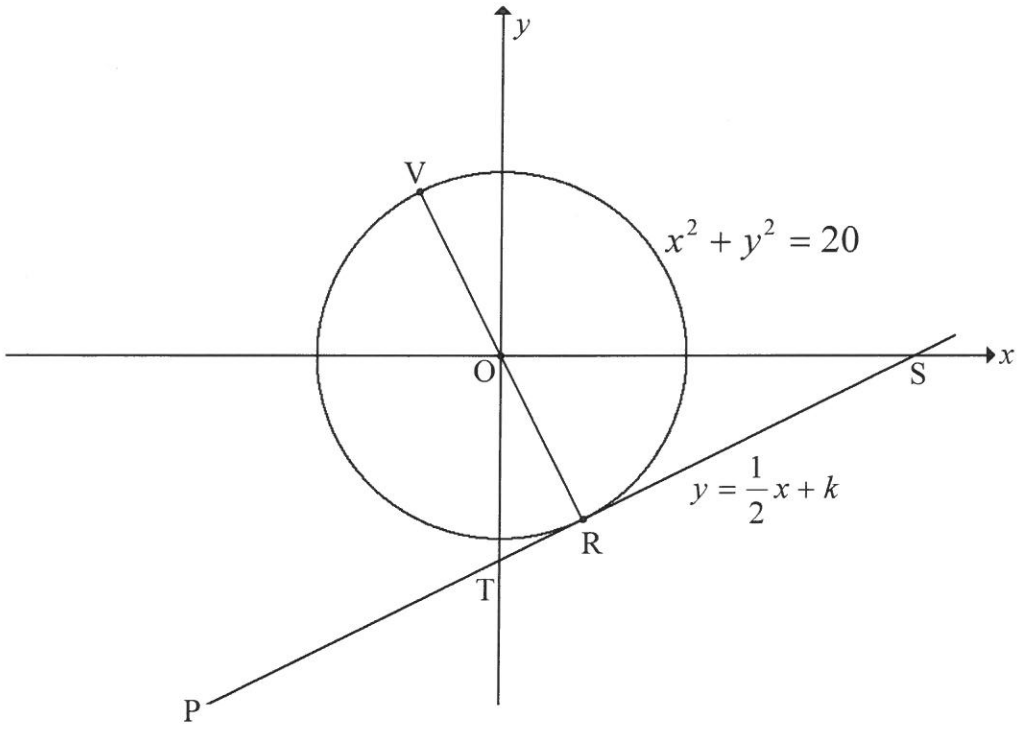
	Solution/*Oplossing*	**Marks/ *Punte***
3.1		(2)
3.2		(2)

	Solution/*Oplossing*	**Marks/ *Punte***
3.3		(3)
3.4		(2)
3.5		(6)
3.6		(3)

	Solution/*Oplossing*	**Marks/ *Punte***
3.7		(5)
		[23]

	Additional space/*Bykomende ruimte*	**Marks/ *Punte***

QUESTION/*VRAAG* 4

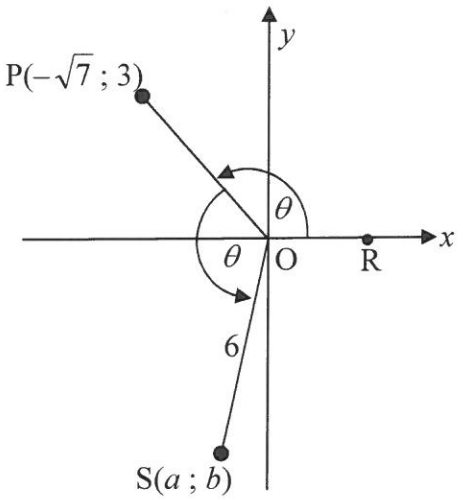


	Solution/*Oplossing*	**Marks/ *Punte***
4.1		(3)
4.2		(4)

	Solution/*Oplossing*	**Marks/ *Punte***
4.3		(6)
4.4		(4)
		[17]

QUESTION/*VRAAG* 5

5.1



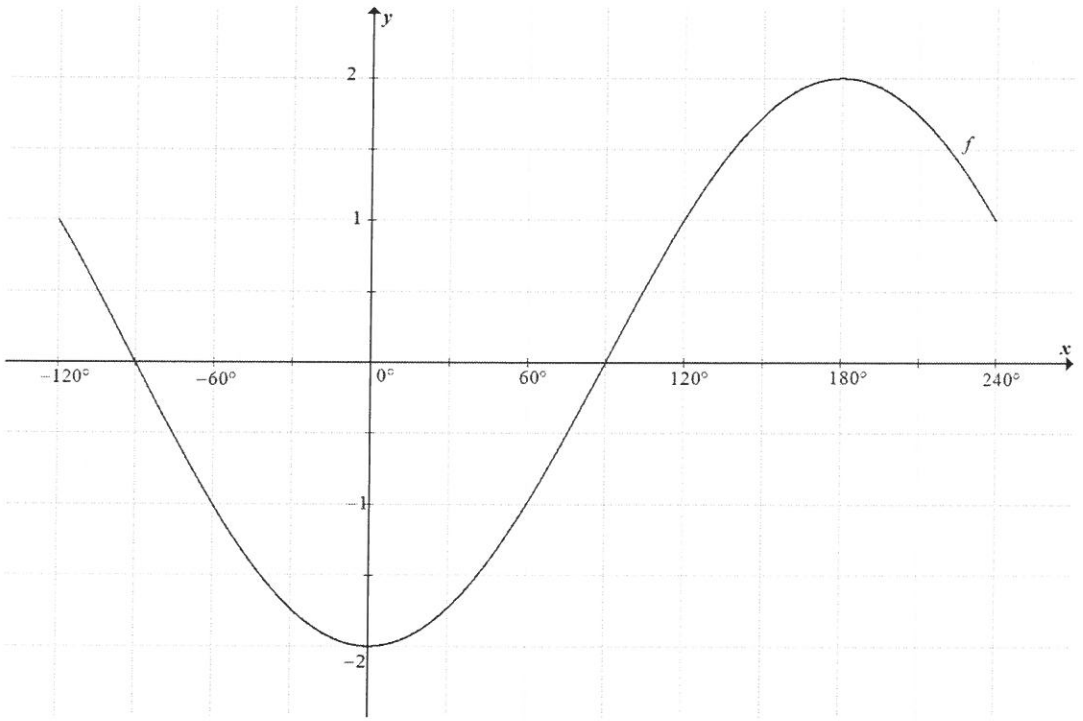
	Solution/*Oplossing*	**Marks/ *Punte***
5.1.1		(1)
5.1.2		(3)
5.1.3		(4)

	Solution/*Oplossing*	**Marks/ *Punte***
5.2.1		(3)
5.2.2		(2)
		[13]

	Additional space/*Bykomende ruimte*	**Marks/ *Punte***

QUESTION/*VRAAG* 6

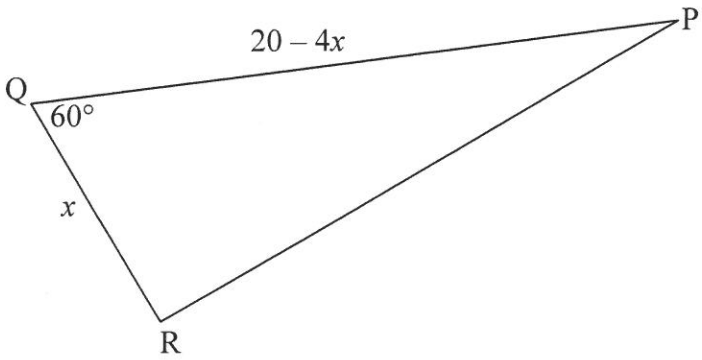
	Solution/*Oplossing*	**Marks/ *Punte***
6.1		(4)
6.2		(3)

	Solution/*Oplossing*	**Marks/ *Punte***
6.3.1		(3)
6.3.2		(3)
		[13]

	Additional space/*Bykomende ruimte*	**Marks/ *Punte***

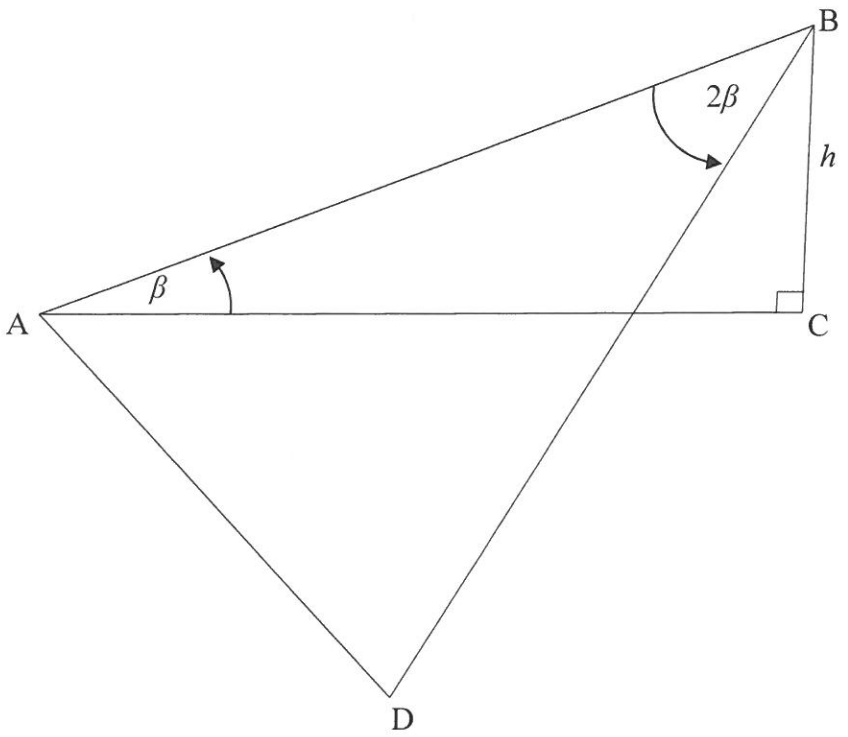
QUESTION/*VRAAG* 7

7.1



	Solution/*Oplossing*	**Marks/ *Punte***
7.1.1		(2)
7.1.2		(3)
7.1.3		(3)

7.2

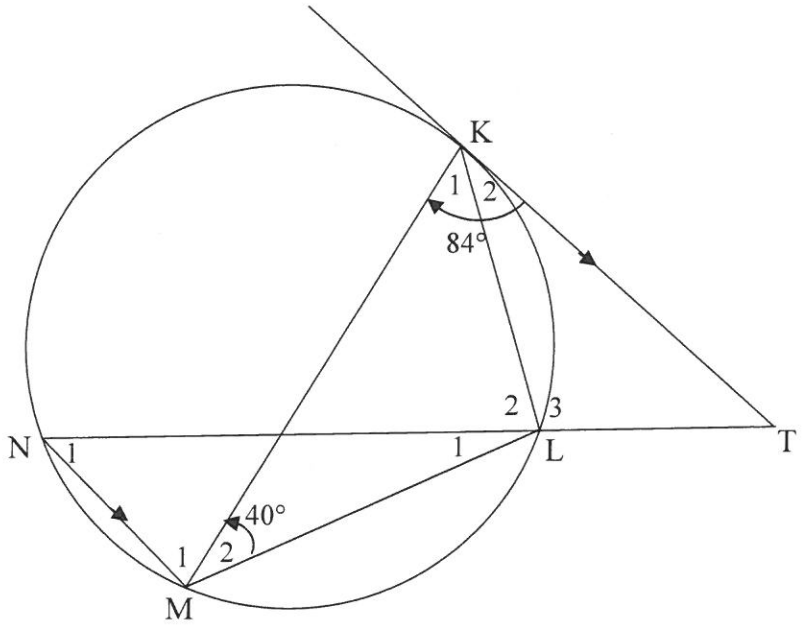


	Solution/*Oplossing*	**Marks/ *Punte***
		(7)
		[15]

Give reasons for ALL statements in QUESTIONS 8, 9 and 10.
Gee redes vir ALLE bewerings in VRAAG 8, 9 en 10.

QUESTION/*VRAAG* 8

8.1

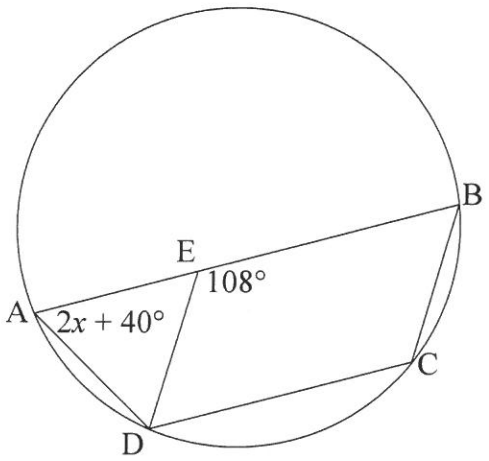


	Solution/*Oplossing*	**Marks/ *Punte***
8.1.1		(2)
8.1.2		(3)
8.1.3		(2)

	Solution/*Oplossing*	**Marks/ *Punte***
8.1.4		(2)
8.1.5		(1)

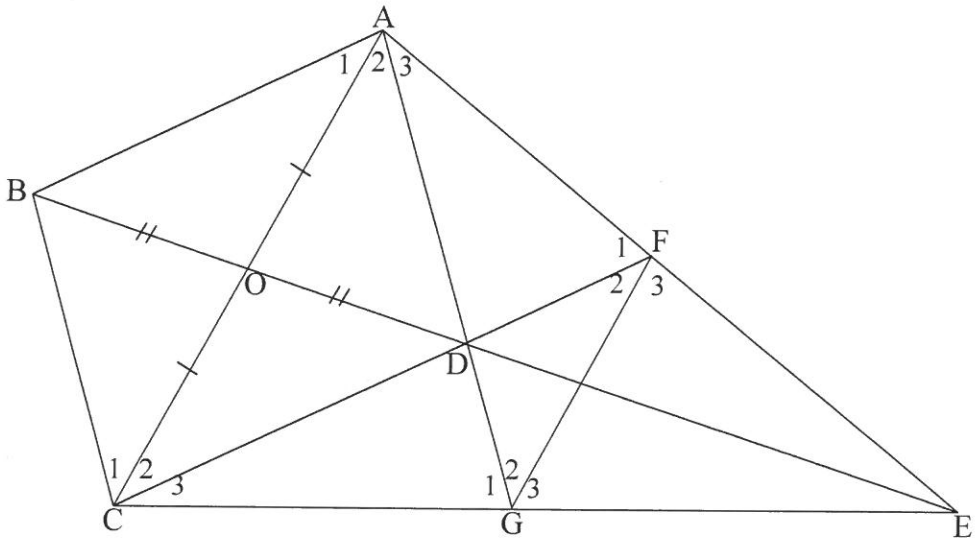
	Additional space/*Bykomende ruimte*	**Marks/ *Punte***

8.2



	Solution/*Oplossing*	**Marks/ *Punte***
8.2		(5)
		[15]

QUESTION/*VRAAG* 9



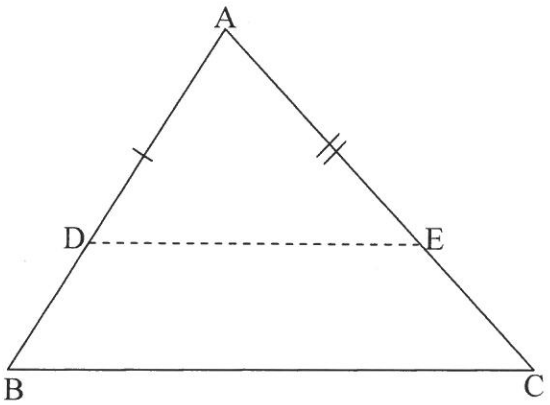
	Solution/*Oplossing*	**Marks/ *Punte***
9.1		(1)
9.2		(4)
9.3		(5)

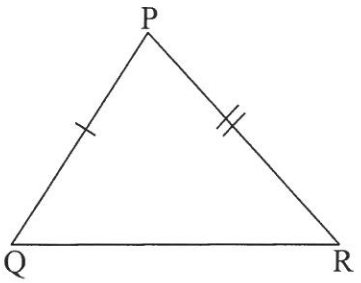
	Solution/*Oplossing*	**Marks/ *Punte***
9.4		(3)
		[13]

	Additional space/*Bykomende ruimte*	**Marks/ *Punte***

QUESTION/*VRAAG* 10

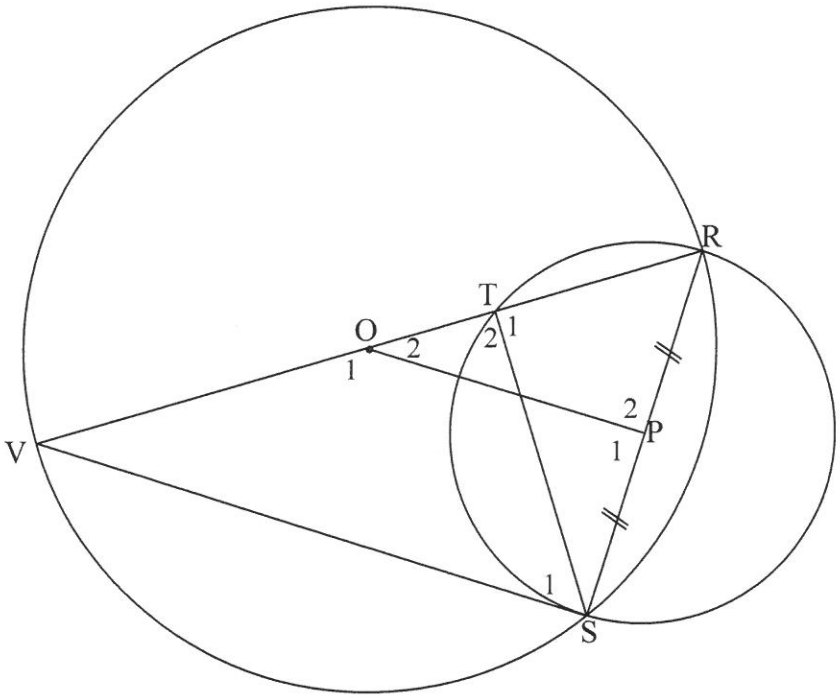
10.1





	Solution/*Oplossing*	**Marks/ *Punte***
10.1.1		(2)
10.1.2		(3)
10.1.3		(2)

10.2



	Solution/*Oplossing*	**Marks/ *Punte***
10.2.1		(1)
10.2.2		(4)

	Solution/*Oplossing*	**Marks/ *Punte***
10.2.3		(3)
10.2.4		(6)
		[21]

	Additional space/*Bykomende ruimte*	**Marks/ *Punte***

	Additional space/*Bykomende ruimte*	**Marks/ *Punte***

TOTAL/*TOTAAL*: 150

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

FEBRUARY/MARCH/*FEBRUARIE/MAART* 2016

MEMORANDUM

MARKS: 150
PUNTE: 150

This memorandum consists of 21 pages./
Hierdie memorandum bestaan uit 21 bladsye.

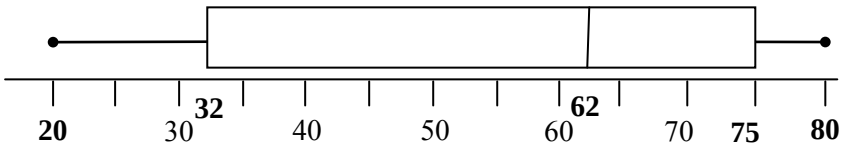
NOTE:

- If a candidate answers a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

LET WEL:

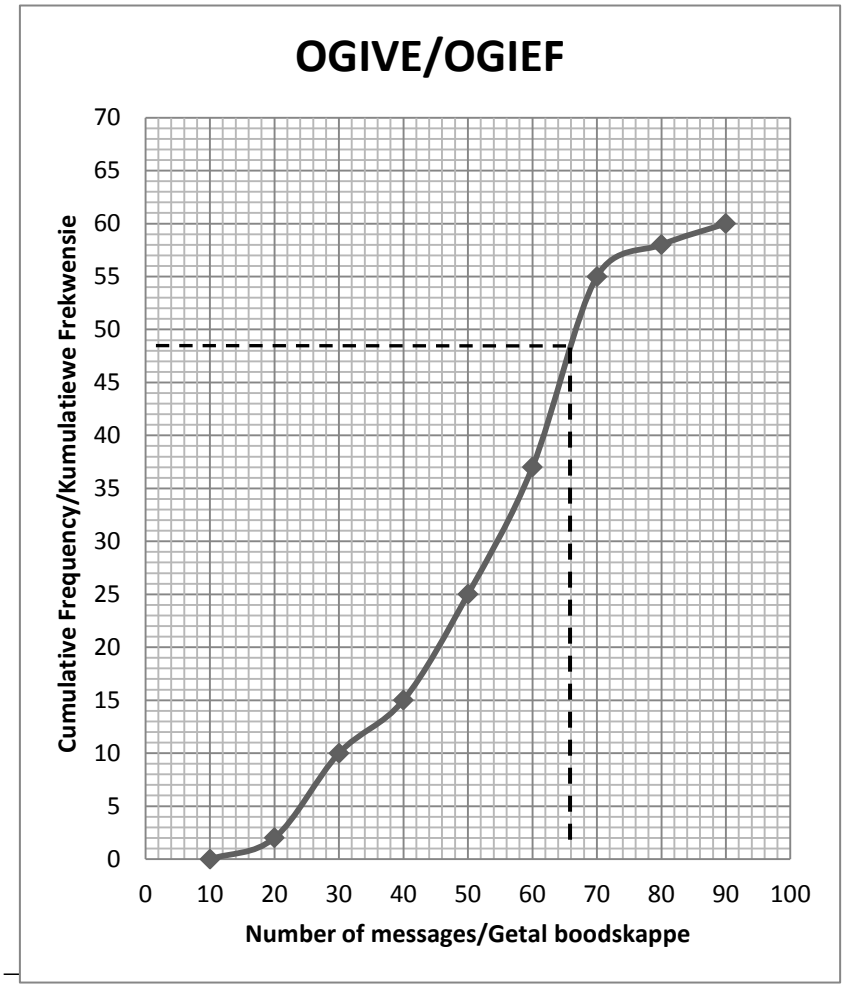
- Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.
- Indien 'n kandidaat 'n antwoord doodgetrek en nie oorgedoen het nie, sien die doodgetrekte poging na.
- Volgehoue akkuraatheid word in ALLE aspekte van die memorandum toegepas. Hou op nasien by die tweede berekeningsfout.
- Om antwoorde/waardes om 'n probleem op te los, te veronderstel, word NIE toegelaat NIE.

QUESTION/*VRAAG* 1

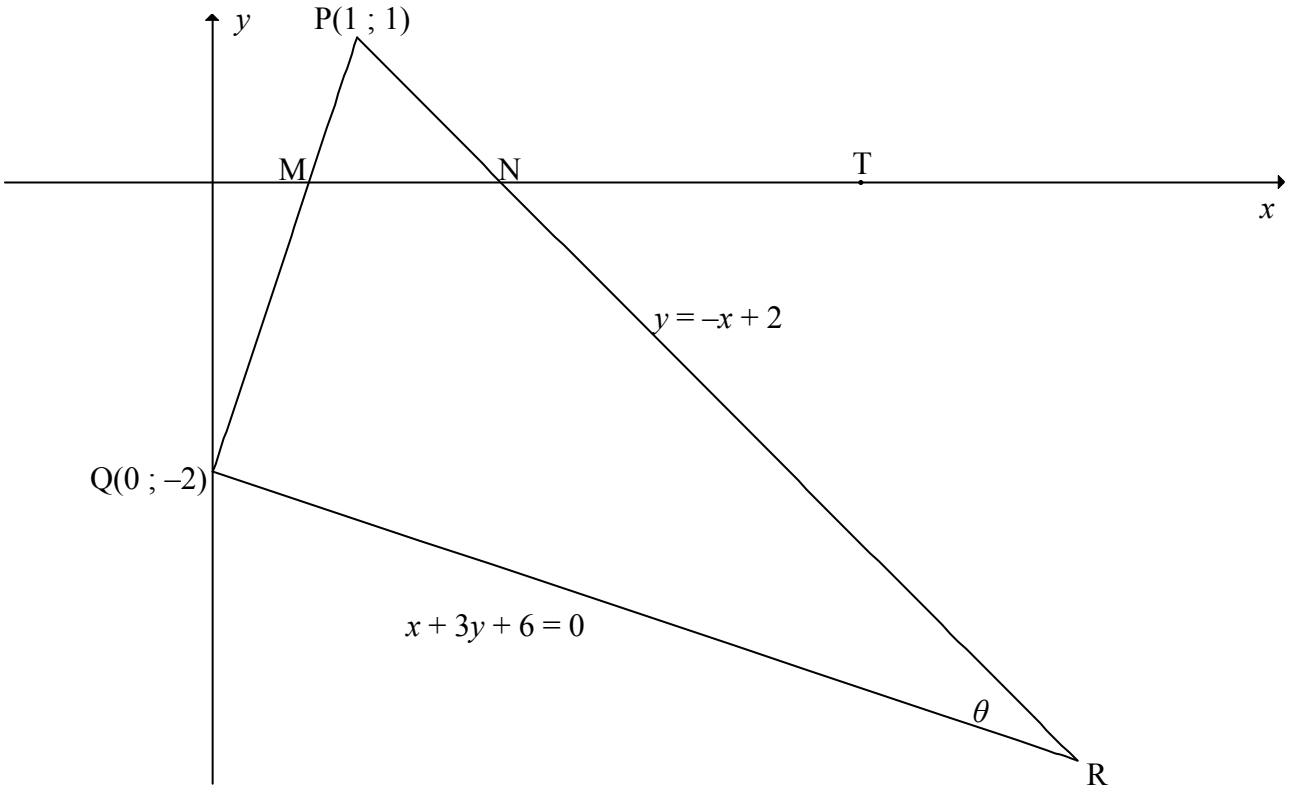


1.1	The data is skewed to the left/*Die data is skeef na links.* **OR/*OF*** The data is negatively skewed/*Die data is negatief skeef.*	✓answ/*antw* ✓answ/*antw* (1)
1.2	Range/*Omvang* = 80 – 20 = 60	✓ max. – min. ✓answ/*antw* (2)
1.3	25% of the learners failed/*van die leerders het gedruip*	✓ ✓answ/*antw* (2)
1.4	$54 = \frac{445 + T_4}{9}$ $T_4 = 41$ **20** 28 36 **41** **62** 69 **75** **75** **80**	✓ 20 ✓✓ 41 ✓ 62 ✓ 75 ✓ 80 (6) **[11]**

QUESTION/*VRAAG* 2

2.1	Mean/*Gemiddelde* = $\frac{2(15)+8(25)+...2(85)}{60}=\frac{3080}{60}$ = 51,33 messages per day/*boodskappe per dag*	✓ 3 080 ✓ $\frac{3080}{60}$ ✓answ/*antw* (3)
2.2		✓grounding at (10 ; 0) ✓plotting at upper limits ✓plotting. cumulative *f* ✓smooth shape of curve ✓*geanker by* (10 ; 0) ✓*stip by boonste limiete* ✓*plot kumulatiewe f* ✓*gladde vorm van kurwe* (4)
2.3	Number of days/*Getal dae* = 60 – 46 (see on graph above/*sien op grafiek hierbo*) = 14 days/*dae* **OR/*OF*** Number of days/*Getal dae* = 2 + 3 + $\frac{1}{2}\times 18$ = 14 days/*dae*	✓46 (accept 45 – 49) ✓answ/*antw* (accept 11 – 15) (2) ✓add correct values/*tel korrekte waardes by* ✓answ/*antw* (2) **[9]**

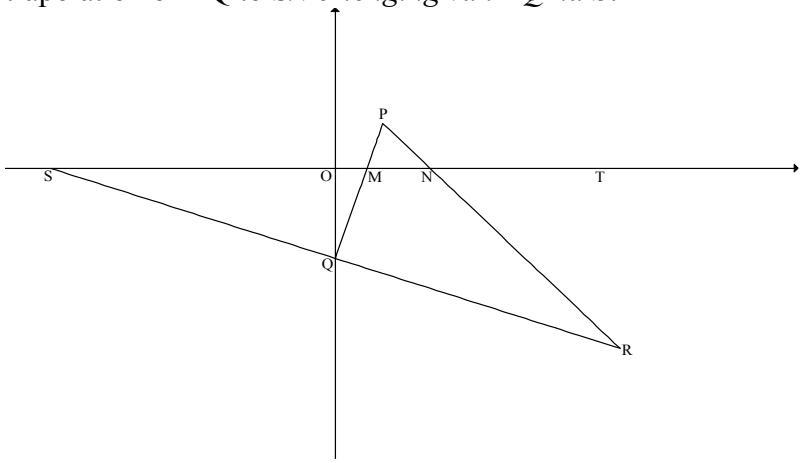
QUESTION/*VRAAG* 3



3.1	$m_{PQ} = \frac{1-(-2)}{1-0}$ $= 3$	✓subst (1 ; 1) & (0 ; –2) ✓answ/*antw* (2)
3.2	QR: $y = -\frac{1}{3}x - 2$ $\therefore m_{QR} = -\frac{1}{3}$ $m_{PQ} \times m_{QR} = 3 \times -\frac{1}{3}$ $= -1$ $\therefore PQ \perp QR \quad \therefore P\hat{Q}R = 90°$	✓ $m_{QR} = -\frac{1}{3}$ ✓ $m_{PQ} \times m_{QR} = -1$ (2)

3.3	$-\frac{1}{3}x-2=-x+2$ $\frac{2}{3}x=4$ $x=6$ $y=-4$ $\therefore$ R(6 ; –4)	✓equating/*gelyk stel* ✓*x*-value/*waarde* ✓*y*-value/*waarde* (3)
3.4	$PR=\sqrt{(1-6)^2+(1-(-4))^2}$ $=\sqrt{50}=5\sqrt{2}$ **OR/*OF*** $PR^2=(1-6)^2+(1-(-4))^2$ $=50$ $\therefore PR=\sqrt{50}=5\sqrt{2}$	✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2) ✓subst into/*in* distance formula/ *afstandsformule* ✓answ/*antw* in surd form/ *wortelvorm* (2)
3.5	PR is a diameter/*'n middellyn* [chord subtends/*kd onderspan* 90°] Centre of circle/*Midpt v sirkel*: $\left(\frac{1+6}{2};\frac{1-4}{2}\right)$ $=\left(3\frac{1}{2};-1\frac{1}{2}\right)$ $r=\frac{\sqrt{50}}{2}$ OR $\frac{5\sqrt{2}}{2}$ OR 3,54 $\therefore\left(x-\frac{7}{2}\right)^2+\left(y+\frac{3}{2}\right)^2=\frac{50}{4}$ OR $\frac{25}{2}$ OR 12,5	✓✓S ✓✓ $\left(3\frac{1}{2};-1\frac{1}{2}\right)$ ✓*r*-value/*waarde* ✓answ/*antw* (6)
3.6	*m* of/*van* radius = –1 $\therefore$ *m* of/*van* tangent/*raaklyn* = 1 Equation of tangent/*Vgl van raaklyn*: $y-y_1=(x-x_1)$ $y-1=x-1$ $\therefore y=x$ **OR/*OF*** $y=x+c$ $1=1+c$ $y=x$	✓*m* of tang/*rkl* ✓subst *m* & P(1 ; 1) into/*in* eq of line/*vgl v lyn* ✓answ/*antw* (3)
3.7	$\tan P\hat{N}T=m_{PR}=-1$ $\therefore P\hat{N}T=135°$ $\tan P\hat{M}T=m_{PQ}=3$ $\therefore P\hat{M}T=71{,}57°$ $\hat{P}=63{,}43°$ [ext $\angle$ of Δ/*buite* $\angle v\ \Delta$] $\therefore\theta=26{,}57°$ [sum of $\angle$s in Δ/*som v* $\angle e$ *in* Δ] **OR/*OF***	✓ $\tan P\hat{N}T=-1$ ✓ $P\hat{N}T=135°$ ✓ $P\hat{M}T=71{,}57°$ ✓ $\hat{P}=63{,}43°$ ✓answ/*antw* (5)

Extrapolation of RQ to S/*Verlenging van RQ na S*:



$\tan \hat{PNT} = m_{PR} = -1$
$\therefore \hat{SNR} = 135°$
$\tan \hat{NSR} = m_{RS} = -\frac{1}{3}$
$\therefore \hat{NSR} = 18,43°$
$\theta = 180° - (135° + 18,43°)$ [sum of ∠s in Δ/*som v ∠e in* Δ]
$= 26,57°$

✓ $\tan \hat{PNT} = -1$
✓ $\hat{SNR} = 135°$
✓ $\tan \hat{NSR} = -\frac{1}{3}$
✓ $\hat{NSR} = 18,43°$
✓answ/*antw*
(5)

OR/*OF*

$PQ^2 = 1^2 + 3^2 = 10$
$PQ = \sqrt{10}$
$\therefore \sin\theta = \frac{PQ}{PR} = \frac{\sqrt{10}}{\sqrt{50}} = \frac{1}{\sqrt{5}}$
$\therefore \theta = 26,57°$

✓subst into/*in* distance formula/ *afstandsformule*
✓distance/*afst* PQ
✓correct trig ratio/ *korrekte trig vh*
✓correct trig eq/ *korrekte trig vgl*
✓answ/*antw*
(5)

OR/*OF*

$QR^2 = 6^2 + 2^2 = 40$
$QR = 2\sqrt{10}$
$\therefore \cos\theta = \frac{2\sqrt{10}}{\sqrt{50}} = \frac{2}{\sqrt{5}}$
$\therefore \theta = 26,57°$

✓subst into/*in* distance formula/ *afstandsformule*
✓distance/*afst* PQ
✓correct trig ratio/ *korrekte trig vh*
✓correct trig eq/ *korrekte trig vgl*
✓answ/*antw*
(5)

OR/*OF*

$$\tan\theta = \frac{m_{RQ} - m_{PR}}{1 + m_{RQ}.m_{PR}}$$

✓correct formula/ *korrekte formule*

$$= \frac{-\frac{1}{3} - (-1)}{1 + (-\frac{1}{3})(-1)}$$

✓ $m_{RQ} = -\frac{1}{3}$

✓correct subst/ *subst korrek*

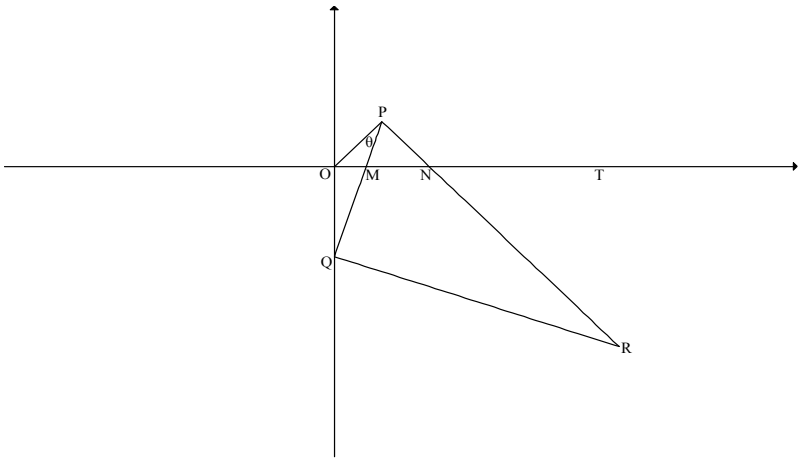
$$= \frac{1}{2}$$

✓ $\tan\theta = \frac{1}{2}$

$\therefore\ \theta = 26{,}57°$

✓ $\theta = 26{,}57°$

(5)



tangent OP goes through the origin/*raakl OP gaan deur oorsprong*

✓ $P\hat{O}M = 45°$

✓R

$P\hat{O}M = 45°$

$O\hat{P}M = \theta = \hat{P}$ [tan-chord theorem/*raakl-kdst*]

$\tan P\hat{M}T = m_{PQ} = 3$

✓ $P\hat{M}T = 71{,}57°$

$\therefore P\hat{M}T = 71{,}57°$

✓S

$\therefore\ \theta + 45° = 71{,}57°$ [ext ∠ of Δ/*buite-∠ v* Δ]

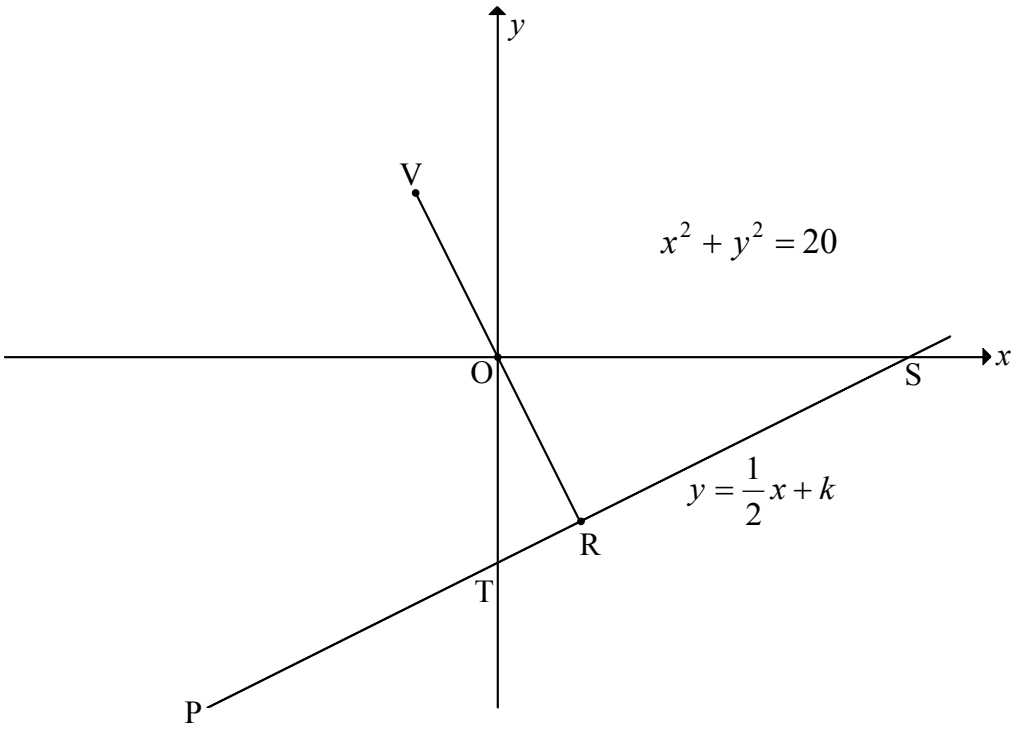
✓ $\theta = 26{,}57°$

$\therefore\ \theta = 26{,}57°$

(5)

[23]

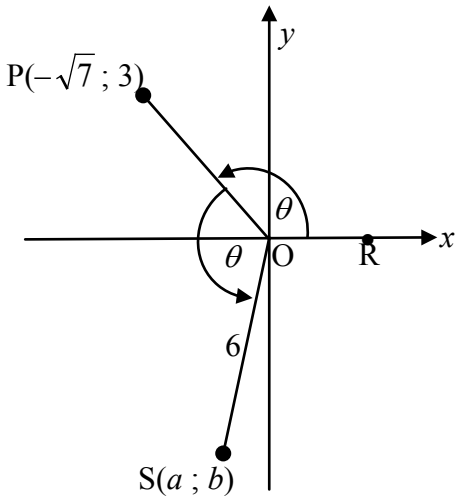
QUESTION/*VRAAG* 4



4.1	OR ⊥ TR [radius ⊥ tangent/*raakl*] $\therefore\ m_{TR} \times m_{OR} = -1$ $\therefore\ m_{OR} = -2$ $\therefore\ y = -2x$	✓S/R ✓*m* of/*van* OR ✓equation/*vgl* (3)
4.2	$x^2 + (-2x)^2 = 20$ $x^2 + 4x^2 = 20$ $5x^2 - 20 = 0$ $x^2 - 4 = 0$ $(x + 2)(x - 2) = 0$ $\therefore x = 2$ $y = -2(2) = -4$ $\therefore$ R(2 ; –4)	✓subst eq of OR into circle eq/ *subst vgl OR in sirkelvgl* ✓st. form/*st. vorm* ✓*x*-value/*waarde* ✓*y*-value/*waarde* (4)

4.3	Subst R(2 ; –4) into the equation of/*in vgl van* PRS: $-4 = \frac{1}{2}(2) + k$ $k = -5$ $\therefore$ OT = 5 $0 = \frac{1}{2}x - 5$ $x = 10$ $\therefore$ OS = 10 Area/*Oppervlakte* = $\frac{1}{2}$ OS . OT = $\frac{1}{2}(10)(5)$ = 25 sq units/*vk eenh*	✓correct subst/ *korrekte subst* ✓value of k ✓$y = 0$ ✓x-intercept/*afsnit* ✓correct subst into area form/ *subst korrek in opp-formule* ✓answ/*antw* (6)
4.4	$0 = \frac{x_V + 2}{2}$ and/*en* $0 = \frac{y_V - 4}{2}$ $\therefore$ V(–2 ; 4) T(0 ; –5) .... from/*van* 4.3 VT = $\sqrt{(-2-0)^2 + (4-(-5))^2}$ = $\sqrt{4+81}$ = $\sqrt{85}$	✓x-value/*waarde*V ✓y-value/*waarde*V ✓subst of points V and T into distance formula/ *subst punte V en T in afst-form* ✓answ/*antw* (4) **[17]**

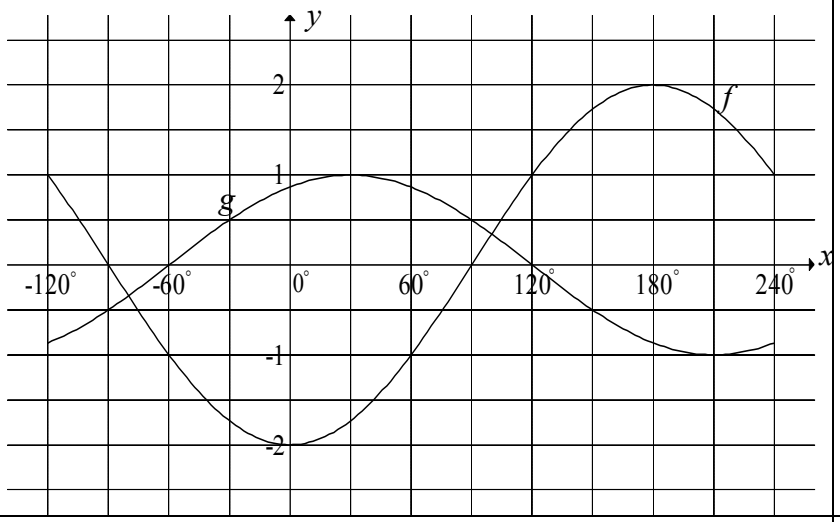
QUESTION/*VRAAG* 5



5.1.1	$\tan\theta = -\frac{3}{\sqrt{7}}$	✓answ/*antw* (1)
5.1.2	$\sin(-\theta) = -\sin\theta$ $OP^2 = \left(-\sqrt{7}\right)^2 + 3^2$ $OP^2 = 16$ $OP = 4$ $\sin(-\theta) = -\frac{3}{4}$	✓reduction/ *reduksie* ✓ OP = 4 ✓answ/*antw* (3)
5.1.3	$\frac{a}{6} = \cos 2\theta$ $a = 6(1 - 2\sin^2\theta)$ $= 6 - 12\left(\frac{3}{4}\right)^2$ $= \frac{24}{4} - \frac{27}{4}$ $= -\frac{3}{4}$ **OR/*OF*** $\frac{a}{6} = \cos 2\theta$ $a = 6(2\cos^2\theta - 1)$ $= 12\left(\frac{-\sqrt{7}}{4}\right)^2 - 6$ $= \frac{21}{4} - \frac{24}{4}$ $= -\frac{3}{4}$ **OR/*OF***	✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4) ✓trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ ✓answ/*antw* (4)

	$\frac{a}{6} = \cos 2\theta$ $a = 6(\cos^2\theta - \sin^2\theta)$ $= 6\left[\left(\frac{-\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]$ $= 6\left(-\frac{2}{16}\right)$ $= -\frac{3}{4}$	✓ trig ratio/*verh* ✓expansion/ *uitbreiding* ✓ $\cos\theta = \frac{-\sqrt{7}}{4}$ & $\sin\theta = \frac{3}{4}$ ✓answ/*antw* (4)
5.2.1	$\frac{4\sin x.\cos x}{2\sin^2 x - 1} = \frac{2(2\sin x.\cos x)}{-(1-2\sin^2 x)}$ $= \frac{2\sin 2x}{-\cos 2x}$ $= -2\tan 2x$	✓2sin 2*x* ✓–cos 2*x* ✓answ/*antw* (3)
5.2.2	$\frac{4\sin 15°\cos 15°}{2\sin^2 15° - 1} = -2\tan 2(15°)$ $= -2\tan 30°$ $= -2\left(\frac{1}{\sqrt{3}}\right)$ $= -\frac{2}{\sqrt{3}}$ **OR/*OF*** $-\frac{2\sqrt{3}}{3}$	✓ $-2\tan 2(15°)$ ✓answ/*antw* (2) **[13]**

QUESTION/*VRAAG* 6

6.1	$\sin(x + 60°) + 2\cos x = 0$ $\sin x \cos 60° + \cos x \sin 60° + 2\cos x = 0$ $\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x + 2\cos x = 0$ $\frac{1}{2}\sin x = -2\cos x - \frac{\sqrt{3}}{2}\cos x$ $\sin x = -4\cos x - \sqrt{3}\cos x$ $\sin x = \cos x(-4 - \sqrt{3})$ $\frac{\sin x}{\cos x} = \frac{\cos x(-4 - \sqrt{3})}{\cos x}$ $\therefore \tan x = -4 - \sqrt{3}$	✓expansion/*uitbreiding* ✓special angle values/ *spesiale ∠-waardes* ✓simpl/*vereenv* ✓ $\sin x = \cos x(-4 - \sqrt{3})$ (4)
6.2	$\tan x = -4 - \sqrt{3}$ $\tan x = -(4 + \sqrt{3})$ *ref* $\angle$ = 80,10° $x = -80{,}1°$ or/*of* 99,9°	✓80,10° ✓99,90° ✓–80,1° (3)
6.3.1		✓(30° ; 1) ✓(–60° ; 0) ✓shape/*vorm* (3)
6.3.2	$\therefore \sin(x + 60°) > -2\cos x$ $x \in (-80{,}10° ; 99{,}90°)$ **OR/*OF*** $-80{,}10° < x < 99{,}90°$	✓✓critical values/ *kritiese waardes* ✓notation/*notasie* (3) **[13]**

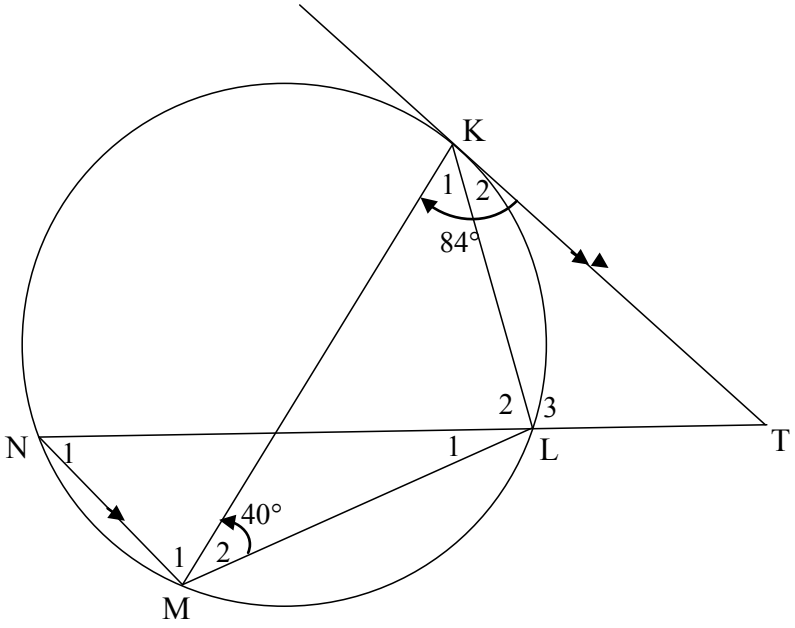
QUESTION/*VRAAG* 7

7.1.1	Area of/*Oppervlakte van* $\Delta PQR = \frac{1}{2}PQ.QR.\sin\hat{Q}$ $= \frac{1}{2}x(20-4x)(\sin 60°)$ $= 10x - 2x^2\left(\frac{\sqrt{3}}{2}\right)$ $= 5\sqrt{3}x - \sqrt{3}x^2$	✓subst into area rule/ *subst in opp-reël* ✓subst & simpl/ *subst en vereenv* (2)
7.1.2	For maximum area/*Vir maksimum opp:* $(\text{Area } \Delta PQR)' = 0$ $5\sqrt{3} - 2\sqrt{3}x = 0$ $2\sqrt{3}x = 5\sqrt{3}$ $\therefore\ x_{\max} = \frac{5}{2}$ or $2\frac{1}{2}$ or/*of* 2,5 **OR/*OF*** $x_{\max} = -\frac{b}{2a}$ $= -\frac{5\sqrt{3}}{2(-\sqrt{3})} = \frac{5}{2}$ or $2\frac{1}{2}$ or 2,5 **OR/*OF*** $5\sqrt{3}x - \sqrt{3}x^2 = 0$ $\sqrt{3}x(5-x) = 0$ $\therefore\ x = 0$ or 5 $\therefore\ x_{\max} = \frac{0+5}{2} = \frac{5}{2}$ or/*of* 2,5	✓$(\text{Area } \Delta PQR)' = 0$ ✓$5\sqrt{3} - 2\sqrt{3}x$ ✓answ/*antw* (3) ✓formula/*e* ✓ subst ✓answ/*antw* (3) ✓*x*-intercepts/ *x-afsnitte* ✓subst ✓answ/*antw* (3)
7.1.3	$RP^2 = QP^2 + QR^2 - 2.QP.QR.\cos Q$ $= 10^2 + 2{,}5^2 - 2(10)(2{,}5)\cos 60°$ $= 81{,}25$ $\therefore\ RP = 9{,}01$	✓subst into cosine rule/*in cos-reël* ✓simpl/*vereenv* ✓answ/*antw* (3)

7.2	In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: AB = BD and/*en* $A\hat{D}B = 90° - \beta$ [$\angle$s of/*v* $\Delta = 180°$] $\frac{\sin 2\beta}{AD} = \frac{\sin(90° - \beta)}{AB}$ $AD = \frac{AB.\sin 2\beta}{\sin(90° - \beta)}$ $= \frac{h}{\sin\beta} \times \frac{2\sin\beta.\cos\beta}{\cos\beta}$ $= 2h$	✓AB ito *h* and/*en* β ✓ $A\hat{D}B = 90° - \beta$ ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓AD as subject/ *onderwerp* ✓expansion/*uitbrei* ✓sin (90°– β) = cos β ✓answer ito *h* (7)
	OR/*OF*	
	In ΔABC: $\sin\beta = \frac{h}{AB}$ $\therefore AB = \frac{h}{\sin\beta}$ In ΔABD: AB = BD $AD^2 = AB^2 + AB^2 - 2AB.AB.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2.\cos 2\beta$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2 (1 - 2\sin^2\beta)$ $= \left(\frac{h}{\sin\beta}\right)^2 + \left(\frac{h}{\sin\beta}\right)^2 - 2\left(\frac{h}{\sin\beta}\right)^2 + 4h^2$ $= 4h^2$ $\therefore AD = 2h$	✓AB ito *h* and/*en* β ✓correct subst into cosine rule/*subst korrek in cos-reël* ✓expansion/*uitbrei* ✓multiplication/ *vermenigv* ✓simpl/*vereenv* ✓answer ito *h* (7)
	OR/*OF*	
	Split isosceles triangle ABQ into two congruent triangles AEB and DEB. Then $\Delta ABC \equiv \Delta BAE$ (AB = AC, $A\hat{B}E = B\hat{A}C = \beta$, *h*) $\therefore AE = ED = BC = h$ $\therefore AD = 2h$	(7)
		[15]

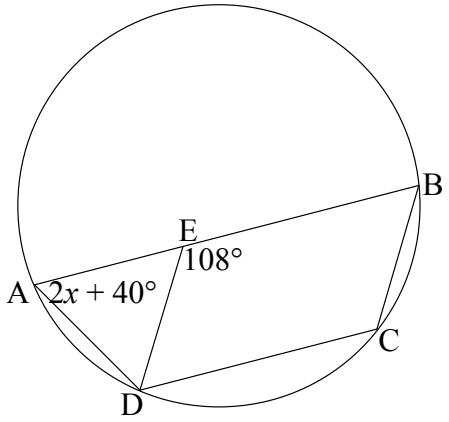
QUESTION/*VRAAG* 8

8.1



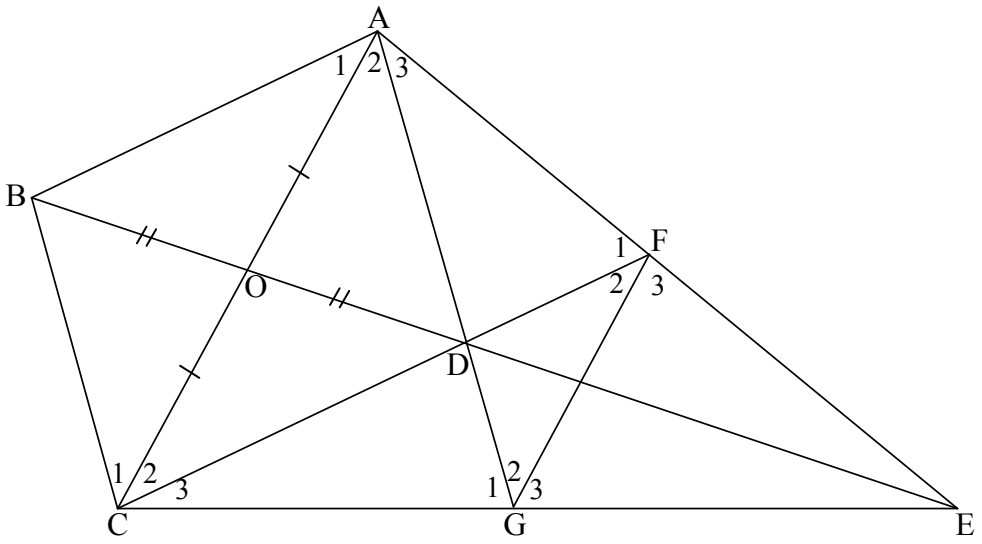
8.1.1	$\hat{K}_2 = \hat{M}_2 = 40°$ [tan chord theorem/*raakl-kdst*]	✓S ✓R (2)
8.1.2	$\hat{N}_1 = \hat{K}_1$ [∠s in the same seg/*∠e in dies segm*] $\hat{K}_1 = 84° - 40° = 44°$ $\therefore\ \hat{N}_1 = 44°$	✓S ✓R ✓S (3)
8.1.3	$\hat{T} = \hat{N}_1 = 44°$ [alt/*verw* ∠s/*e*; KT | | NM]	✓S ✓R (2)
8.1.4	$\hat{L}_2 = \hat{K}_2 + \hat{T}$ [ext ∠ of Δ/*buite ∠ v Δ*] $= 40° + 44°$ $= 84°$	✓R ✓S (2)
8.1.5	In ΔKLM: $44° + 84° + 40° + \hat{L}_1 = 180°$ [∠s sum in Δ/*∠e som in* Δ] $\therefore \hat{L}_1 = 12°$	✓S (1)

8.2



8.2	$\hat{C} = 108°$	[opp∠s of ‖m/*tos ∠e v ‖m*]	✓S ✓R
	$2x + 40° + 108° = 180°$	[opp∠s of cyc quad/*tos∠e v kdvh*]	✓S ✓R
	$2x = 32°$		
	$x = 16°$		✓answ/*antw*
			(5)
		OR/*OF*	
	$\hat{C} = 180° - (2x + 40°)$	[opp∠s of cyc quad/*tos∠e v kdvh*]	✓S ✓R
	$180° - (2x + 40°) = 108°$	[opp∠s of ‖m/*tos ∠e v ‖m*]	✓S ✓R
	$2x = 32°$		
	$x = 16°$		✓answ/*antw*
			(5)
			[15]

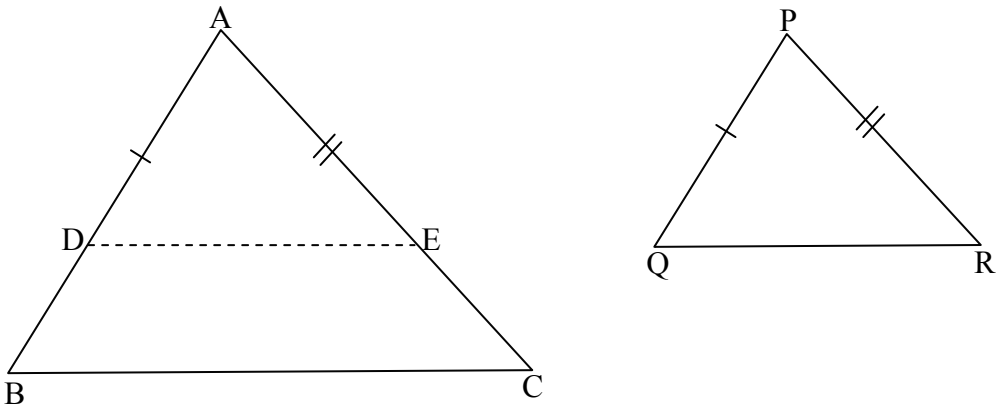
QUESTION/*VRAAG* 9



9.1	ABCD is a ||m [diags of quad bisect each other/ *hoekl v vh halveer mekaar*]	✓ R (1)
9.2	$\frac{ED}{DB}=\frac{FE}{AF}$ [Prop Th/*Eweredigh st*; DF || BA] $\frac{ED}{DB}=\frac{GE}{CG}$ [Prop Th/*Eweredigh st*; DG || BC]	✓S ✓ R ✓S ✓ R (4)
9.3	$\frac{FE}{AF}=\frac{GE}{CG}$ [proved/*bewys*] $\therefore$ AC || FG [line divides two sides of Δ in prop/ *lyn verdeel 2 sye van Δ eweredig*] $\hat{C}_2=\hat{F}_2$ [alt/*verw* ∠s/*e*; AC || FG] $\hat{A}_1=\hat{C}_2$ [alt/*verw* ∠s/*e*; AB || CD] $\therefore\ \hat{A}_1=\hat{F}_2$	✓S ✓ S ✓R ✓S ✓ S (5)
9.4	$\hat{A}_1=\hat{A}_2$ [diags of rhombus/*hoekl v ruit*] $\hat{A}_2=\hat{F}_2$ [$\hat{A}_1=\hat{F}_2$] $\therefore$ ACGF = cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =] **OR/*OF*** $\hat{C}_2=\hat{A}_2$ [∠s opp equal sides of rhombus/ *∠e to gelyke sye v ruit*] $\hat{A}_2=\hat{G}_2$ [alt/*verw*-∠s/*e*; AC || FG] $\therefore\ \hat{C}_2=\hat{G}_2$ $\therefore$ ACGF is a cyc quad/*kdvh* [∠s in the same seg =/ *∠e in dies segm* =]	✓S ✓ S ✓R (3) ✓S ✓ S ✓R (3) **[13]**

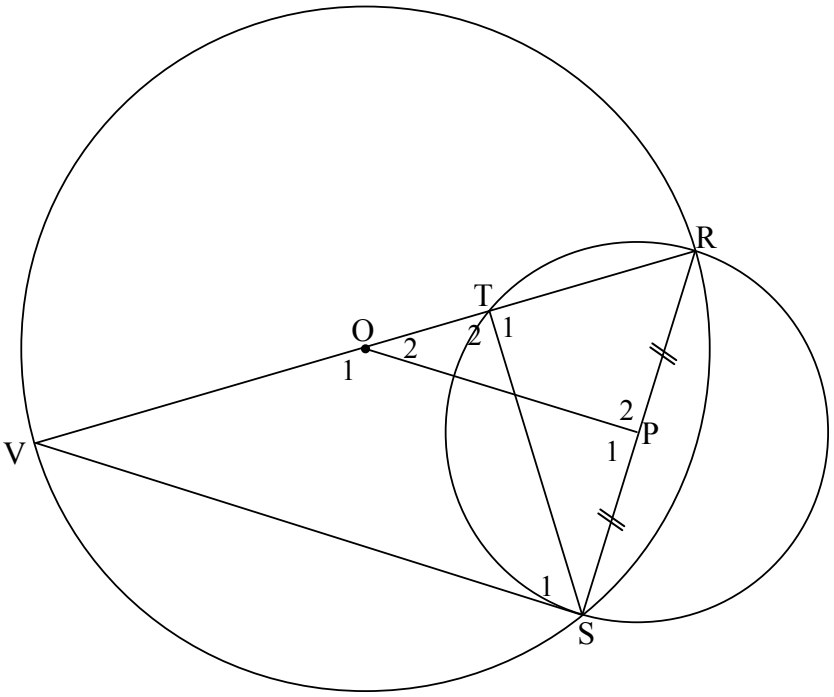
QUESTION/*VRAAG* 10

10.1



10.1.1	In ΔADE and/*en* ΔPQR: AD = PQ [construction/*konstr*] $\hat{A} = \hat{P}$ [given/*gegee*] AE = PR [construction/*konstr*] ∴ΔADE ≡ ΔPQR [S∠S]	✓all/*al* 3 S's/*e* ✓reason/*rede* (2)
10.1.2	$A\hat{D}E = \hat{Q}$ [Δs ≡ ∴ corres/*ooreenk* ∠s/*e* =] But $\hat{B} = \hat{Q}$ [given/*gegee*] ∴ $A\hat{D}E = \hat{B}$ ∴ DE || BC [corres/*ooreenk* ∠s/*e* =]	✓ $A\hat{D}E = \hat{Q}$ ✓ $A\hat{D}E = \hat{B}$ ✓reason/*rede* (3)
10.1.3	$\frac{AB}{AD} = \frac{AC}{AE}$ [Prop Th/*Eweredigh st*; DE || BC] But/*Maar* AD = PQ and/*en* AE = PR [construction/*konstr*] ∴ $\frac{AB}{PQ} = \frac{AC}{PR}$	✓S/R ✓S (2)

10.2



10.2.1	line from centre to midpt of chord/*lyn van midpt na midpt van koord*	✓answ/*antw* (1)
10.2.2	OP || VS [Midpt Theorem/*Midpt-stelling*] In ΔROP and/*en* ΔRVS: $\hat{R} = \hat{R}$ [common/*gemeen*] $\hat{O}_2 = \hat{V}$ [corresp/*ooreenk* ∠s/*e*; OP || VS] $\therefore$ΔROP ||| ΔRVS [∠,∠,∠] **OR/*OF*** In ΔROP and/*en* ΔRVS: $\hat{P}_2 = V\hat{S}R$ [corresponding ∠s/ *ooreenkomstige* ∠'e] $\hat{R} = \hat{R}$ [common/*gemeen*] $\therefore$ΔROP ||| ΔRVS [∠,∠,∠]	✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4) ✓S ✓R ✓S ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (4)

10.2.3	In ΔRVS and/*en* ΔRST: $V\hat{S}R = S\hat{T}R = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\hat{R}$ is common/*gemeen* $\hat{V} = T\hat{S}R$ $\therefore \Delta RVS \mid\mid\mid \Delta RST$ [∠,∠,∠]	✓S ✓ R ✓S & ∠;∠;∠ **OR/*OF*** 3 angles/*hoeke* (3)
10.2.4	In ΔRTS and/*en* ΔSTV: $R\hat{T}S = V\hat{T}S = 90°$ [∠ s on straight line/∠*e op rt lyn*] $\hat{R} = 90° - T\hat{S}R$ $= T\hat{S}V$ $T\hat{S}R = \hat{V}$ $\therefore \Delta RTS \mid\mid\mid \Delta STV$ [∠,∠,∠] $\therefore \frac{RT}{ST} = \frac{TS}{VT}$ $\therefore ST^2 = VT.TR$	✓ΔRTS & ΔSTV ✓S ✓S ✓S (with justification/*met motivering*) ✓ΔRTS $\mid\mid\mid$ ΔSTV ✓ratio/*verh* (6)
		[21]

TOTAL/*TOTAAL*: 150